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GYROSCOPIC EFFECT ON MODE SPLITTING IN ROTATING DISK: HDD SPINDLE SYSTEM VIBRATIONS

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ABSTRACT

A rotating rigid disk, attached on a flexible shaft or supported by a torsional spring, experiences precessional whirling due to gyroscopic moment loading. It is well known in rotor dynamics area that, as the rotational speed increases, the precessional mode of the rotating rigid disk starts splitting into two: forward and backward precessional modes. On the other hand, it is also well known in disk vibration area that a rotating flexible disk also shows another kind of mode splitting phenomenon due to the rotation, resulting in forward and backward traveling waves. When rotating multiple flexible disks are coupled in vibration with the supporting flexible shaft, the associated mode splitting should be compatible with the two seemingly different vibration analysis methods. This paper investigates the possibility of fusing the precessional and traveling wave mode splittings so that the bending coupled disk vibrations in HDD spindle systems can be better understood.

1. INTRODUCTION

The components of rotating machines nowadays tend to become lighter and more flexible due to the modern design trend toward high performance and efficiency. In particular, the disk as well as shaft elements are no longer considered to be rigid in many applications. Among others, the vibration characteristics of a flexible rotor with multiple flexible disks then becomes considerably complicated under certain conditions, due to the effects of dynamic coupling between the flexible elements.

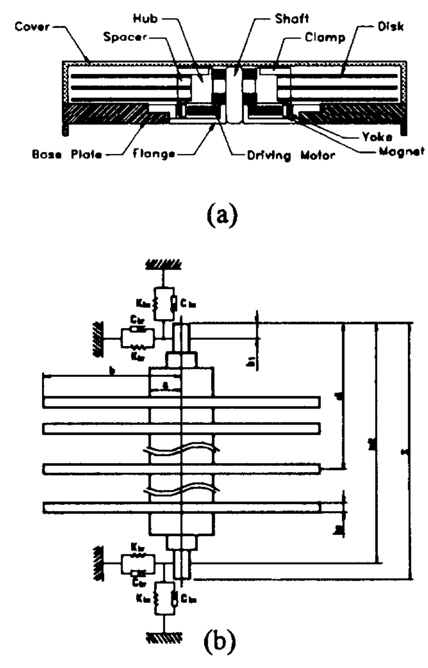

Fig. 1. Typical HDD spindle system: (a) Schematics and (b) Mathematical model[1]

The HDD spindle system shown in Fig.1 is essentially an assemblage of a shaft with multi-disks and supporting bearings. In the past, the most popular analysis model has been the flexible shaft-rigid disk model: no flexible coupling effects are allowed between the disks and shaft, except the rigid rotation effects such as the rotary inertia effect and the gyroscopic moments. However, as the modern HDD systems tend to become lighter and more flexible with a higher operating speed and a higher data storage capacity, the flexible shaft-rigid disk model can no longer properly represent the actual vibration characteristics. Particularly, there has been a strong tendency for relatively large and thin disks in HDD systems, so that the disk flexibility plays an important role in the dynamics of the spindle systems [1,2].

The rotordynamic analysis of the flexible shaft-flexible disk model is intricate due to the presence of coupled vibrations between the shaft and disks. According to the nature of coupling with the shaft, disk vibratory modes are commonly classified into three groups: uncoupled disk modes with more than one nodal diameters, disk modes with a single nodal diameter coupled with the shaft bending vibrations and umbrella modes coupled with the shaft longitudinal (axial) vibrations. The natural frequencies and mode shapes for each group are normally calculated by employing different solution techniques.

The uncoupled disk vibration has been a popular research subject for a long time, which is well documented in the literature [3,4,5,6,7]. For example, Lamb et al.[4] and Southwell [5] studied the vibration of thin disks of uniform cross section and Mote [6,7] investigated the dynamic behavior of thin disks under uniform in-plane tension, which can stabilize or strengthen thin annular plates during rotation, using the Ritz method. In the early works, the closed form solutions were derived for simplistic disk models.

Whereas few works have addressed the longitudinal coupled vibrations [8,9], the bending coupled vibrations have recently been studied by many researchers such as Dopkin & Shoup [10], Chivens & Nelson [11], Wilgen & Schlack [12], Shahab & Thomas [13], Wu & Flowers [14], and Sakata, et al.[15]. Recently, Chun and Lee [1] investigated the effects of a flexible bladed disk on the vibrational modes of the flexible rotor system using the substructure synthesis and assumed modes method.

