A STUDY ON CHARACTERISTICS OF DEFUZZIFICATION METHODS IN FUZZY CONTROL

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Abstract -- Defuzzification plays a great role in fuzzy control system. Defuzzification is a process which maps from a space defined over an output universe of discourse into a space of nonfuzzy(crisp) number. But, it's impossible to convert a fuzzy set into a numeric value without losing some information during defuzzification. Also it's very hard to find a number that best represents a fuzzy set. Many methods have been used for defuzzification but most of them were problem dependent. There has been no rule which guides how to select a method that is suitable to solve given problem. Here, we have investigated most widely used methods and we have analyzed their characteristics and evaluated them. D. Driankov and Mizumoto have suggested 5 criteria which the 'ideal' defuzzification method should satisfy. But, they didn't considered about control action. Output fuzzy set is not only a fuzzy set but also a sequence of control action. We suggested 4 new criteria which describe sequence of control action from some experiments. In addition, we have compared each method in simple adaptive fuzzy control. COG(Center of Gravity), or COS(Center of Sums) methods were successful in fuzzy control. However, at transition region, MOM(Mean of Maxima) was best among others in adaptive fuzzy control.
Keyword -- defuzzification, characteristics of defuzzification methods

1. Introduction

In 1965, Prof. Lofti A. Zadeh have introduced fuzzy set theory[22]. Classical logic or set theory was successful in "On-Off" logic. But, there are many things which can not be described by true or false. Many applications based on his theory were boomed during late 1980s. But, researchers countered serious problem. Fuzzy control was powerful and successful in many applications but it has a critical drawback what is called defuzzification. Defuzzification is a process to find a number which best represents a fuzzy set. The chosen number should really "represent" a fuzzy set. If anyone who have a experience of voting for a candidate who would be the president, may feel that it is a hard work to find a best representative among counterparts. Many methods like COG(Center-of-Gravity) method or MOM(Mean-of-Maxima) method have been used for defuzzification process.

When the output fuzzy set is normalized and convex, all of the following methods are good enough. But, in case of non-convex, or not normalized fuzzy set, the situation is quite different. So, comparing each defuzzification method is not an easy work. D. Driankov and Mizumoto have suggested 5 indices which the "ideal" defuzzification methods should satisfy[1,2,3,8,22]. But, those indices are based on only output fuzzy set. In defuzzification process, the most important thing which should be considered is "sequence of control action". Many researchers have been stick to how to solve problem of non-convex fuzzy set or worst cases. So, some of them suggested methods which can be tailored to a specific system using parameters. We have evaluated several methods which are widely used in fuzzy control when the input of the fuzzy controller is unit step function or square wave with 5Hz frequency.

Adaptive fuzzy control is a field extensively studied in these days. Some well known defuzzification methods like COG, MOM methods were used without comparison in adaptive fuzzy control. One of adaptive fuzzy control, model based adaptive fuzzy control has a stage which updates fuzzy control rules. We have thought that in case of adaptive fuzzy control each method would have influence on system performance.

We shall show various defuzzification methods, then we will evaluate each method in SISO(Single Input Single Output) system. We will investigate the general performance of defuzzification method with respect to COG method. At that time, the study was focused on 1st, 2nd, and 3rd order systems. Next, we will discuss the characteristics of defuzzification methods in model based adaptive fuzzy control. Finally, we compared each defuzzification method in simple fuzzy control system.

2. Defuzzification Methods

2.1. Basic Defuzzification Methods[1,2,3,8,9,22]

2.1.1. Center of Gravity Method

The Center of Gravity method is one of the most widely used techniques since it has some advantages. The defuzzified value z_o has tends to move smoothly around the output fuzzy region. It's relatively easy to calculate. But, this method is rather complex computationally and leads to unwanted results if the output fuzzy set is not unimodal. Anyway, this method is the most physically appealing.

2.1.2. Center of Sums Method

Using this method, it is possible to consider distribution of the area of inference results from each fuzzy rule individually. Inference results are overlapping and each area is counted more than once. This method is the same as COG method.

