Variable Structure Control for Mechatronics Application
메카트로닉스에의 적용을 위한 가변구조제어

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Abstract

In this paper, a new variable structure controller (VSC) is presented. The presented VSC can be applicable to most mechatronic systems such as robotics. A VSC (or also called sliding mode control; SMC) algorithm is presented first, and next, a VSC with nonlinear integral control algorithms is presented. The algorithms use no linear approximation for the derivation of the control law or in the stability proof. It is shown that the robustness of the developed algorithms are guaranteed by the sliding mode control and that the algorithms are globally convergent.

Keywords: Variable structure control(VSC), Sliding mode control(SMC), Mechatronics

Ⅰ. Introduction

Recently, methodology, which uses in its idealized form piecewise continuous feedback control laws, resulting in the state trajectory 'sliding' along a discontinuity or sliding surface in the state space has been researched. This methodology is known as variable structure control(VSC) (or, also known as sliding mode control; SMC). The concept of VSC has been studied in detail in [2,3,4], where it has been used to stabilize a class of non-linear systems.

Earlier sliding mode control exhibited problems, particularly: (i) there is a 'reaching' phase in which the trajectories starting from a given initial condition off the sliding surface tend towards the sliding surface, but the trajectories in this phase are sensitive to parameter variations; (ii) small imperfections in switching between control laws at the discontinuity surface result in the trajectory chattering rather than sliding along the switching surface. Young and Kwatny [5] suggested alleviating the first difficulties by the use of high-gain feedback to speed-up the reaching phase. However, this has the usual drawbacks associated with high-gain feedback, ie. extreme sensitivity to unmodeled dynamics, actuator saturation, etc. The above drawbacks have been removed in Slotine and Sastry [6] by using the concept of a time-varying sliding surface in the state space.

Sliding mode control is able to be effectively used in mechatronics (such as the tracking control of robot arms) and has been studied by many researchers in recent years(for example [1~7]). However, obstacles still exist to practical implementation such as: parameter variations are limited to a fairly small range [7]; the complexity of the strategies requires large calculation times for multi-input-multi-output systems, etc..

In this paper, we present a SMC algorithms for mechatronics application. To enhance the performance, a nonlinear integral control law is added to the algorithm. The algorithm uses no linear approximation for the derivation of the control law or in the stability proof. It is shown that the robustness of the developed algorithms are guaranteed by the sliding mode control law. Only position error and velocity error measurements are required and the computational load required is roughly the same as that of a PID controller even for multi-input-multi-output applications.

The organization of this paper is as follows: section 2 gives some mathematical formulations which will be useful to develop the algorithm; section 3 presents a sliding mode control algorithm; in section 4, a nonlinear integral control law is added to the algorithm developed in section 3 to generate better tracking performance; section 5 concludes the paper.

II. Problem Formulation

Consider a second-order mechatronic system

$$M(x)\ddot{x}+F(\boldsymbol{x})=u \qquad (1)$$

where $\boldsymbol{x}=[x\ \ \dot{x}]^{T}$ is the state vector, M is positive definite and a bounded differentiable inertia matrix function of x, and $F(\boldsymbol{x})$ is the nonlinear function which is not exactly known but the extent of the imprecision on $F(\boldsymbol{x})$ is upper bounded by a known continuous function of $\boldsymbol{x}$.

The control problem is to synthesize a control law for u such that the state $\boldsymbol{x}$ traces the desired trajectory, $\boldsymbol{x}_d=[x_d\ \ \dot{x}_d]^{T}$, with a certain precision defined by

$$\|x_d-x\|\le\gamma_1,\ \ \|\dot{x}_d-\dot{x}\|\le\gamma_2,\ \ \gamma_1>0,\ \ \gamma_2>0$$

It is assumed that $x_d(t)$, $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are well defined and bounded for all operational time t.

Define

$$e=x_d-x, \qquad z=\dot{e}+\Lambda e \qquad (2)$$

with $\Lambda=diag(\Lambda_1,\Lambda_2,\cdots,\Lambda_n)$, $\Lambda_i>0$.

Then, from (1) and (2),

$$M\dot{z}=M(\Lambda\dot{e}+\ddot{x}_d)+F-u \qquad (3)$$

Eq.(3) cannot be given exactly due to the disturbances, modelling uncertainties and unknown parameters, but upper bounds for the norms of (3) can be estimated as follows.

