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Stability of Intervalwise Receding Horizon Control for Linear Time-Varying Systems

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Abstracts In this paper, an intervalwise receding horizon control (IRHC) is proposed which stabilizes linear continuous and discrete time-varying systems each other by means of a feedback control stemming from a receding horizon concept and a minimum quadratic cost. The results parallel those obtained for continuous [4], [9] and discrete time varying system [5], [15] each other.

Keywords Intervalwise Receding Horizon Control

1 Introduction

The receding horizon control has received much attention as a powerful tool for the control of the industrial process systems since it can overcome the limitation of the existing control method such as LQ or LQG, which considers an infinite future horizon without constraints. There are two kinds of receding horizon controls, pointwise and intervalwise. In pointwise receding horizon control, the terminal point of a fixed-length finite cost horizon recedes continuously. In intervalwise receding horizon control, the terminal point is kept fixed for a period of a finite cost horizon and, after one period, the terminal point moves by one period and is fixed for the next period.

It is known that the intervalwise receding horizon control (IRHC) provides a much lower computation cost, a better tracking performance [16], and an easier way [3] than the pointwise receding horizon control (PRHC). In addition, the gain of IRHC tends to be smaller than that of PRHC since the terminal point of IRHC is fixed for a period, while that of PRHC always recedes.

IRHC has only been developed for time-invariant and periodic systems [1], [2], [3], [16], [17], while PRHC have been developed for time-invariant and time-varying systems [4], [5], [6], [7], [8], [9], [10], [15].

In this paper, we investigate IRHC for linear time-varying systems. Conditions are proposed which guarantee closed-loop stability of linear time-varying systems with IRHC.

In Section 2, a stabilizing H_2 IRHC for continuous time-varying systems is proposed. In Section 3, a stabilizing H_2 IRHC for discrete time-varying systems is also proposed. In Section 4, a stabilizing H_∞ IRHC for continuous time-varying systems is proposed. In Section 5, a stabilizing H_∞ IRHC for discrete time-varying systems is also proposed. Finally, conclusions are presented in Section 6.

2 H_2 intervalwise receding horizon control for continuous systems

In this session, we suggest an intervalwise receding horizon control and stabilizing conditions for continuous systems.

Consider the following continuous time-varying system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^p$. Consider also a cost function

$$J = \int_{t_0}^{t_f} (y^T(t)Q(t)y(t) + u^T(t)R(t)u(t))dt \tag{2.2}$$

where $Q(t)$ and $R(t)$ are piecewise continuous with

$Q(t) \geq 0$, $R(t) > 0$, and $x(t_f) = 0$. Then the optimal closed-loop control is given by

$$u^*(t) = -R^{-1}(t)B^T(t)P^{-1}(t,t_f)x(t) \quad (2.3)$$

where $P^{-1}(\tau,\sigma)$ is the inverse of a symmetric positive definite matrix $P(\tau,\sigma)$ satisfying

$$\begin{aligned} -\frac{\partial P(\tau,\sigma)}{\partial \tau} &= -A(\tau)P(\tau,\sigma) - P(\tau,\sigma)A^T(\tau) \\ &-P(\tau,\sigma)C^T(\tau)Q(\tau)C(\tau)P(\tau,\sigma) \\ &+B(\tau)R^{-1}(\tau)B^T(\tau), \quad \tau \leq \sigma \quad (2.4) \end{aligned}$$

with the boundary condition $P(\sigma,\sigma) = 0$. We assume there exist fixed positive real numbers α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8, such that

$$\alpha_1 I \leq Q(t) \leq \alpha_2 I \quad (2.5)$$

$$\alpha_3 I \leq R(t) \leq \alpha_4 I \quad (2.6)$$

$$\alpha_5 I \leq \int_t^{t+\delta} \Phi(t,\tau)B(\tau)B^T(\tau)\Phi^T(t,\tau)d\tau \leq \alpha_6 I \quad (2.7)$$

$$\alpha_7 I \leq \int_t^{t+\delta} \Phi^T(\tau,t)C^T(\tau)C(\tau)\Phi(\tau,t)d\tau \leq \alpha_8 I \quad (2.8)$$

for $\forall$ t where Φ is the state transition matrix. Now we propose an intervalwise receding horizon control (IRHC) for continuous time-varying systems.

$$u^*(t) = -R^{-1}(t)B^T(t)P^{-1}(t,T,T_f)x(t) \quad (2.9)$$

where $T_f - T \geq \delta$, $P(t,T,T_f)$ is obtained by integrating (2.4) backward from $\sigma = t_0 + (k+1)T + T_f$ to $\tau = t$ for $k = 0, 1, \cdots$, and $t \in [t_0 + kT, t_0 + (k+1)T)$. The following result is needed to prove the stability of (2.9).