2.1.3. Mean of Maxima Method

The defuzzified value z_o is an average of the elements which reach the maximal grade in output fuzzy set C. This method and following Center of Maxima Method is generally applicable to some specific problems. The reason is that the expected value is very sensitive to a single rule that dominates the rule set. If the fuzzy region changes the expected value tend to jump from one membership function to next.

2.1.4. Center of Maxima Method

This method is a simplified version of the Mean of Maxima method. Instead of taking all elements which give the maximal grade, the smallest element z' and the largest element z'' among them are picked up and the midpoint of z' and z'' is given as the representative point. z_o

2.1.5. First (or Last) of Maxima Method

First of Maxima takes the smallest number of the domain with maximal membership grade in output fuzzy set. And Last of Maxima method takes the largest one. When the fuzzy sets are truncated at either of the domain, this method is useful sometimes.

2.1.6. Center of Area Method

In this method, the defuzzified value is defined as the value for which the area of output fuzzy set is divided equal sub areas. Many cases, center of area can be approximated by COG method.

2.1.7. Center of Largest Area Method

This method is used in the case when inference result is non-convex, i.e., it has at least two convex fuzzy subsets. Then the method defines the crisp value z_* to be the center of gravity of the convex fuzzy subset with the largest area.

2.1.8. Height Method

This method called moment method, fuzzy-mean method, or weighted average method, is a method which uses the individual clipped or scaled output z_i of C_i Thus neither the support or shape of C_i play a role in the computation of z_o. The method obtains z_o as the weighted average of the representative points z_i of C_i with the heights h_i of C_i.

2.1.9. Maximal Height Method

A representative point z_j of C_j which corresponds to the maximal height h_j among h_i $(i = 1, \Lambda, n)$ is adopted as z_o.

$$z_o = z_j \quad (h_j \text{ is the maximal height})$$

2.2. Recently Published Defuzzification Methods

Nowadays, several analytic defuzzification methods like Mabuchi's method[4]. Yager's methods[5, 23], Set-theoretical method[6]. Saade's Method[7], and other methods[14,15,16, 17] have been proposed. These methods are based on neural network[13] or probability theory[5] etc. Most of these methods are performed by systematic procedures[4].

2.3. Previous Evaluation Indices for Defuzzification Methods[22]

2.3.1. Continuity

A small change in the input of the FKBC(Fuzzy Knowledge Based Control) should not result in a large change in the output. for example. in the case of a two-input, one-output FKBC, when two inputs $(e_1^*, \dot{e}_1^*)$ and $(e_2^*, \dot{e}_2^*)$ differ slightly, then the corresponding output values u_1^* and u_2^* should differ slightly too, i.e.

$$\forall \varepsilon > 0\ \exists \delta > 0 : |e_1^* - e_2^*| < \delta \text{ and } |\dot{e}_1^* - \dot{e}_2^*| < \delta \text{ then } |u_1^* - u_2^*| < \varepsilon$$

2.3.2. Disambiguity

There are two equally large areas covered by the two fuzzy convex subsets constituting the overall control output for both max-min composition based inference and scaled inference. Thus, the defuzzification method cannot choose between these two areas. In other words it is ambiguous. This criterion is not satisfied by the Center-of-Largest-Area defuzzification method.

2.3.3. Plausibility

Every defuzzified control output has a horizontal component $u^* \in U$, and vertical control output has a horizontal component $\mu_{\tilde{U}}(u^*) \in [0,1]$. We define u^* to be plausible if it lies approximately in the middle of the support of $\tilde{U}$ and has a high degree of membership in $\tilde{U}$. The Center-of-Area method, does not satisfy these properties: although the Center-of-Area lies in the middle of the support set, its membership degree in some the lowest possible.

2.3.4. Computational Complexity

This criterion is particularly important in practical application of FKBC. The Height method, together with the Middle and First-of-Maxima are fast methods, whereas the Center-of Area method is slower. The computational complexity of Center-Sums depends on the shape of the output membership functions and whether max-min composition based inference or scaled inference is chosen. The Middle-of-Maxima method, for example, is faster with scaled inference. There is also the issue of the representation of the fuzzy sets. In case of the Center-of-Area or Center-of-Sums defuzzification is very slow.

