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Effect of Rotary Inertia of Concentrated Masses on Natural Vibration of Simply Supported - Simply Supported Fluid Conveying Pipe

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Abstract

The effect of rotary inertia of concentrated masses on the natural vibration of the simply supported-simply supported fluid conveying pipe has been studied. For the analysis Galerkin's method is used for transformation of the governing equation to the eigenvalue problem and the natural frequencies and mode shapes for the system have been found. Introduction of rotary inertia results in lots of change on the natural frequencies and mode shapes and its effect is highly noticed at the higher natural frequencies and mode shapes. Consideration of rotary inertia results in much decrease on the natural frequencies and its neglect could lead to erroneous results.

1. Introduction

The flow-induced vibration in the industry field has being more and more studied since it always contain the possibility of the severe accidents by the several types of the flow-induced vibration. At this field, knowledge on the natural frequency and mode shape of a system is most important and is the basis of the aseismatic design, vibration analysis, and noise analysis. G. W. Housner [1] was the first who had derived the correct governing equation of motion of a fluid conveying pipe. For several decades, many investigators have studied about this problem by assuming several boundary conditions. Although the vibration analysis of the pipe which having some concentrated masses without fluid flow has been studied [2-5], the vibration analysis of that case when fluid flows through the pipe was not until 1970 when J. L. Hill et al. published their paper on ASME [6]. Since then, the effect of concentrated masses has been studied in the flow-induced vibration field [1,8] without regard to rotary inertia effect. Although some interesting papers [4,5] regarding rotary inertia effect are published, no one is introduced it into the flow-induced vibration field. In this paper, effect of rotary inertia of the concentrated masses is newly introduced in the fluid conveying pipe system and is accordingly included in the governing equation. The fluid conveying pipe which having simply-supported simply-supported boundary condition is taken for the analysis.

2. Theory and Mathematical Development

The well known governing equation for the pipe conveying incompressible fluid is [1]

$$EI\frac{\partial^4 y}{\partial x^4}+2m_fU\frac{\partial^2 y}{\partial t\partial x}+m_fU^2\frac{\partial^2 y}{\partial x^2}+(m_f+m_t)\frac{\partial^2 y}{\partial t^2}=0. \tag{1}$$

According to H. H. Pan et al. [4,5], the effect of concentrated masses placed at $x=x_i$ can be written as follows :

$$\sum_{i=1}^{M} m_i\delta(x-x_i)\frac{\partial^2 y}{\partial t^2}-\frac{\partial}{\partial x}\left\{\sum_{i=1}^{M} J_i\delta(x-x_i)\frac{\partial^3 y}{\partial x\partial t^2}\right\},\quad J_i=m_ix_i^2 \tag{2}$$

Finally, the governing equation for the simply supported - simply supported fluid conveying pipe as shown in Fig. 1 becomes

$$EI\frac{\partial^4 y}{\partial x^4}+2m_fU\frac{\partial^2 y}{\partial t\partial x}+m_fU^2\frac{\partial^2 y}{\partial x^2}+\left\{m_f+m_t+\sum_{i=1}^{M} m_i\delta(x-x_i)\right\}\frac{\partial^2 y}{\partial t^2}$$

$$-\frac{\partial}{\partial x}\left\{\sum_{i=1}^{M} J_i\delta(x-x_i)\frac{\partial^3 y}{\partial x\partial t^2}\right\}=0, \tag{3}$$

where,

$EI\frac{\partial^4 y}{\partial x^4}$; elastic force, the usual distributed load with bending

$2m_fU\frac{\partial^2 y}{\partial t\partial x}$; the inertia force associated with the Coliolis acceleration due to the relative motion of the fluid inside the pipe which has an angular velocity $\frac{\partial^2 y}{\partial t\partial x}$ at any point along its length

$m_fU^2\frac{\partial^2 y}{\partial x^2}$; the inertia force associated with the fluid flowing a curved path

$\left\{m_f+m_t+\sum_{i=1}^{M} m_i\delta(x-x_i)\right\}\frac{\partial^2 y}{\partial t^2}$; the inertia force due to lateral acceleration of the pipe including the moving medium and the concentrated masses

$\frac{\partial}{\partial x}\left\{\sum_{i=1}^{M} J_i\delta(x-x_i)\frac{\partial^3 y}{\partial x\partial t^2}\right\}$; rotary inertia force due to the concentrated masses.

