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VARIABLE STRUCTURE CONTROL FOR MATCHED AND UNMATCHED UNCERTAINTY WITH QUADRATIC CRITERION

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Abstract In this paper, we propose a variable structure control approach for the system with matched and unmatched uncertainty. By using time-varying slidng mode, the reaching mode is removed, and the design methodology represents a realistic design approach with quadratic criterion for systems incorporating both matched and unmatched uncertainties. The criterion contains states and linear part of input for all time. The practical application of the control strategy is presented in the design of a stability augmentation system for an aircraft is presented.

Keywords Variable Structure Control, Unmatched Uncertainties, Time-varying Sliding Surface, Quadratic Criterion, Carathéodory function

1. INTRODUCTION

Variable structure controller is one approach to control uncertain dynamical systems. Variable structure control system(VSCS) is historically characterized by a control structure which is switched as the system state crosses certain discontinuous surfaces in the state-space. Within the context of VSCS the so-called sliding mode behavior has received particular attention. This is characterized by an initial motion onto the switching surfaces and is said to have attained a sliding mode.

The advantage of VSCS is totally insensitive to the matched uncertainty in the problem and the dynamics of the sliding system are completely defined by the choice of switching surfaces[1][4][5]. In terms of transient response, an undesired transient response may occur before sliding mode begins, so called in the reaching mode, because the sliding mode parameters are involved in the control gain matrices which have great effects on the transient response in the reaching mode. To remove the reaching mode, time-varying sliding surfaces are considered[6][7][8][9]. In terms of robustness, the major disadvantage is that the sliding system is sensitive to unmatched uncertainty which will directly affect the dynamic performance. In other words the dynamic performance prescribed by the switching surface is directly dependent upon the unmatched uncertainty. A number of approaches[2][3][10] have been proposed to handle unmatched undertainty. In Ryan and Corless[3] and Spurgeon and Davies[10], unmatched uncertainty is dealt with by choosing a suitable sliding surface. This method allows a free structure for the uncertainty, but limit its magnitude. Moreover, the performance criterion of sliding surfaces has not been considered. In Kwan[2], a dynamical sliding surface which can handle large modelling uncertainty is used, if certain extended matching conditions are satisfied. The requried dynamics implied by the sliding surface are obtained by using adaptive methods: the idea underlying this method is similar to that of back-stepping control.

This paper considers VSCS of linear systems with matched and unmatched uncertainties. It is assumed that all states of the systems are available and the quadratic criterion for the system without any uncertainty is given. The design approaches with quadratic criterion have been considered by Dorling and Zinober[1] and Young and Özgüner[6], but only for states without inputs in the criterion. Moreover, the robustness to unmatched uncertainty is not considered. In this paper, VSCS is designed with a quadratic criterion of states and inputs for the linear systems with matched and unmatched uncertainties. To remove the reaching mode, the time-varying sliding surfaces are adopted by using auxiliary states.

2. A DYNAMICAL SYSTEM WITH UNCERTAINTY

Consider an uncertain dynamical system of the form [10]

$$\dot{x} = Ax(t) + Bu(t) + \Delta(t, x, u) \tag{1}$$

in which $x \in R^n$ and $u \in R^m$ represent the state and control vectors respectively and the constant matrices A and B are compatibly dimensioned. It is assumed that $n > m$, B is of full rank m and the pair (A, B) is completely controllable. The unknown function $\Delta(.,.,.)$: $R \times R^n \times R^m \to R^n$ is uncertainty in the system. Δ is assumed to belong to a class of functions, $\mathcal{F}$, whereby for each $\Delta \in \mathcal{F}$ the matched and unmatched uncertainty components can be decomposed in the following form

$$\left.\begin{array}{l} \Delta = \Delta_1(t,x) + \Delta_2(t,x,u) \\ \Delta_1(.,.) : R \times R^n \to (I_m(B))^\perp \\ \Delta_2(.,.,.) : R \times R^n \times R^m \to I_m(B) \end{array}\right\} \tag{2}$$

where $I_m(.)$ denotes the range of the matrix (.), the operation $(.)^{\perp}$ denotes the orthogonal complement of (.) and Δ_1 and Δ_2 are Carathéodory functions.