2.3.5. Weighting counting

A defuzzification method is weight counting if it sums up the overlapping parts in the overall output fuzzy set. Center of Area method, Height method, and Area method are weight counting.

2.4. Proposed New Evaluation Indices for Defuzzification Methods

In above subsection, D. Driankov and Mizumoto have suggested 5 criteria which the 'ideal' defuzzification method should satisfy for several defuzzification method. They only took their attention to the output fuzzy set. So, they didn't considered about control action. Output fuzzy set is not only a fuzzy set but also a sequence of control action. We suggested 4 new criteria which describe sequence of control action from some experiments. These indices help us to classify the characteristics of defuzzification methods.

2.4.1. Confidence Level Loss

If the output fuzzy set is convex and asymmetric but the support is small, then each defuzzification method can produce significant differences in the system performance. For example, in the sharp and asymmetric case, the results of Center-of-Gravity method will be different depending on the shape of membership function. But Mean-of-Maxima method will produce same results for the shape.

Confidence level loss occurs if following condition is satisfied.

$$|\mu_{max} - \mu_o| > \varepsilon$$

Here, μ_{max} stands for maximal membership degree and ε is small positive number depending on the system.

2.4.2. Trajectory Following

If the input signal to FKBC varies very fast and in a broad range, some defuzzification method can follow the input but other can not do it in the same condition. Specially, if the input signal is periodic some defuzzification methods shows excellent performance but other's don't. Sometimes, each defuzzification method shows similar results and it may be hard to distinguish between two cases. In order to distinguish which method is adequate for given signal swing following condition should be satisfied.

$$\int_T (u(t) - y(t))^2 dt < \xi$$

where T is period of input signal and ξ is the range within which the value of definite integral should converge. u(t) and y(t) are input signal and output signal of the plant.

2.4.3. Discrete version availability

In the implementation of FKBC, One chip controller, PC, or other computerized equipments are generally used. To operate those equipments, discretized signals are generated and processed. So, whether discrete version of each defuzzification method is available or not is an important factor. Recently many papers are published and new defuzzification methods were suggested. But, some of them are too theoretical to implement in digital computer. So, discrete version availability is a critical factor.

2.4.4. Parameter Variation

Can we customize the method for the system? In some defuzzification methods have parameter which must be adjusted by user.

3. Characteristics of Defuzzification Methods in General SISO(Single Input Single Output) Control System

We are familiar with SISO Control system. In case of SISO system control action is usually divided into 2 groups. First is regulating of the output and second is tracking of the input signal. We used PD-like fuzzy controller which uses error and change of error as a input to the controller. We have experimented with 14 methods and evaluated each method. We have varied parameters alpha and beta to see performance differences in BADD(Basic Defuzzification Distribution) Transformation, SLIDE(Semi Linear Defuzzification) and M-SLIDE(Modified-SLIDE) methods. All of test results was not shown here because there are so many graphs. The following graphs was tested with these parameters.

For BADD, alpha = 0.01 or 2
For SLIDE alpha = 0.1, beta = 1
For M-SLIDE alpha = 0.1

As we previously described, BADD, SLIDE and M-SLIDE method is based on probability distribution and can be adjusted to satisfy given conditioned. So, above parameter values are not absolute.

We have simulated first, second, and third order systems. The forward transfer function of each order is as follows.

1st order system: $G(S) = \frac{1}{s+1}$

2nd order system: $G(s) = \frac{1}{s(s+1)}$

3rd order system: $G(S) = \frac{25.04(s+0.2)}{s(s+5.02)(s+0.01247)}$

3.1. Simulation

3.1.1. 1st order System

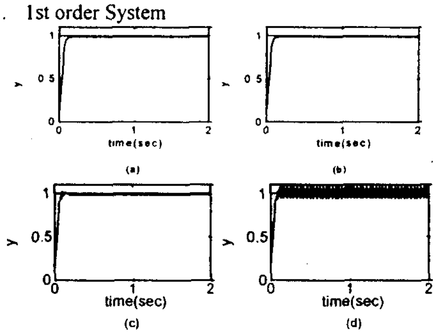


Fig. 1 Unit step input response of first order system: (a) COG (b) COS (c) FOM (d) LOM