2. BACKGROUND

Let us consider an analysis model for typical HDD spindle systems, which consists of a flexible shaft with varying cross section and N identical flexible, uniform circular disks rigidly attached to the shaft as shown in Fig. 1(b). The torsional modes are not considered here, since they are not observed within the frequency range of interest from practical HDD spindle systems.

It is well known that only the disk modes with one nodal diameter give contribution to the coupling between modes of the disk and spindle [1,2,14]. This is due to net inertia moment produced by the one nodal diameter modes of disk. This moment causes the interaction between the disk and spindle modes, leading to change in system natural frequencies. The moment is canceled out for m-nodal diameter modes except m=1 and thus no such interaction will be found. Accordingly, in the following discussion, the vibration analysis will be divided into three groups: bending coupled vibration (m = 1), uncoupled disk vibration ($m \geq 2$) and longitudinal (axial) coupled vibration (m = 0), in accordance with the order of importance in this work.

A rotating flexible circular disk, uncoupled with the shaft or support bending motion, is free of gyroscopic moment loading and the mode splitting due to rotation associated with the resulting forward and backward traveling wave frequencies, ω^f_{mn} and ω^b_{mn}, can be expressed, with respect to the stationary frame, as

$$\omega^f_{mn} - |\omega^b_{mn}| = 2m\Omega, \quad m = 0,1,2,3,\cdots\cdots (1)$$

where m and n indicate the number of nodal diameters and circles, respectively. On the other hand, a rigid rotor experiencing a gyroscopic motion is characterized by the mode splitting between the forward and backward precessional modes as

$$\omega^f_{mn} - |\omega^b_{mn}| = \alpha\Omega \qquad (2)$$

where $0 < \alpha = J_P/J_T \leq 2$. Here J_P and J_T are the polar and diametrical mass moments of inertia of the rotor. Note that $\alpha \cong 2$ for a thin disk and $\alpha \to 0$, as the rotor becomes a long cylinder. Another interesting point is that the mean frequency defined by $(|\omega^f_{mn}| + |\omega^b_{mn}|)/2$ is greatly affected by the gyroscopic moment effect as well as the centrifugal force effect, as the rotational speed increases. Figure 2 shows the forward and backward precessional modes of a typical gyroscopic rigid rotor, where the modal frequencies are normalized with respect to the non-rotating system natural frequency[16].

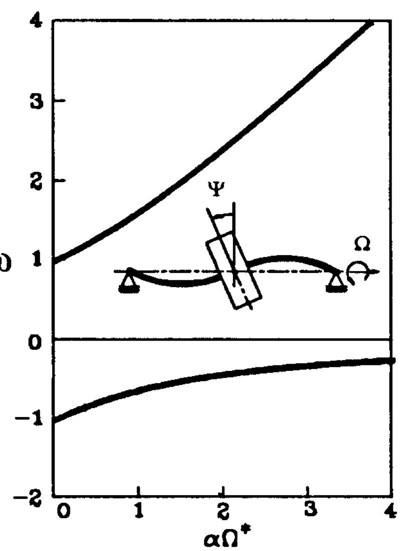

Fig. 2 A simple gyroscopic rotor supported by torsional spring [16]

When a flexible disk is subjected to a gyroscopic motion, the associated mode splitting will naturally become

$$\omega^f_{mn} - |\omega^b_{mn}| = \begin{cases} 2m\Omega & \text{for } m \neq 1 \\ \alpha'\Omega & \text{for } m = 1 \end{cases} \qquad (3)$$

where α' will normally be different in value from α.

3. COUPLED VIBRATIONS IN HDD SPINDLE SYSTEMS

Figure 3 shows the natural frequencies of a commercial three-disks HDD spindle system as the rotational speed, Ω, varies. In the figure, the frequencies and rotational speed have been normalized with respect to the nominal spindle operating speed Ω_0 = 75 Hz. Note that the split in the uncoupled disk and bending coupled modes is clearly observed, unlike the (0, n) umbrella modes.