It is assumed that the matched and unmatched components of each $\Delta \in \mathcal{F}$ can be expressed in the form

$$\left.\begin{aligned}\Delta_1(t,x) &= W_{u1}(t,x)x + W_{u2}(t,x)\\ \Delta_2(t,x,u) &= W_{m1}(t,x,u)u + W_{m2}(t,x)\end{aligned}\right\} \quad (3)$$

such that

$$\|W_{u1}\| < K_{u1},\ \ \|W_{u2}\| < K_{u2}$$
$$\|W_{m1}\| < K_{m1},\ \|W_{m2}\| < \kappa(t,x)$$

which implies

$$\left.\begin{aligned}\|\Delta_1(t,x)\| &\le K_{u1}\|x\| + K_{u2}\\ \|\Delta_2(t,x,u)\| &\le K_{m1}\|u\| + \kappa(t,x)\end{aligned}\right\}. \quad (4)$$

Here $\|.\|$ denotes the Euclidean norm, K_{u1}, K_{u2} and K_{m1} are known constants and κ is a known, strongly Carathéodory function.

3. TIME-VARYING SLIDING SURFACE FOR VSCS DESIGN

The first task is to specify a particular canonical form for the system in order to simplify the development of the design scheme. This form is the same as that used in [1] and [10].

By the assumption, rank$B = m$, there exists an orthogonal $n \times n$ transformation matrix T[1] such that

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (5)$$

where B_2 is a nonsingular $m \times m$ matrix . And define

$$y = Tx = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} x = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (6)$$

with $y_1 \in R^{n-m}$, $y_2 \in R^m$ and T_1, T_2 compatibly dimensioned. Then, the system representation becomes

$$\left.\begin{aligned}\dot{y}_1(t) &= A_{11}y_1(t) + A_{12}y_2(t) + \bar{\Delta}_1(t,y)\\ \dot{y}_2(t) &= A_{21}y_1(t) + A_{22}y_2(t) + B_2u(t)\\ &\quad +\bar{\Delta}_2(t,y,u)\end{aligned}\right\} \quad (7)$$

where

$$TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (8)$$

$$\begin{aligned}\bar{\Delta}_1 &= T_1\Delta_1(t,T^Ty)\\ &= T_1W_{u1}(t,T^Ty)T^Ty + T_1W_{u2}(t,T^Ty) \quad (9)\\ \bar{\Delta}_2 &= T_2\Delta_2(t,T^Ty,u)\\ &= T_2W_{m1}(t,T^Ty,u)u + T_2W_{m2}(t,T^Ty) \ .(10)\end{aligned}$$

For the given F, let's the time-varying sliding surface be the set such that

$$S = \{x \in R^n, \alpha \in R^m \mid s = [F\ I]Tx + \alpha = 0\} \quad (11)$$

i.e.

$$S = \{y \in R^n, \alpha \in R^m \mid s = Fy_1 + y_2 + \alpha = 0\}$$

where I denotes a compatibly dimensioned identity matrix, and the dynamic equation of the auxiliary state α and a virtual input β is

$$\dot{\alpha} = \beta \quad (12)$$

$$\beta = K \begin{bmatrix} y_1 \\ \alpha \end{bmatrix} \quad (13)$$

where α, $\beta \in R^m$ and $K \in R^{m\times n}$. If we set initial value of α as $\alpha(t_0) = -[F\ I]Tx(t_0)$, the initial value of s(t) becomes zero, so that the reaching mode of sliding mode control can be removed easily.

On the sliding surface, equation (7) becomes

$$\dot{z} = \begin{bmatrix} A_{11} - A_{12}F & -A_{12} \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ I \end{bmatrix}\beta + \begin{bmatrix} \bar{\Delta}_1 \\ 0 \end{bmatrix} \quad (14)$$

$$y_2 = -[F\ I]z \quad (15)$$

where $z^T = [y_1^T\ \alpha^T]$.

4. THE CONTROL INPUT OF VSCS

In general, VSCS control law consists of linear part and nonlinar part. A control strategy of the form

$$u(t) = u_l(y(t),\alpha(t),s) + u_n(t,y(t),\alpha(t),s(t)) \quad (16)$$

is hypothesized where u_l is a linear feedback control component and u_n is a nonlinear feedback effort. The linear feedback control component is defined as

$$\begin{aligned}u_l = -B_2^{-1}&[(FA_{11} + A_{21})y_1 + (FA_{12} + A_{22})y_2\\ &+\beta + \Lambda s] \quad (17)\end{aligned}$$

where $\Lambda > 0$.