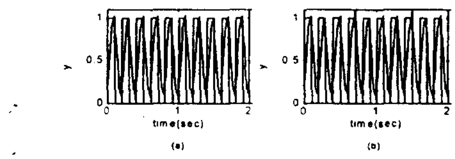


Fig. 2 Square wave (5Hz) traking response of first order system: (a) COG (b) COS

3.1.2. 2nd order System

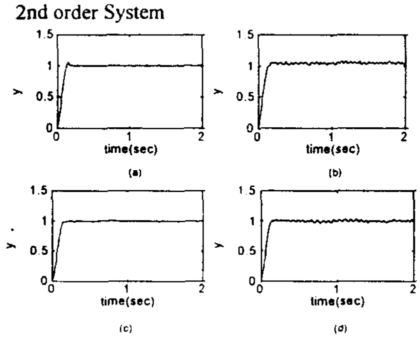


Fig. 3 Unit step input response of first order system: (a) Height (b) Maximal Height (c) SLIDE(alpha=0.1, beta=1) (d) M-SLIDE (alpha=0.1)

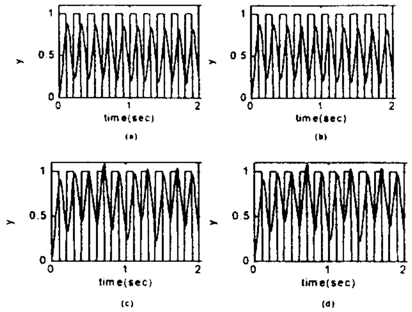


Fig. 4 Square wave (5Hz) tracking response of first order system: (a) COG (b) COS (c) MOM (d) COM

3.1.3. 3rd order System

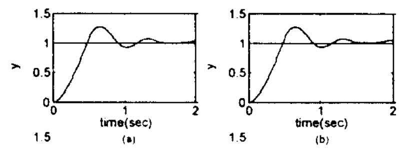


Fig. 5 Unit step input response of first order system: (a) FOM (b) LOM

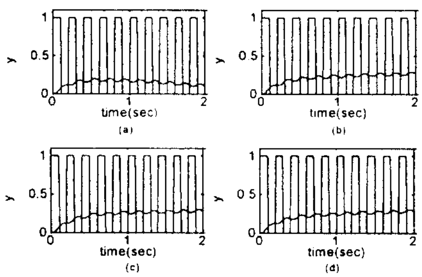


Fig. 6 Square wave (5Hz) tracking response of first order system: (a) COA (b) BADD (alpha=2) (c) BADD(alpha=0.01) (d) SLIDE (alpha =0.1, beta=1)

3.2. Discussion

1. There is no standard defuzzification method. so every defuzzification method were compared with respect to COG method.
2. COS(Center of Sums) method shows no difference with respect to COG method.
3. For unit step input. in case of 2nd order system, almost every method shows hunting in steady state because rules are not enough. MOM has superior to other methods in transient response.
4. For 2nd order system. COM. FOM. LOM shows no difference in steady state.
5. BADD. SLIDE. M-SLIDE can be tailored to given system for better performance.
6. In order to view input signal tracking, 5Hz square wave was used as a test signal.
7. COG, COS methods follows reference signal very well relative to other methods.
8. Methods using maxima shows much fluctuation. So, those methods are sensitive to high frequencies.
9. For the same parameter SLIDE method shows good response for unit step and square wave. But M-SLIDE shows much difference between two cases.
10. If the input signal varies very fast. We can not say that the output signal of a plant controlled by FKBC is reliable.
11. Though COG, or MOM was very successful in first and second order system. There was not so much difference in third order system. So, for higher order system, we don't have to concentrate on choosing a good method.