Since the deflection and the bending moment of the simply supported - simply supported pipe are zero at x=0 and L, the boundary conditions for the system are as follows

$$y(0,t)=y(L,t)=0,\quad \frac{\partial^2 y}{\partial t^2}(0,t)=\frac{\partial^2 y}{\partial t^2}(L,t)=0. \tag{4}$$

Here y is the pipe displacement, x the axial coordinate, t the time, m_f and m_t the mass per unit length of the fluid and pipe, and L the tube length, respectively ; U the constant flow velocity, m_i and J_i the concentrated mass and its rotary inertia placed at $x=x_i$, respectively ; M number of concentrated masses, δ Dirac delta function, and EI the flexural rigidity of the uniform pipe.

To derive the governing equation some assumptions are made as follows: Euler-Bernoulli pipe, the small lateral motion about the equilibrium position, neglect effect of the gravity, uniform pipe except the concentrated masses, neglect rotary inertia and shear forces of the pipe itself, and steady state uniform flow.

Introducing dimensionless parameters, eq. (3) is converted into

$$\frac{\partial^4 \eta}{\partial \xi^4}+2u\beta^{\frac{1}{2}}\frac{\partial^2 \eta}{\partial \xi \partial \tau}+u^2\frac{\partial^2 \eta}{\partial \xi^2}+\left\{1+\sum_{i=1}^{M}\alpha_i\delta(\xi-\xi_i)\right\}\frac{\partial^2 \eta}{\partial \tau^2}$$
$$-\frac{\partial}{\partial \xi}\left\{\sum_{i=1}^{M}\mu_i\delta(\xi-\xi_i)\frac{\partial^3 \eta}{\partial \xi \partial \tau^2}\right\}=0, \tag{5}$$

where the dimensionless parameters are

$$\xi=\frac{x}{L} \qquad \xi_i=\frac{x_i}{L} \qquad \eta=\frac{y}{L} \qquad \tau=\left[\frac{EI}{m_f+m_t}\right]^{\frac{1}{2}}\frac{t}{L^2}$$

$$\omega=\left[\frac{m_f+m_t}{EI}\right]^{\frac{1}{2}}L^2\Omega \qquad u=\left[\frac{m_f}{EI}\right]^{\frac{1}{2}}UL \qquad \beta=\frac{m_f}{m_f+m_t}$$

$$\alpha_i=\frac{m_i}{L(m_f+m_t)} \qquad \mu_i=\frac{J_i}{(m_f+m_t)L^3}=\alpha_i\xi_i^2.$$

The motion $\eta(\xi,\tau)$ can be written as follows

$$\eta(\xi,\tau)=a_m(\tau)\Phi_m(\xi). \tag{6}$$

By inserting eq. (6) into eq. (4), the governing equation becomes

$$\left[\left\{1+\sum_{i=1}^{M}\alpha_i\delta(\xi-\xi_i)\right\}\Phi_m(\xi)-\sum_{i=1}^{M}\mu_i\delta'(\xi-\xi_i)\Phi_m'(\xi)-\sum_{i=1}^{M}\mu_i\delta(\xi-\xi_i)\Phi_m''(\xi)\right]\ddot{a}_m(\tau)$$
$$+2u\beta^{\frac{1}{2}}\Phi_m'(\xi)\ \dot{a}_m(\tau)+\{\Phi_m''''(\xi)+u^2\Phi_m''(\xi)\}a_m(\tau)=0. \tag{7}$$

Galerkin's method [6,9] says that $\Phi_m(\xi)$ can be represented by the superposition of $\phi_m(\xi)$ which is the mode shape function of the pipe without fluid and the concentrated masses. Then eq. (6) becomes

$$\eta(\xi,\tau)=\sum_{m=1}^{\infty}a_m(\tau)\phi_m(\xi) \tag{8}$$

$$\phi_m(\xi)=\sin(\lambda_m\xi) \text{ and } \lambda_m=m\pi, \quad m=1,2,3,\ldots$$

Introducing the orthogonality of the functions, multiplying eq. (7) by $\Phi_n(\xi)$, and integrating it about ξ from $\xi=0$ to $\xi=1$, we finally obtain the governing equation in the matrix form

$$[A_{mn}]\ \ddot{a}_m(\tau)+[B_{mn}]\ \dot{a}_m(\tau)+[C_{mn}]a_m(\tau)=0, \tag{9}$$

$$[A_{mn}] = \delta_{mn} + 2\sum_{i=1}^{M}\{\alpha_i\phi_m(\xi_i)\phi_n(\xi_i) + \mu_i\phi_m'(\xi_i)\phi_n'(\xi_i)\},$$

$$[B_{mn}] = 4u\beta^{\frac{1}{2}}\frac{mn[1-(-1)^{m+n}]}{n^2-m^2}, \text{ and } [C_{mn}] = (\lambda_m^4 - u^2\lambda_m^2)\delta_{mn}.$$