Lemma *1 :* a) Assume (2.6), (2.7), $0 \leq Q(t) \leq \alpha_9 I$, and $\|C(t)\| \leq \alpha_{10}$ for $\forall t$, then there exist positive scalars α_{11} and α_{12} such that $\alpha_{11} I \leq P(t,T,T_f) \leq \alpha_{12} I$, for $\forall t$. b) Assume (2.5), (2.6), (2.7), and (2.8), then there exist positive constants α_{13} and α_{14} such that $\alpha_{13} I \leq P(t,T,T_f) \leq \alpha_{14} I$ for $\forall t$

proof: a) This result may be found in [4]. b) This result may be found in [12]. ■

THEOREM *1 :* 1) Assume (2.6), (2.7), $0 \leq Q(t) \leq \alpha_9 I$, and $\|C(t)\| \leq \alpha_{10}$ for $\forall t$, then the system (2.1) with feedback control (2.9) is uniformly asymptotically stable. 2) Assume (2.5), (2.6), (2.7), and (2.8), then the system (2.1) with the control law (2.9) is uniformly asymptotically stable.

proof: The proof procedure parallels that of [4] using Theorem 4 of [13]. The only difference between the proof of Theorem 2.1 of [4] and that of this Theorem is to use $P^{-1}(t,T,T_f)$ instead of $P^{-1}(t,t+T)$. ■

In the following session, we investigate IRHC for discrete systems

3 H_2 intervalwise receding horizon control for discrete systems

Consider the following discrete time-varying system

$$\begin{aligned} x_{i+1} &= A_i x_i + B_i u_i \quad (3.1) \\ y_i &= C_i x_i \end{aligned}$$

where $x_i \in R^n$, $u_i \in R^m$, and $y_i \in R^p$. Consider also a cost function

$$J = \sum_{i=i_0}^{i_f-1} [y_i^T Q_i y_i + u_i^T R_i u_i] \quad (3.2)$$

where $Q_i \geq 0$ and $R_i > 0$, and $x_{i_f} = 0$. Then the optimal closed-loop control is given by

$$u_i^* = -R_i^{-1} B_i^T P_{i+1,i_f}^{-1} A_i x_i \quad (3.3)$$

where P^{-1}-matrix is the inverse of a symmetric positive definite matrix P-matrix satisfying

$$\begin{aligned} P_{i,j} &= A_i^{-1} P_{i+1,j} A_i^{T^{-1}} - A_i^{-1} P_{i+1,j} A_i^{T^{-1}} C_i^T D_i^T \\ &[I + D_i C_i A_i^{-1} P_{i+1,j} A_i^{T^{-1}} C_i^T D_i^T]^{-1} D_i C_i A_i^{-1} \\ &P_{i+1,j} A_i^{T^{-1}} + B_{i-1} R_{i-1}^{-1} B_{i-1}^T, \quad i \leq j \quad (3.4) \end{aligned}$$

with the boundary condition $P_{j,j} = 0$. We assume there exist fixed positive real numbers β_1, β_2, β_3, β_4, β_5, β_6, β_7, β_8, such that

$$\beta_1 I \leq Q_i \leq \beta_2 I \quad (3.5)$$

$$\beta_3 I \leq R_i \leq \beta_4 I \quad (3.6)$$

$$\beta_5 I \leq \sum_{j=i}^{i+l} [A_{i,j+1} B_i B_i^T A_{i,j+1}^T] \leq \beta_6 I \quad (3.7)$$

$$\beta_7 I \leq \sum_{j=i}^{i_f} [A_{i,j+1}^T C_i^T C_i A_{i,j+1}] \leq \beta_8 I \quad (3.8)$$

for $\forall$ i where $A_{i,j}$ is the state transition matrix. We also propose H_2 IRHC for discrete time-varying systems.

$$u_i^* = -R_i^{-1} B_i^T P_{i+1,L,L_f}^{-1} A_i x_i, \quad L_f - L \geq l \quad (3.9)$$

where P_{i+1,L,L_f} is obtained by calculating (3.4) backward from $i_0 + (k+1)L + L_f$ to $i+1$ for $k = 0, 1, \cdots$, and $i \in [i_0 + kL, i_0 + (k+1)L - 1]$.