4. Characteristics of Defuzzification Methods in Adaptive FKBC(Fuzzy Knowledge based Control)

4.1. Model based Adaptive Fuzzy Control

FKBC contain a number of sets of parameters that can be altered to modify the controller performance. These are the scaling factors for each variable, the fuzzy set representing the meaning of linguistic values, and the if-then rules. A non-adaptive FKBC is one in which these parameters do not change once the controller is being used on-line. If any of these parameters are altered on-line. we will called the controller an adaptive FKBC. The main approaches to the design of adaptive FKBC consist as follows: membership function tuning using gradient descent[18], membership function tuning using performance criteria, the self-organizing controller[19,20,21]. and model based controller[11,12].

In the adaptive fuzzy controller. we deal with model based adaptive fuzzy controller. A self-organizing system that uses on-line identification of a fuzzy process model for adaptation has been developed by Gramham and Newell.

This controller consists of three parts: a fuzzy process model, a controller performance measure, and a decision maker. The fuzzy process model consists of a set of if-then rules that are the inverse of the rules found in a Mamdani FKBC. Instead of specifying the desired control-output for a given process state, they predict the process-output to be expected in time due to the current and previous process-states and control-outputs. The aim of the decision maker is to choose, from a predefined. finite set of control-outputs, which action will maximize the performance of the process if no future control-outputs are taken. A set of used control outputs for a single control-variable might be {LARGE DECREASE, SMALL DECRESE. NO CHANGE, SMALL INCRESE, LARGE INCRESE}.

Adaptation of the controller is achieved by the use of on-line fuzzy identification of the process model.

At time 0:

1. An initial relation matrix $\tilde{R}_0$ is defined.
2. The initial process state $L\tilde{x}_0$. process-output $L\tilde{y}_0$. and
3. control-output $L\tilde{u}_0$ are established.

At time k+1:

1. The relation matrix is updated using fuzzy identification via:

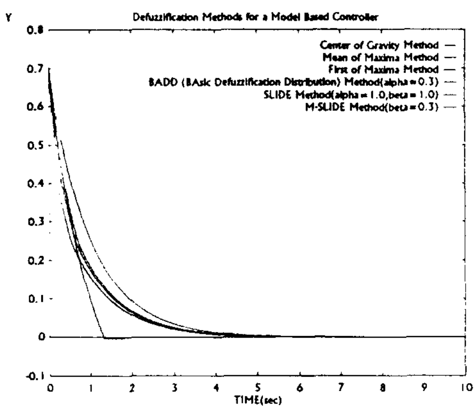
$$\tilde{R}_{k+1} = \tilde{R}_k \cup (L\tilde{X}_u \times \Lambda \times L\tilde{X}_m \times L\tilde{U}_u \times \Lambda \times L\tilde{U}_m \times L\tilde{Y}_u \times \times L\tilde{Y}_k)$$

2. The model-based controller calculates a new control-output using predictions from the fuzzy model: $L\tilde{y} = L\tilde{x} \circ L\tilde{u} \circ R_{k+1}$
3. In this controller, defuzzification process performed at convert predicted fuzzy value $L\tilde{y}$ to crisp value.

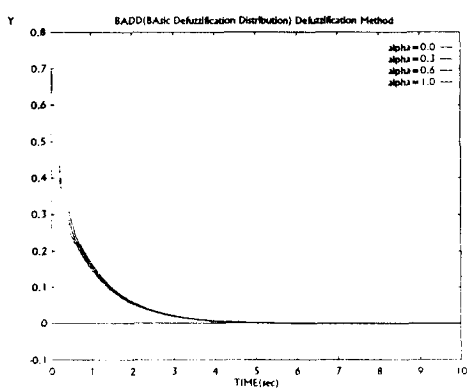
4.2. Simulation

We used simple first order system $b/(s+a)$ for simulations. $(a = 1, b = 1)$ And, for the a single control output, we used to 5 different control variable. At defuzzification stage, we applied several kinds of defuzzification methods. Center of gravity method, mean of maxima method, BADD mothod, SLIDE method. and M-SLIDE method. Simulation results are represented in Fig. 7.