The nonlinear control component is defined as

$$u_n(t,y(t),\alpha(t),s(t)) = -\rho(t,y,s)\frac{B_2^{-1}s}{\|s\| + \delta} \quad (18)$$

where

$$\rho = \frac{\gamma_1}{\sigma}(\eta(t,y) + \gamma_2) \quad (19)$$

with

$$\sigma \le \inf_{W_{m1}} \lambda_{min}\left(\frac{1}{2}\left[T_2W_{m1}B_2^{-1} + B_2^{-T}W_{m1}^TT_2^T\right] + I\right) \quad (20)$$

$$\eta(t,y) \ge \left\|FT_1\left(W_{u1}T^{-1}y + W_{u2}\right) + T_2W_{m1}u_l + T_2W_{m2}\right\|. \quad (21)$$

where $\lambda_{min}(.)$ denotes the minimum eigenvalue of (.), $\gamma_1 \ge 1$, $\gamma_2 > 0$ are design parameters, and $\sigma > 0$.

Theorem 1. The nonlinear control structure (16) - (21) is such that the uncertain system is globally uniformly ultimately bounded with respect to

$$\mathcal{R} = \left\{y, s : V_1(s) = \frac{1}{2}s^Ts \le r_1\right\} \quad (22)$$

where

$$r_1 = \frac{1}{2}\left(\frac{\delta}{\gamma_1 - 1}\right)^2 .$$

PROOF. Substitute the control effort (16), (17) and (18) into the system equations (7), (11) and (12) to obtain the following closed-loop system representation

$$\dot{s}(t) = -\Lambda s + B_2 u_n + F\bar{\Delta}_1 + \bar{\Delta}_2 . \quad (23)$$

Consider the Lyapunov function

$$V_1(s) = \frac{1}{2} s^T s .$$

Then

$$\dot{V}_1(s) = -s^T \Lambda s - s^T \left[\bar{\sigma}\frac{\rho s}{||s|| + \delta} - \gamma(t, y, u_l)\right] \quad (24)$$

where

$$\bar{\sigma}(t, y, u) = \frac{1}{2}\left(T_2 W_{m1} B_2^{-1} + B_2^{-T} W_{m1}^T T_2^T\right) + I \quad (25)$$

$$\gamma(t, y, u_l) = F T_1 \left(W_{u1} T^{-1} y + W_{u2}\right) + T_2 W_{m1} u_l + T_2 W_{m2} . \quad (26)$$

From (20) and (21)

$$\dot{V}_1(s) \leq -\lambda_{min}(\Lambda)||s||^2 - ||s|| \left(\frac{\sigma\rho||s||}{||s|| + \delta} - \eta\right) . (27)$$

Expressing $\eta(t, y, s)$ in terms of ρ, σ, γ_1 and γ_2 from (19) and substituting in (27), we have

$$\dot{V}_1(s) \leq -\lambda_{min}(\Lambda)||s||^2 - \gamma_2||s|| - \sigma\rho||s|| \left(\frac{||s||}{||s|| + \delta} - \frac{1}{\gamma_1}\right) . \quad (28)$$

It follows that if

$$||s|| \geq \frac{\delta}{\gamma_1 - 1} \quad (29)$$

then

$$\dot{V}_1 < 0$$

and (29) is satisfied if

$$V_1(s) > \frac{1}{2}\left(\frac{\delta}{\gamma_1 - 1}\right)^2 = r_1 \quad (30)$$

where r_1 is defined in (22).

□

5. TIME-VARYING SLIDING SURFACE DESIGN BY QUADRATIC MINIMIZATION FOR THE MATCHED UNCERTAINTY

First, assume that there exists only matched uncertainty(i.e. $\Delta_1 = 0$). If $\delta = 0$, then all states are on the sliding surface $s = 0$, $y_2 = -Fy_1 - \alpha$, then

$$u_l = -B_2^{-1}\left([K_1 \; K_2] z + \beta\right) \quad (31)$$

where

$$K_1 = (FA_{11} + A_{21} - FA_{12}F - A_{22}F) \quad (32)$$

$$K_2 = -(FA_{12} + A_{22}) . \quad (33)$$

Let a cost function be

$$J = \int_0^\infty x^T Q x + u_l^T R u_l \, dt \quad (34)$$

where Q and R are constant symmetric positive definite. Partition the following product as

$$TQT^T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} .$$

On the sliding surface, the cost function becomes

$$J = \int_0^\infty z^T \hat{Q} z + u_l^T R u_l \, dt \quad (35)$$

where

$$\hat{Q} = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ \hat{Q}_{21} & \hat{Q}_{22} \end{bmatrix} > 0 \quad (36)$$

with

$$\hat{Q}_{11} = Q_{11} - F^T Q_{21} - Q_{12} F + F^T Q_{22} F$$
$$\hat{Q}_{12} = -Q_{12} + F^T Q_{22}$$
$$\hat{Q}_{21} = -Q_{21} + Q_{22} F$$
$$\hat{Q}_{22} = Q_{22} .$$