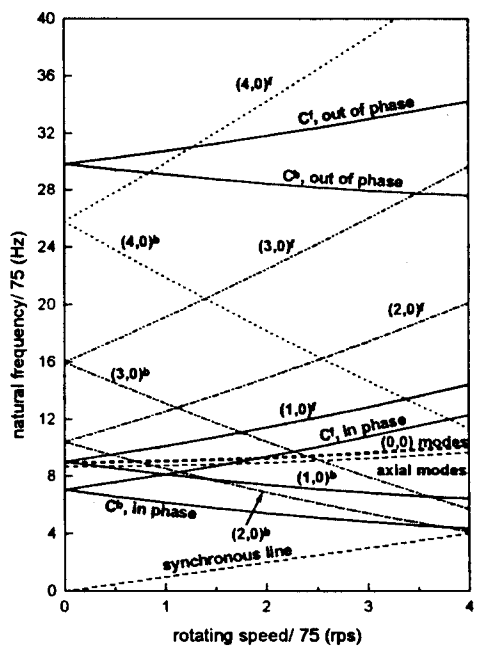

Fig.3 Natural frequencies of the three-disks HDD spindle system with rigid fixture [17]

Figure 4 emphasizes only the five lowest natural frequencies associated with the bending coupled vibration of the three-disks HDD spindle system, along with the corresponding mode shapes. The first bending coupled (in-phase unbalanced) mode is associated with the conical motion of the rotor assembly, which is typically

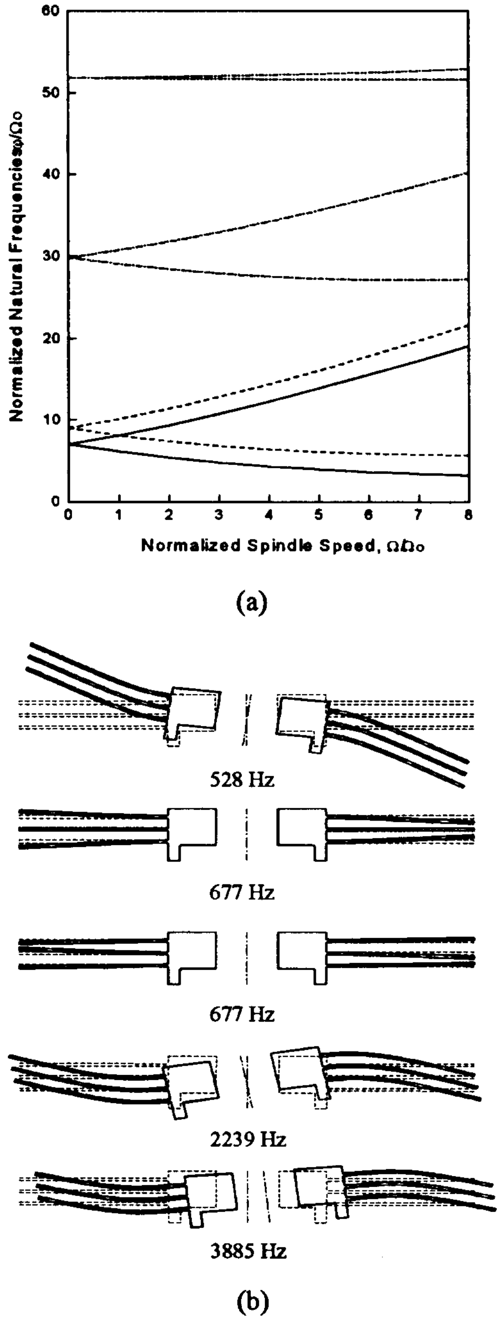

Fig. 4 Bending coupled (a) modal frequencies and (b) mode shapes of the three-disks HDD spindle system with rigid fixture [18].

observed from a rigid rotor as shown in Fig.2. Note that the in-phase mode shape of the three disks tends to intensify the net inertia moment to the spindle so that the conical motion of system is easily induced. The second and third modes (balanced modes) are degenerate, which behave like uncoupled disk modes. In fact, these modes can be well predicted from the individual flexible disk analysis without considering the shaft and support flexibility. Note that the out-of-phase mode shapes of the three disks tend to nullify the net inertia moment for the second and third bending coupled modes. The fourth (out-of-phase unbalanced) mode has the disk flexural motion plus the gyroscopic motion of the assembly. Each elements have the local mode shapes similar to those of the first mode but the motions associated with the spindle and the three disks are now out-of-phase. It is very interesting that this mode can not be predicted from the individual flexible disk or spindle analysis without considering the coupling effect. The fifth mode is associated with the predominant translational mode of spindle, where the three disks are of the (1,1) modes, resulting in relatively small inertia moment. Among others, the first and fourth (in-phase and out-of-phase unbalanced) modes are of a primary concern in this work.