Lemma 1

(a) There exists bounded differentiable functions $\theta(t)\in \boldsymbol{R}^{n}$ *and bounded*

nonlinear functions $l_1(t), l_2(t) \in \boldsymbol{R}^n$ *such that*

$$z^{\mathrm{T}}[M(\Lambda \dot{e} + \dot{x}_d) + F] + \frac{1}{2} z^{\mathrm{T}} \dot{M} z = \theta \|z\| + (l_1 + l_2 \|z\|) \|z\|^2, \quad \forall x, \dot{x} \tag{4}$$

(b) There exists constant $\eta > 0$ *such that*

$$z^{\mathrm{T}}[M(\Lambda \dot{e} + \dot{x}_d) + F] + \frac{1}{2} z^{\mathrm{T}} \dot{M} z \le \phi \eta \|z\|, \quad \forall x, \dot{x} \tag{5}$$

with

$$\phi = 1 + \|z\| + \|z\|^2 \tag{6}$$

Proof : Noting that x_d and $\dot{x}_d$ are bounded, $F(\boldsymbol{x})$ is a bounded function at most quadratic in $\dot{x}$, we have that x and $\dot{x}$ are bounded on $\|e\|$ and $\|\dot{e}\|$ respectively. From (2), $\dot{e} = z - \Lambda e$ and thus $\dot{e}(s) = [I - T(s)]z(s)$ with $T(s) = \Lambda(sI + \Lambda)^{-1}$. Denote the H_∞ norm of a stable transfer function by $\|\cdot\|_\infty$. Then, it follows that $\|\dot{e}\| \le \|z\| + \|T\|_\infty \|z\|$. Since $T(s)$ is stable, $\|T\|_\infty$ is bounded and thus $\|\dot{e}\|$ is bounded on $\|z\|$. Now, it is clear that there exist bounded nonlinear functions $\theta(t), r_1(t), r_2(t) \in \boldsymbol{R}^n$ such that

$$M(\Lambda \dot{e} + \dot{x}_d) + F = \theta + r_1 \|z\| + r_2 \|z\|^2 \tag{7}$$

Moreover, without loss of generality, θ can be chosen as differentiable functions. Since M is bounded differentiable function of x, $\dot{M}$ is bounded on $\dot{x}$. Thus, there exist bounded nonlinear functions $l_1(t), l_2(t) \in R^n$ such that (a) holds. Let η be the largest value of $\sup_t \|\theta(t)\|$, $\sup_t \|l_1(t)\|$ and $\sup_t \|l_2(t)\|$. Then, clearly (b) holds. △△△

Note that $z(s) = (sI + \Lambda)e$, and $(sI + \Lambda)^{-1}$ is a strictly proper and stable transfer function. Thus $z(t) = 0, \forall t > t_0$, $e(t) \to 0$ and $\dot{e}(t) \to 0$. Therefore, the control problem becomes how to choose the control law, such that $\|z\| \le \varepsilon$, where $\varepsilon > 0$ is a predefined tracking error precision (see Slotine and Li, 1991).

III. Configurations of Proposed Sliding Mode Control

In this section, we propose a variable structured control law to compute the control input torque for the uncertain system (1) as follows

$$\begin{aligned} \tau &= u_1 + u_2 \\ u_1 &= k_1 z, \quad k_1 > 0 \\ u_2 &= \begin{cases} \phi k_2 \dfrac{z}{\|z\|}, & \text{if} \|z\| > \varepsilon \\ & \quad ; k_2 > \eta, \ 0 < \varepsilon < 1 \\ \phi k_2 \dfrac{z}{\varepsilon}, & \text{else} \end{cases} \end{aligned} \tag{8}$$

with ϕ defined by (6). In (8), we see that u_1 is the feedback error torque vector, and u_2 is the sliding-mode torque vector with z as the sliding surface.

Then, we have the following result.

Theorem 1 *Consider the system (1) with the control law (8). Let* η *be given by Lemma 1. If* $k_2 > \eta$ *is chosen, the closed-loop system is globally stable in the sense that the tracking error* z *is globally bounded by*

$$\|z\| \le \frac{\phi \varepsilon \eta}{\varepsilon k_1 + \phi k_2} < \varepsilon \tag{9}$$

Proof: Choose a Lyapunov function

$$v = \frac{1}{2} z^{\top} M z \tag{10}$$

Differentiating (10) with respect to t and substituting (3) gives

$$\dot{v} = \frac{1}{2} z^{\top} \dot{M} z + z^{\top} [M(\Lambda \dot{e} + \ddot{x}_d) + F - u] \tag{11}$$

If k_2 is chosen to satisfy $k_2 > \eta$, from Lemma 1(b) and (8), $\dot{v}$ can be expressed as

$$\dot{v} \le -(k_1 + \phi k_2 / \varepsilon) \|z\|^2 + \phi \eta \|z\| \tag{12}$$

In (12), we see that z is bounded by $\|z\| \le \varepsilon \phi \eta / (\varepsilon k_1 + \phi k_2)$, and that, when $k_2 > \eta$,

$\varepsilon\phi\eta < \varepsilon k_1 + \phi k_2$. Therefore, we can conclude that $\|z\|$ is globally bounded by (9).