Then, this cost function becomes to optimize the system, such that

$$\dot{z} = \hat{A} z + \hat{B} u_l \quad (37)$$

where

$$\hat{A} = \begin{bmatrix} A_{11} - A_{12}F & -A_{12} \\ -K_1 & -K_2 \end{bmatrix} \quad (38)$$

$$\hat{B} = \begin{bmatrix} 0 \\ -B_2 \end{bmatrix} \quad (39)$$

Theorem 2. If there exist $P_1 > 0$ that satisfies

$$P_1 \hat{A} + \hat{A}^T P_1 - P_1 \hat{B} R^{-1} \hat{B}^T P_1 + \hat{Q} = 0 \quad (40)$$

then, the virtual input β that minimize the cost function (34) becomes

$$\beta = -\left[K_1 + \hat{R} P_{21}, \; K_2 + \hat{R} P_{22}\right] z \quad (41)$$

where

$$\hat{R} = B_2 R^{-1} B_2^T$$
$$P_1 = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

where P_{ij} $(i, j = 1, 2)$ are compatibly dimensioned.

PROOF. From (40), u_l becomes

$$u_l = -R^{-1} \hat{B}^T P_1 z .$$

Then, from (31), the virtual input β becomes

$$\beta = \left(B_2 R^{-1} \hat{B}^T P_1 - [K_1\ K_2]\right) z$$
$$= -\left[K_1 + \hat{R}P_{21},\ K_2 + \hat{R}P_{22}\right] z\ .$$

□

Now, summarize these things in the x space, then

$$u = u_l + u_n \quad (42)$$
$$u_l = -R^{-1}\hat{B}^T P_1 \begin{bmatrix} T_1 x \\ \alpha \end{bmatrix} - B_2^{-1}\Lambda s$$
$$= R^{-1}B_2^T P_{21} T_1 x + R^{-1} B_2^T P_{22}\alpha - B_2^{-1}\Lambda s \quad (43)$$
$$u_n = -\rho(t,y,s)\frac{B_2^{-1}s}{\|s\| + \delta} \quad (44)$$

and the dynamics of the auxiliary state (12) becomes

$$\dot{\alpha} = -\left(K_1 + \hat{R}P_{21}\right) T_1 x - \left(K_2 + \hat{R}P_{22}\right)\alpha\ . \quad (45)$$

Till now, we are concerned about the design method under matched uncertainty. In the next section, the design technique to reduce the effect of unmatched uncertainty and continuous approximation of nonlinear control is investigated.

6. ROBUST DESIGN FOR UNMATCHED UNCERTAINTY AND CONTINUOUS APPROXIMATION CONTROL

On the sliding surface with unmatched uncertainty, (14) becomes

$$\dot{z} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} z + \begin{bmatrix} \bar{\Delta}_1 \\ 0 \end{bmatrix} \quad (46)$$

where

$$\bar{A}_{11} = A_{11} - A_{12}F \quad (47)$$
$$\bar{A}_{12} = -A_{12} \quad (48)$$
$$\bar{A}_{21} = -K_1 - \hat{R}P_{21} \quad (49)$$
$$\bar{A}_{22} = -K_2 - \hat{R}P_{22}\ . \quad (50)$$

Let $T^{-1} = [\bar{T}_1\ \bar{T}_2]$, then, in the case of continuous approximation nonlinear control (i.e. $\|s\| < \sqrt{r_1}, \delta > 0$), (9) becomes

$$\bar{\Delta}_1 = T_1\left(W_{u1}T^{-1}y + W_{u2}\right)$$
$$= T_1 W_{u1}\left(\bar{T}_1 - \bar{T}_2 F\right) y_1 - T_1 W_{u1}\bar{T}_2\alpha$$
$$+ T_1 W_{u2} + T_1 W_{u1}\bar{T}_2 s \quad (51)$$

and (46) can be written as

$$\dot{z} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} z + \begin{bmatrix} \tilde{\Delta}_1 \\ 0 \end{bmatrix} \quad (52)$$

where

$$\tilde{A}_{11} = \bar{A}_{11} + T_1 W_{u1}\left(\bar{T}_1 - \bar{T}_2 F\right) \quad (53)$$
$$\tilde{A}_{12} = \bar{A}_{12} + T_1 W_{u1}\bar{T}_2 \quad (54)$$
$$\tilde{\Delta}_1 = \left(T_1 W_{u1}\bar{T}_2 + A_{12}\right) s + T_1 W_{u2} \quad (55)$$