THEOREM *2 :* 1) Assume (3.6), (3.7), $0 \leq Q_i \leq \beta_9 I$, and $\|C_i\| \leq \beta_{10}$ for $\forall i$, then the system (3.1) with feedback control (3.9) is uniformly asymptotically stable. 2) Assume (3.5), (3.6), (3.7), and (3.8), then the system (3.1) with the control law (3.9) is uniformly asymptotically stable.

proof: The proof parallels that of [5]. The only difference between the proof of Theorem of [5] and that of this Theorem is to use P_{i,L,L_f}^{-1} instead of $P_{i,i+L}^{-1}$. ■

4 H_∞ intervalwise receding horizon control for continuous systems

Consider the following continuous time-varying system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \\ z(t) &= C(t)x(t) \end{aligned} \tag{4.1}$$

where $x(t_0) = 0$, $x(t) \in R^n$, $u(t) \in R^m$, $w(t) \in R^l$, and $z(t) \in R^p$. Consider also the cost index with $F > 0$:

$$\begin{aligned} J &= x^T(t_f)Fx(t_f) + \int_{t_0}^{t_f} [z^T(t)z(t) \\ &\quad -\gamma^2 w^T(t)w(t)]dt \end{aligned} \tag{4.2}$$

Define the following Riccati equation

$$\begin{aligned} \frac{\partial Z(\tau,\sigma)}{\partial \tau} &= A^T(\tau)Z(\tau,\sigma) + Z(\tau,\sigma)A(\tau) + Z(\tau,\sigma) \\ &\quad (B_2(\tau)B_2^T(\tau) - \gamma^{-2}B_1(\tau)B_1^T(\tau)) \\ &\quad Z(\tau,\sigma) - C^T(\tau)C(\tau), \quad \tau \le \sigma \end{aligned} \tag{4.3}$$

with the boundary condition $Z(\sigma,\sigma) = 0$, and the threshold value

$$\begin{aligned} \hat{\gamma}^{CL} &= inf\{\gamma > 0 : The\ RDE\ (4.3)\ does\ not \\ &\quad have\ a\ conjugatepoint\ on\ [t_0,t_f]\} \end{aligned} \tag{4.4}$$

Then if $\gamma > \hat{\gamma}^{CL}$, the differential game with closed-loop system admits a unique feedback saddle-point solution given by

$$\begin{aligned} u^*(t) &= -B_2^T(t)Z(t,t_f)x(t) \\ w^*(t) &= \gamma^{-2}B_1^T(t)Z(t,t_f)x(t) \end{aligned} \tag{4.5}$$

and the saddle point value of the game is given by $J_\gamma(u^*,w^*) = x^T(t_0)Z(t_0,t_f)x(t_0)$.

Using the above (4.5), we derive an intervalwise receding horizon H_∞ control

$$\begin{aligned} u^*(t) &= -B_2^T(t)Z(t,T)x(t) \\ w^*(t) &= \gamma^{-2}B_1^T(t,T)Z(t)x(t) \end{aligned} \tag{4.6}$$

where $Z(t,T)$ is obtained by integrating (4.3) backward from $\sigma = t_0 + (k+1)T$ to $\tau = t$ for $k = 0, 1, \cdots$, and $t \in [t_0 + kT, t_0 + (k+1)T)$. Consider the controllability gramians

$$G_1(t_0,t_f) = \int_{t_0}^{t_f} \Phi(t_0,t)B_1(t)B_1^T(t)\Phi^T(t_0,t)dt$$

$$G_2(t_0,t_f) = \int_{t_0}^{t_f} \Phi(t_0,t)B_2(t)B_2^T(t)\Phi^T(t_0,t)dt$$

Now, we investigate the stabilty of an intervalwise receding horizon H_∞ control.

THEOREM *3* : Assume $C^TC > \epsilon I$ for $\forall t$ and $G_2^{-1}(t_0,t_1)G_1(t_0,t_1)$ for $\forall\ t_1 - t_0 \le T$. Then the system (4.1) with feedback control (4.6) is exponentially stabilizing and the H_∞-norm bound of the closed-loop system is guaranteed.

proof: Let $F(t) = A(t) - B_2(t)B_2^T(t)Z(t,T)$ and $V(t) = x^T(t)Z(t,T)x(t)$. Then $\dot{V} = x^T(-Z^TB_2B_2P - \gamma^2 ZB_1B_1^TZ - C^TC + \frac{\partial Z(t,\sigma)}{\partial\sigma})x$. Since Z is positive definite and uniformly bounded and $\frac{\partial Z(t,\sigma)}{\partial \sigma} \le 0$ by [9], $\dot{V} < -\epsilon x^T(t)x(t)$. Hence the system (4.6) is exponentially stable.