△△△

Ⅳ. SMC with Nonlinear Integration

Let us consider further (11). From Lemma 1(a) we have

$$z^T[M(\Lambda\dot{e} + \ddot{x}_d) + F] + \frac{1}{2}z^T\dot{M}z \le \|\theta\|\|z\| + (l_1 + l_2\|z\|)\|z\|^2 \tag{13}$$

Applying (8) and (13) into (11), (12) can also be expressed as

$$\dot{v} \le -(k_1 + \phi k_2/\varepsilon)\|z\|^2 + \|\theta\|\|z\| + (l_1 + l_2\|z\|)\|z\|^2 \tag{14}$$

Hence, $\|z\|$ is globally bounded by

$$\|z\| \le \|\theta\|(k_1 + \frac{\phi k_2}{\varepsilon} - l_1 - l_2\|z\|)^{-1} \tag{15}$$

From (15), we see that for any $0<\varepsilon<1$ and finite $k_1>0$, $\|z\|\to 0$ if and only if $\|\theta| = 0$, and that if the unknown θ is compensated, high performance control can be achieved. In this section, the techniques for compensating for unknown θ are developed by adding a nonlinear integral control law to the algorithm of theorem 1, and as a result, high performance control is achieved.

The control law is designed as

$$\begin{aligned} \tau &= u_1 + u_2 + u_3 \\ u_1 &= k_1 z, \quad k_1>0 \\ u_2 &= \begin{cases} \phi k_2 \dfrac{z}{\|z\|}, & \text{if} \|z\| > \varepsilon \\ & ; k_2 > \eta, \ 0<\varepsilon<1 \\ \phi k_2 \dfrac{z}{\varepsilon}, & \text{else} \end{cases} \\ \dot{u}_3 &= \rho z, \ u_3(0) = 0, \ \rho>0 \end{aligned} \tag{16}$$

with ϕ defined by (6).

For further development of the algorithm, we want to evaluate the boundary for θ as follows.

Lemma 2 Let

$$\theta - u_3 := \tilde{\theta} \tag{17}$$

Then, there exist bounded and positive nonlinear functions ϑ, f_3 and $f_4 \in \boldsymbol{R}$ such that

$$\| \dot{\theta}^{\top} \tilde{\theta} \| \leq \vartheta \|z\| + f_3 \|z\|^2 + f_4 \|z\|^3 \tag{18}$$

Denote

$$\vartheta_m := \sup_t \vartheta(t) \tag{19}$$

Then, we have the following result.

Theorem 2 Consider system (1) with control law (16). Then, the closed-loop system is globally stable in the sense that integral signals are bounded, and the tracking error z is globally bounded by

$$\|z\| \leq \frac{\vartheta_m}{d_2 \rho} \tag{20}$$

where d_2 is given by

$$d_2 = k_1 + k_2/\varepsilon - (l_1 + f_3/\rho) - (l_2 + f_4/\rho) \tag{21}$$

with ϑ_m is defined by (19), and $f_{3,}$ f_3 are defined by (18).

Proof: Choose the Lyapunov candidate function

$$v = \frac{1}{2}(z^{\top} Mz + \frac{1}{\rho} \tilde{\theta}^{\top} \tilde{\theta}) \tag{22}$$

with $\tilde{\theta}$ is defined by (17). The time derivative of (22) along the trajectories of (1) shows that z is globally bounded by (20).

The boundness of u_1 and u_2 follows from the boundness of $\|z\|$, and u_3 is bounded since θ and $\theta - u_3$ are bounded.

△△△

It can be seen from (20) that a larger ρ gives a smaller tracking error. However, a large ρ will also result in undesirable transient response when $\|z(0)\|$ is large. Note that the transient response is ensured by u_1 and u_2, and that u_3 is used to improve the static performance. A reasonable way to choose ρ is to set it small when $\|z\|$ is large and to set it large when $\|z\|$ is small. This motivates the use of a time-varying gain for the integral term.

The control law is the same as that for (16), except the update law for u_3 is chosen as

$$\dot{u}_3 = \begin{cases} \dfrac{\rho z}{\|z\|}, & \text{if } \|z\| > \varepsilon \\ \dfrac{\rho z}{\varepsilon}, & else \end{cases} \quad ; u_3(0) = 0, \quad \rho > 0, \; ; 1 \gg \varepsilon > 0 \tag{23}$$

With the modification (23), the gain of the integration is time-varying and nonlinear. When $\|z\| \le \varepsilon$, u_3 works as normal integral control with a large gain of ρ/ε. When $\|z\| > \varepsilon$, u_3 is increasing or decreasing with a fixed ratio ρ according to the direction of $z(t)$. This results in a slow variation of u_3 when the tracking errors are large and a fast variation of u_3 when the errors are small. Note that a large error will excite a large control effort supported by u_1 and u_2. This control scheme can supply a fast and robust correction to the control law.

V. Conclusions

In this paper, a variable structure control algorithms for mechatronics application have been presented. To enhance the performance, a nonlinear integral control law was added to the algorithm. The algorithm uses no linear approximation for the derivation of the control law or in the stability proof. It is shown that the robustness of the developed algorithms are guaranteed by the sliding mode control law and that the algorithms are globally convergent. Only position error and velocity error measurements are required and the computational load required is roughly the same as that of a PID controller even for multi-input-multi-output applications. Therefore, the presented algorithm is model free, and can be applicable to most mechatronic systems such as robotics.

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