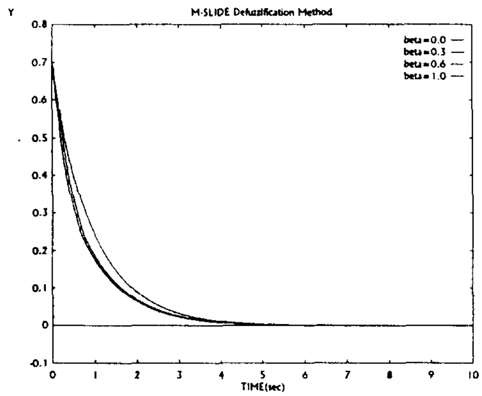
At that time, 5*5*5 rule table was created. And, at each step, rule table will be updated, continually.



(a)



(b)



(c)

Fig. 7 Simulation result of various defuzzification methods in adaptive fuzzy control

4.3. Discussion

Generally, adaptive stage have trend of decreasing effect of parameters for defuzzification method. Because model based fuzzy adaptive controller has learning ability. In our simulation results, the parameter of BADD method or SLIDE method did not importance on in defuzzification process. Similar to PD liked fuzzy control, center of gravity method represented reliable performance and mean of maxima method have faster response in transition region. Most of defuzzification methods have similar results each other. But, some defuzzification method(BADD, M-SLIDE) have good performance with respect to center of gravity method.

5. Conclusions and Further Works

5.1. Conclusions

Center of gravity method and center of sums method is widely used. In our study, these methods have reliable characteristics. In addition to existing five indices, we proposed new four indices as follows: confidence level loss, trajectory following, discrete version availability, and parameter variation. Especially. trajectory following represent some methods have a frequency limitation. So, we can say that each method is not reliable in the worst case.

The effect of model based adaptive fuzzy controller in various defuzzification methods is investigated by simple plant simulation. In this simulation, some parameters of defuzzification methods did not have effect.

In this study, we considered about 14 defuzzification methods. We summarize characteristics of defuzzification method in Table 1 .

At first, There exist confidence level loss in Center of Gravity and Center of Sum. etc because those methods takes defuzzified value from near middle of the support of output fuzzy set. This criterion is very useful for output fuzzy with sharp membership function.

Second, trajectory following is a useful guide for choosing a method to check the reliability of the method.

For example, Center of Area method shows poor performance. So, in the worst case we can't believe the performance of COA method.

Third, the discrete version availability is important factor to implement in discretized systems. Sadde's method did not have discrete version, yet.

Finally BADD, SLIDE, M-SLIDE, and Saade's methods have parameters which could be adjusted for a given system. Specially, those methods provides systematic approach to defuzzification. So, if we have abundant knowledge of the system, we can change the parameters to achieve best performance.

5.2. Further Works

To complex plants which have inconsistent rules[14], selection guides of defuzzification methods are needed. This is important to apply for real complex plants.

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Table 1 Characteristics of defuzzification methods under criteria

	Continuity	Disambiguity	Plausibility	Comp. Complexity	Weight Counting	Confidence Level Loss	Trajectory Following	Discrete Version	Parameter Variation.
Center of Gravity	Yes	Yes	No	Bad	No	Yes	Good	Yes	No
Center of Sums	Yes	Yes	No	Bad	Yes	Yes	Good	Yes	No
Mean of Maxima	No	Yes	No	Good	No	No	Avg.	Yes	No
Center of Maxima	No	Yes	No	Good	No	No	Avg.	Yes	No
First of Maxima	No	Yes	No	Good	No	No	Avg.	Yes	No
Last of Maxima	No	Yes	No	Good	No	No	Avg.	Yes	No
Center of Area	Yes	Yes	No	Bad	No	Yes	Poor	Yes	No
Center of Largest Area	No	No	Yes	Bad	No	Yes	Bad	Yes	No
Height	Yes	Yes	No	Good	Yes	Yes	Bad	Yes	No
Maximal Height	No	No	Yes	Good	No	No	Avg.	Yes	No
BADD	Yes	Yes	No	Bad	No	Yes	Avg.	Yes	Yes
SLIDE	Yes	Yes	No	Bad	Yes	Yes	Good	Yes	Yes
M-SLIDE	Yes	Yes	No	Bad	Yes	Yes	Poor	Yes	Yes
Saade's Method	Yes	Yes	Yes	Bad	Yes	Yes		No	Yes