The proof of the H_∞-norm bound may be also found in [9]. The only difference between the proof of Theorem 1 of [9] and that of this Theorem is to use $Z^{-1}(t,T)$ instead of $P^{-1}(t,t+T)$. ■

5 H_∞ intervalwise receding horizon control for discrete systems

Consider the following discrete time-varying system

$$\begin{aligned} x_{i+1} &= A_ix_i + B_{1i}w_i + B_{2i}u_i \\ z_i &= C_ix_i \end{aligned} \tag{5.1}$$

where $x_i \in R^n$, $u_i \in R^m$, $w_i \in R^l$, and $z_i \in R^p$. Consider also a cost function

$$\begin{aligned} J &= x_{i_f}^TQ_{i_f}x_{i_f} + \sum_{i=i_0}^{i_f-1}[x_i^TC_i^TC_ix_i \\ &\quad + u_i^Tu_i - \gamma^2 w_i^Tw_i]. \end{aligned} \tag{5.2}$$

Then the dynamic game theory described by (5.1) and (5.2) admits a unique feedback saddle-point solution, if and only if $[I - \gamma^{-2}B_{1i}^TM_{i+1,i_f}B_{1i}] > 0$ over $i \in [i_0, i_f - 1]$ [14]:

$$\begin{aligned} u_i^* &= -B_{2i}^TM_{i+1,i_f}\Lambda_{i+1,i_f}^{-1}A_ix_i \\ w_i^* &= \gamma^{-2}B_{1i}^TM_{i+1,i_f}\Lambda_{i+1,i_f}^{-1}A_ix_i \end{aligned} \tag{5.3}$$

where

$$\begin{aligned} M_{i,i_f} &= C_i^TC_i + A_i^TM_{i,i_f}\Lambda_{i,i_f}^{-1}A_i \\ M_{i_f} &= Q_{i_f} \\ \Lambda_{i,i_f} &= I + [B_{2i}B_{2i}^T - \gamma^{-2}B_{1i}B_{1i}^T]M_{i,i_f} \end{aligned} \tag{5.4}$$

Using the above (5.3), we also derive an intervalwise receding horizon H_∞ control for discrete time-varying system.

$$u_i^* = -B_{2i}^TM_{i+1,L,L_f}\Lambda_{i+1,i_f}^{-1}A_ix_i \tag{5.5}$$

$$w_i^* = \gamma^{-2}B_{1i}^TM_{i+1,L,L_f}\Lambda_{i+1,i_f}^{-1}A_ix_i \tag{5.6}$$

where $L_f - L \geq 2$ and $M(i+1, L, L_f)$ is obtained by calculating (5.4) backward from $i_0 + (k+1)L + L_f$ to $i+1$ for $k = 0, 1, \cdots$, and $i \in [i_0 + kL, i_0 + (k+1)L - 1]$.

THEOREM *4* : 1) Assume $Q_{i_f} \geq \Psi_{i_f} + C_{i_f}^T C_{i_f} + H_{i_f}^T H_{i_f}, \forall i_f \in [L_f, \infty]$ and $I - \gamma^{-2} B_{1i_f}^T Q_{i_f+1} B_{1i_f} > 0$ where $\Psi_{i_f} = F_{i_f}^T Q_{i_f+1} B_{1i_f} (I - B_{1i_f}^T Q_{i_f+1} B_{1i_f})^{-1} B_{1i_f}^T Q_{i_f+1} F_{i_f} + F_{i_f}^T Q_{i_f+1} F_{i_f}$ and $F_{i_f} = A_{i_f} - B_{i_f} H_{i_f}$. 2) Assume the system (5.1) is uniformly detectable. Then the system (5.1) with the control law (5.5) is stable and the H_∞-norm bound of the closed-loop system is guaranteed.

proof: The proof procedure parallels that of [15]. The only difference between the proof of Theorem of [15] and that of this Theorem is to use M_{i,L,L_f}^{-1} instead of $M_{i,i+L}^{-1}$. ■

6 Conclusion

In this paper, an intervalwise receding horizon control (IRHC) is proposed for linear continuous and discrete time-varying systems each other. It is shown that the proposed IRHCs guarantee closed-loop stability under the proposed stabilizing conditions.

Our results enable us to take advatages of IRHC for linear time-varying systems. In the same way, an intervalwise receding horizon concept can be applied to various systems such as nonlinear, delay, and constrained system.

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