Lemma 3. For any $M \in R^{n\times m}$, $N \in R^{m\times p}$, $x \in R^n$, $y \in R^p$ and $\varepsilon > 0$

$$x^T M N y \le \varepsilon x^T M M^T x + \varepsilon^{-1} y^T N^T N y\ . \quad (56)$$

Theorem 4. If there exist P_2, P_3 and F such that

$$\tilde{A}_{11}P_2 + P_2\tilde{A}_{11} + 2\varepsilon_1 P_2^2 + 2\varepsilon_2 \bar{A}_{21}^T \bar{A}_{21} \le -v_1 I \quad (57)$$

$$\bar{A}_{22}^T P_3 + P_3\bar{A}_{22} + 2\varepsilon_1^{-1}\tilde{A}_{12}^T\tilde{A}_{12} + 2\varepsilon_2^{-1}P_3^2 \le -v_2 I \quad (58)$$

for some real scalar $v_1 > 0$, $v_2 > 0$ and positive definite matrices P_2, P_3. Then, the the dynamic equation (52) is such that the uncertain system is globally uniformly ultimately bounded with respect to

$$\mathcal{R} = \{y_1 : \|y_1\| \le r_2\} \quad (59)$$

where

$$r_2 = \frac{2}{v_1}\sigma_1(t,y,s) \quad (60)$$

with

$$\sigma_1(t,y,s) \ge \lambda_{max}(P_2)\|\left(T_1 W_{u1}\bar{T}_2 + A_{12}\right) s + T_1 W_{u2}\|\ . \quad (61)$$

PROOF. Consider the Lyapunov function

$$V_2(z) = \frac{1}{2}y_1^T P_2 y_1 + \frac{1}{2}\alpha^T P_3\alpha\ . \quad (62)$$

Then

$$\dot{V}_2(z) = \frac{1}{2}y_1^T\left[\tilde{A}_{11}^T P_2 + P_2\tilde{A}_{11}\right] y_1 + \frac{1}{2}\alpha^T\left[\bar{A}_{22}^T P_3 + P_3\bar{A}_{22}\right]\alpha + y_1^T\left[P_2\tilde{A}_{12} + \bar{A}_{21}^T P_3\right]\alpha + y_1^T P_2\tilde{\Delta}_1\ .$$

From (57), (58), (49) and lemma 3

$$\dot{V}_2(z) \le \frac{1}{2}y_1^T\left[\tilde{A}_{11}^T P_1 + P_1\tilde{A}_{11} + 2\varepsilon_1 P_2^2 + 2\varepsilon_2\bar{A}_{21}^T\bar{A}_{21}\right] y_1 + \frac{1}{2}\alpha^T\left[\bar{A}_{22}^T P_2 + P_2\bar{A}_{22} + 2\varepsilon_1^{-1}\tilde{A}_{12}^T\tilde{A}_{12} + 2\varepsilon_2^{-1}P_3^2\right]\alpha + y_1^T P_2\tilde{\Delta}_1$$
$$\le -\|y_1\|\left(\frac{1}{2}v_1\|y_1\| - \sigma_1\right) - \frac{1}{2}v_2\|\alpha\|^2\ . \quad (63)$$

It follows that if

$$\|y_1\| \ge \frac{2}{v_1}\sigma_1(t,y,s) \quad (64)$$

then

$$\dot{V}_2 \le 0\ .$$

□

7. SIMULATION RESULTS

To illustrate the design technigque suggested in this paper, consider VSTOL aircraft autostabilization problem[1].