Figure 5 compares the first bending coupled modes of the three-disks HDD system with α = 1.94 when the disks are assumed to be rigid and flexible. Note that the disk flexibility introduces many new modes into the system and significantly lowers the bending coupled modal frequencies. As expected, the mode split in balanced modes is equal to 2Ω [16]. On the other hand, the mode splits in in-phase and out-of-phase unbalanced modes are 1.99Ω and 1.67Ω. It implies that it becomes extremely difficult to identify the in-phase unbalanced mode by the amount of mode split only, requiring a check of the corresponding mode shapes. In practice, extensive modal testing to obtain mode shapes is a burden to testing engineers. Unlike the in-phase unbalanced mode, it is quite easy to single out the out-of-phase unbalanced mode from the rest of modes by the amount of mode split. But, the out-of-phase unbalanced mode is normally considered to be far less important than other lower modes.

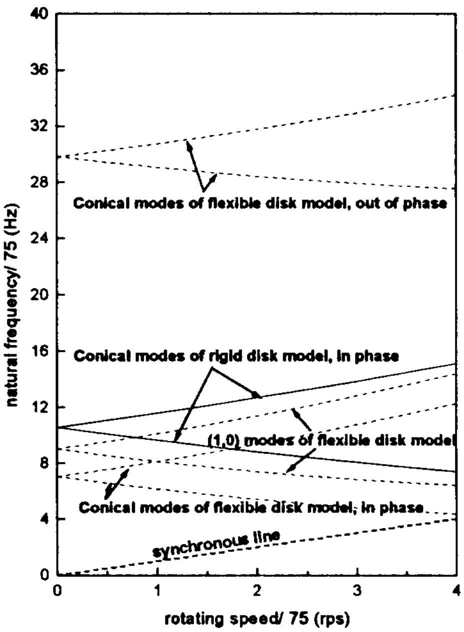

Fig. 5 Influence of disk flexibility on the bending coupled modes: three- disks HDD spindle system with rigid fixture.

Figure 6 shows the first bending (conical) modes of a commercial ten-disks HDD system with $\alpha = 1.74$, when the disks are assumed to be rigid and flexible as before. Now the mode splits in the in-phase and out-of-phase unbalanced modes for . flexible disk case are 1.98Ω and 1.10Ω, respectively, whereas the nine degenerate balanced modes still have mode split of 2Ω. Although the value of α for the ten-disks HDD spindle system is 1.74, which is not as close to 2 as for the three-disks spindle system, the mode split in the in-phase unbalanced mode is still close to 2Ω. Figure 7 shows the natural frequencies of

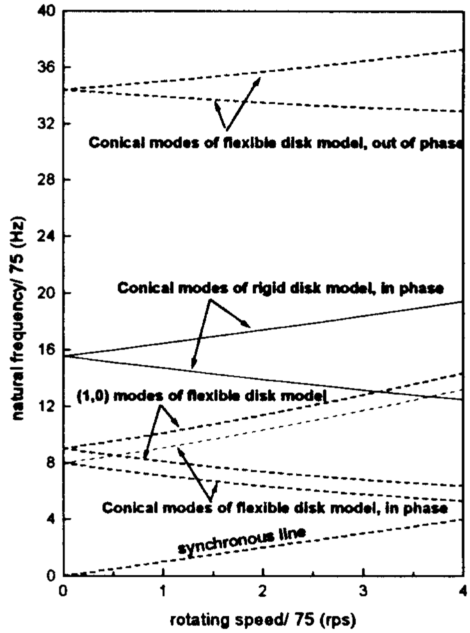

Fig. 6 Influence of disk flexibility on the bending coupled modes: ten-disks HDD spindle system with rigid fixture.