Rearranging eq. (9) we get

$$[M]\{\dot{p}(\tau)\} + [K]\{p(\tau)\} = 0. \tag{10}$$

where

$$\{p(\tau)\} = \begin{Bmatrix} \{\dot{a}_m(\tau)\} \\ \{a_m(\tau)\} \end{Bmatrix}, [M] = \begin{bmatrix} [0] & [A] \\ [A] & [B] \end{bmatrix}, \text{ and } [K] = \begin{bmatrix} -[A] & [0] \\ [0] & [C] \end{bmatrix}.$$

Multiplying eq (9) by $[M]^{-1}$, we get

$$[I]\{\dot{p}(\tau)\} + [M]^{-1}[K]\{p(\tau)\} = \{0\}. \tag{11}$$

Since $a_m(\tau) = e^{j\omega\tau}\Psi$ where $j = \sqrt{-1}$ and Ψ are constants,

$$\{p(\tau)\} = \begin{Bmatrix} \{\dot{a}_m(\tau)\} \\ \{a_m(\tau)\} \end{Bmatrix} = e^{j\omega\tau}\{\Psi\} \tag{12}$$

Introducing eq. (12) into eq. (11)

$$j\omega[I]\{\Psi\} + [M]^{-1}[K]\{\Psi\} = \{0\}, \tag{13}$$

$$[D]\{\Psi\} - \nu[I]\{\Psi\} = \{0\}\ ; \tag{14}$$

where $\nu = -j\omega$, $[D] = [M]^{-1}[K] = \begin{bmatrix} [A]^{-1}[B] & [A]^{-1}[C] \\ -[I] & [0] \end{bmatrix}.$

Eq. (14) can , therefore , be written as

$$[f(\nu)]\{\Psi\} = \{0\}, \tag{15}$$

where $[f(\nu)] = [D] - \nu[I]$.

The eigenvalue problem (15) has a nontrivial solution only if the characteristic determinant, i.e. the determinant of the matrix $[f(\nu)]$, vanishes [9] :

$$|f(\nu)| = 0. \tag{16}$$

Generally, eigenvalue of the determinant has its real and imaginary part as follows :

$$\omega = \mathrm{Re}(\omega) + j\mathrm{Im}(\omega). \tag{17}$$

The real component, $\mathrm{Re}(\omega)$, corresponds to the frequency of oscillation, whereas the imaginary component, $\mathrm{Im}(\omega)$, is associated with stability of the system. In this paper, analysis only on $\mathrm{Re}(\omega)$ is done to find natural frequency related vibration and stability related issue will be treated in another paper.

3. Results

The natural frequencies and mode shapes are calculated by the computer code developed and the effect of rotary inertia can be identified by comparing the results with or without rotary inertia. Some results or the study are as follows:

Figure 2 is the natural mode shapes of the system. Real lines are results of the system without rotary inertia and dotted lines are with rotary inertia. As shown in the figure, rotary inertia gives small effect on the first mode shape while it gives much on the second and third mode shapes. Special interest is on the third mode shape. By introducing rotary inertia, one node is disappeared and only one node is shown whereas there are two nodes when the system has no rotary inertia effect.

Figure 3 shows variation of the natural frequencies when the fluid velocity is changing. Three natural frequencies are shown and the dotted lines are results of the system with rotary inertia while real lines are results without rotary inertia. Introduction of rotary inertia gives much change for the second and third natural frequencies while it gives small effect on the first natural frequency. By increasing the fluid velocity the natural frequencies are decreased and go to zero. When the dimensionless fluid velocity gets π, 2π, and 3π the first, second, and third natural frequencies have zero value, respectively. Zero natural frequency is related with the system stability problem and is not considered in this paper.

The system frequencies with or without rotary inertia are shown at Fig. 4 according to the mass ratio change. For the case, rotary inertia results in much change for the higher natural frequencies. As shown in the figure, change of the mass ratio gives very small effect on the natural frequencies.

4. Conclusion

Through the computer simulation and experiment, it has been identified that rotary inertia changes not only the natural frequencies but also the natural mode shapes and the following conclusions can be achieved.

1) Rotary inertia gives very much change on the higher natural frequencies and mode shapes.

2) The number and location of nodes can be changed by rotary inertia effect.