Let the states x_1 to x_4 be the sideslip angle, the roll rate, the yaw rate and the bank angle respectively. Two inputs are available to control these states: the rudder angle u_1 and the aileron angle u_2. The system is linearized about a nominal operating point equivalent to an airspeed of 80 knots at sea level. The linearized system is

$$A = \begin{bmatrix} -0.0506 & 0 & -1 & 0.2380 \\ -0.7374 & -1.3345 & 0.3696 & 0 \\ 0.0100 & 0.1074 & -0.3320 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0409 & 0 \\ 1.2714 & -20.3106 \\ -2.0625 & 1.3350 \\ 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.9998 & 0.0014 & 0.0207 & 0 \\ 0.0120 & -0.8513 & -0.5245 & 0 \\ -0.0169 & -0.5247 & 0.8511 & 0 \end{bmatrix}.$$

It is assumed that there are the uncetainty such that

$$\begin{aligned} K_{u1} &= 0.0541 \\ K_{u2} &= 0.0107 \\ K_{m1} &= 1.1383 \\ \kappa(t,x) &= 0.2438\|x\| + 0.7319 \end{aligned}$$

and the initial states $x(t_0) = [1\ 0\ -1\ 0]^T$.

In the Dutch roll mode, let

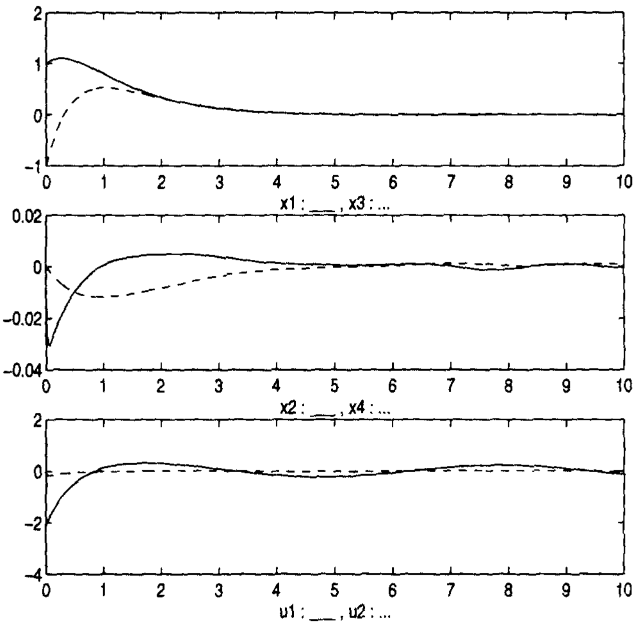

Fig. 1. All states and control input.

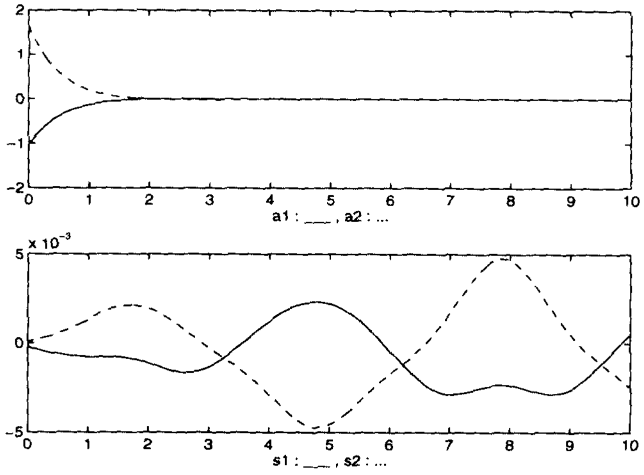

Fig. 2. Auxiliary states and sliding surfaces

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the control parameter be $\Lambda = 3I_2$, $\delta = 0.01$, $\gamma_1 = 1.5$ and $\gamma_2 = 0.1$. If we select that

$$F = \begin{bmatrix} -0.7273 & 0.4833 \\ -0.7258 & -0.7868 \end{bmatrix}, \tag{65}$$

then, $v_1 = 0.1346$, $v_2 = 8.8850$, $r_1 = 0.0002$ and $r_2 = 0.0012$.

Figure 1 shows system response with matched and unmatched uncertainties. This simulation result shows that response of the states are similar to that of a optimal control without any uncertainty. It means that the proposed VSCS is one approach of a robust optimal control problem. Figure 2 shows that the states always stay in the boundary layer of sliding surfaces so that there is no reaching mode.

8. CONCLUSION

This paper has formulated a control strategy which has been shown to prescibe bounded motion for a practically useful uncertainty set including both matched and unmatched uncertainty. The approach with quadratic criterion is also shown, in which not only state but also linear input is included in the criterion. Furthermore, since there is no reaching mode, all states are on the sliding surface for all time t. These facts indicate that the proposed methodology represnets a realistic design approach for systems incoorporating both matched and unmatched uncertainty to attain robust performance with quadratic criterion of states and inputs. The advantages of a sliding mode control scheme are hence extended to optimal problems involving unmatched uncertanty.

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