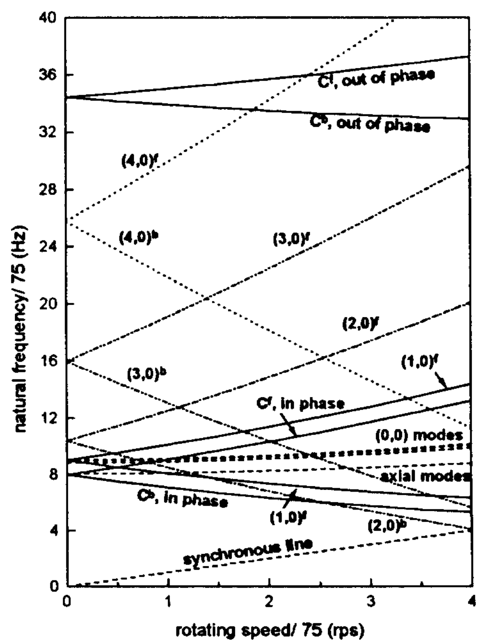

Fig.7 Natural frequencies of the ten-disks HDD spindle system with rigid fixture.

the commercial ten-disks HDD spindle system as the rotational speed varies. Note that the natural frequency of the center-clamped outer-free (1,0) disk mode is about 677 Hz as shown in Fig. 4.

4. DISCUSSION

The frequency equation for the fundamental unbalanced modes associated with a single flexible disk supported by a flexible torsional spring can be expressed as [1,8]

$$\left(\omega^2 - 2\Omega^*\omega - \omega_{mn}^2 + \Omega^{*2}\right)\left(\omega^2 - \alpha\Omega^*\omega - 1\right) - 2\alpha H\omega^2\left(\omega - 2\Omega^*\right)^2 = 0 \quad (4)$$

where

$$H = \frac{\int_0^1 R_{10}(\xi)\xi^2 d\xi}{\int_0^1 R_{10}^2(\xi)\xi d\xi}, \quad \Omega^* = \frac{\Omega}{\omega_t}$$

Here ω_{mn} is the traveling wave frequency associated with the (m, n) mode of the inner-clamped outer-free rotating disk; ω_t is the rocking modal frequency of the rigid stationary disk; $R_{10}(\xi)$ is the radial mode shape of the rotating disk with the unity outer radius. Note that all frequencies including ω are normalized with respect to ω_t. The above equation clearly states that the coupling between the disk and precessional modes becomes significant when (a) $\omega_{mn} \cong 1$, which implies that the disk mode and the precessional mode are close to each other, (b) α increases (up to 2), i.e., the disk gets thinner, and (c) H becomes large, which depends on the radial mode shape, i.e. boundary conditions of the disk. Thus, in case of the commercial HDD spindle systems treated previously where the boundary and support conditions are not much varied between different models and $\omega_{mn} \cong 1$, the mode coupling heavily depends on the value of α.

From Eq.(4), we can derive the sum of frequency splits in a pair of unbalanced modes of a single disk as

$$\frac{\Delta\omega_1 + \Delta\omega_2}{\Omega} = \frac{(\alpha + 2) - 8\alpha H}{1 - 2\alpha H} < \alpha + 2 \quad (5)$$

where $\Delta\omega_1$ and $\Delta\omega_2$ are the mode splits associated with the in-phase and out-of-phase unbalanced modes. The above relation implies that the sum of frequency splits in a pair of unbalanced modes of a single disk tends to decrease as the coupling effect increases, since H normally takes a value near 0.2 - 0.25. For the HDD spindle systems treated so far, $\alpha < \Delta\omega_1 \cong 2$. Thus, to satisfy the relation (5), $\Delta\omega_2$ should be far less than $\Delta\omega_1$. Note that the relation (5) also holds for multiple-disks system.

In summary: it becomes difficult to identify mode splits associated with the balanced and in-phase unbalanced modes by the amount of mode split only, unless α is far less than 2 which a single flexible disk can never achieve. Thus, as shown in the previous section, it is practically possible to differentiate two different mode splits only when a HDD spindle system has many disks. It should also be worth mentioning that the calculated values of H, using the relation (5), for the three and ten disks HDD spindle systems are 0.21 and 0.20, respectively, implying that the boundary conditions indeed remain almost unchanged for the two different systems.

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