3) The fluid velocities for the zero natural frequency are unchanged by introduction of rotary inertia and have the values of π, 2π, and 3π for the first, second, and third natural frequencies, respectively.

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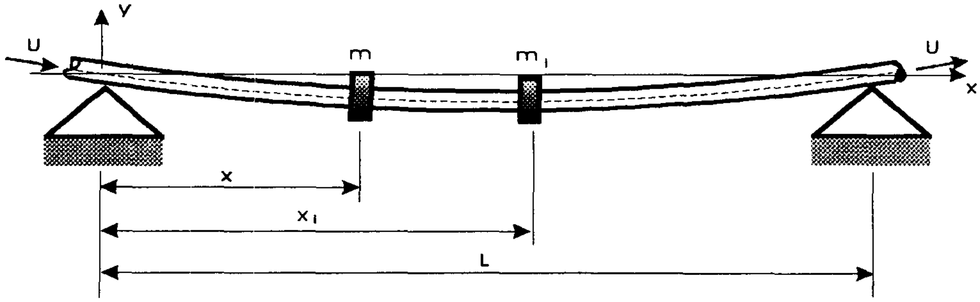
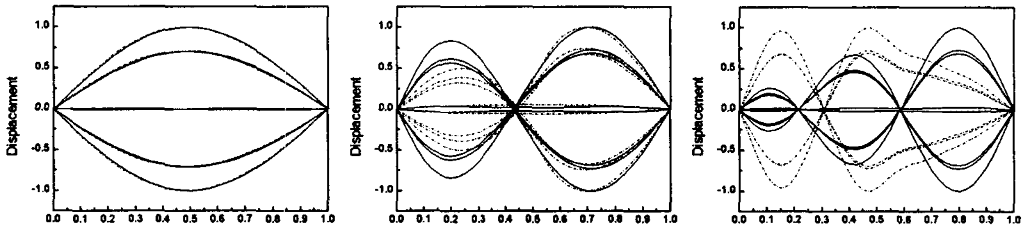


Fig. 1 Shematic diagram of the simply supported - simply supported fluid conveying pipe.



(a) 1st Mode Shape (b) 2nd Mode Shape (c) 3rd Mode Shape

Fig. 2 Natural mode shapes for the simply supported - simply supported fluid conveying pipe: ξ_1=0.1, ξ_2=0.5, α_1=1.0, α_2=0.2, μ_1=0.01, μ_2=0.05, β=0.4, and u=0.5 : real lines are results without rotary inertia and dotted lines are with rotary inertia.

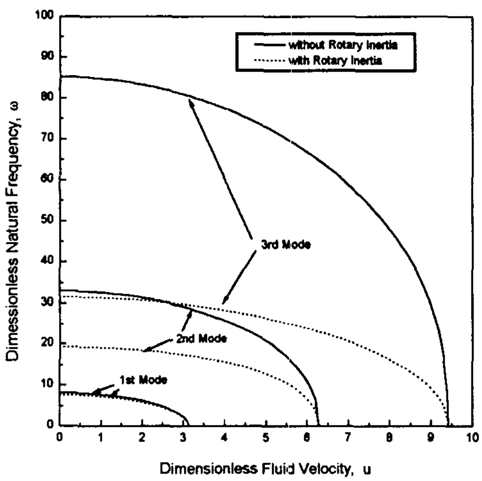


Fig. 3 Natural frequency variation due to fluid velocity change: ξ_1=0.3, ξ_2=0.6, α_1=0.2, α_2=0.1, μ_1=0.018, μ_2=0.036, and β=0.2.

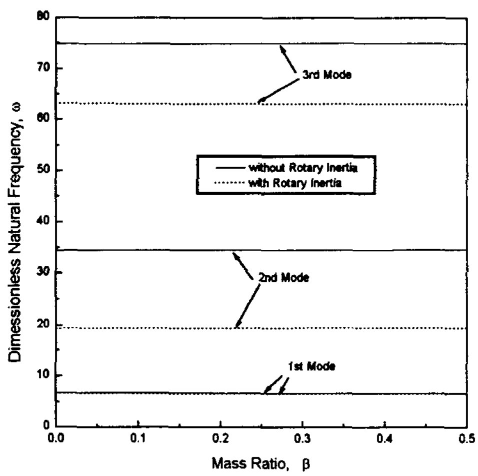


Fig. 4 Natural frequency variation due to mass ratio change: ξ_1=0.2, ξ_2=0.5, α_1=0.1, α_2=0.1, μ_1=0.004, μ_2=0.025, and u=2.0.