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불충분한 작동기를 가진 매니퓰레이터의 비선형제어: 크레인제어에의 응용

Nonlinear Control of Residual Sway of a Container Crane in the Perspective of Controlling an Underactuated System

°김영민*, 홍금식**, 이교일†

* 부산대학교 대학원 (Tel: (051)510-1481, Email: kimymin@hyowon.pusan.ac.kr)
** 부산대학교 기계공학부 (Tel: (051)510-2454, Email: kshong@hyowon.pusan.ac.kr)
† 서울대학교 기계설계학과 (Tel: (02)880-7143, Email: lki@plaza.snu.ac.kr)

Abstracts In this paper the sway-control problem of a container crane is investigated in the perspective of controlling an underactuated mechanical system. For fast loading/unloading of containers from the ship, quick suppression of the remaining swing motion of the container at the end of each trolley stroke is crucial. Known nonlinearities are fully incorporated by feedback linearization. Robustness is enhanced by variable structure control. Compared with the linear LQ control, much better performance can be obtained.

Keywords Crane control, Feedback linearization, Underactuated manipulator, Variable structure control

1. Introduction

In this paper the sway-control problem of a container crane is investigated in the perspective of controlling an underactuated mechanical system. Since fast loading/unloading of containers from the ship is most crucial for the container ship, the time-optimal control with zero terminal conditions is currently employed. However, there always exists some residual swinging left at the end of trolley stroke due to disturbances like winds, therefore it is necessary to switch to another controller to dissipate the residual sway as quickly as possible.

(Boustany and D'Andrea-Novel, 1992) investigated an indirect adaptive control scheme using dynamic feedback linearization and estimation technique for a underactuated mechanical system and applied their control scheme to an overhead crane system. In (Wiklund et al., 1995), the swing up to upright position of Furuta's new inverted pendulum was shown to be done by variable structure controller based on an energy approach.

In this paper a combination of three control methods, time-optimal control, partial feedback linearization, and variable structure controller(VSC) are proposed for the control of crane system which is an underactuated mechanical system with free joint. Each control method is carefully proposed for its specific objectives. The control loop for trolley traveling consists of two stages. The first stage is a time optimal control (Sakawa and Shindo, 1982; Hong et al., 1997a-b), and the second stage is a residual sway control which starts at the end of time optimal control while lowering the container. Due to disturbances like winds some residual sway is always left at the end of each trolley stroke. Therefore, the second stage of residual sway control utilizes partial feedback linearization to account for the nonlinearities as much as possible, and variable structure control to account for the unmodeled dynamics and disturbances. The main contribution of the paper is the first application of combined nonlinear control techniques to the sway-control of container crane systems.

2. Control Design

2.1 Feedback Linearization of Underactuated System

Consider an n degrees of freedom open loop mechanism with joint variables $q^1,\ldots,q^n$. It is assumed that each joint has a single degree of freedom and only $m<n$ joints are active. Each joint which is capable of actuation is called an active joint. The remaining $l=n-m$ joints with no actuation are called passive or free joints depending on the availability of brake.
The equations of motion for the underactuated mechanical system is represented as

$$M_{11}\ddot{q}_1+M_{12}\ddot{q}_2+C_1(q,\dot{q})+G_1(q)=0 \tag{1}$$

$$M_{21}\ddot{q}_1+M_{22}\ddot{q}_2+C_2(q,\dot{q})+G_2(q)=f \tag{2}$$

where the vector functions $C_1(q,\dot{q})\in R^l$ and $C_2(q,\dot{q})\in R^m$ contain Coriolis and centripetal terms, the vector functions $G_1(q)\in R^l$ and $G_2(q)\in R^m$ contain gravitational terms, and $f\in R^m$ represents the input generalized force produced by the m actuators at the active joints. Note that f appears only on the active joints q_2. Hence like a fully actuated robot (i.e., the traditional robot), the dynamic equation itself for an underactuated system can also be written as

$$M(q)\ddot{q}+C(q,\dot{q})+G(q)=Bf \quad (3)$$

where

$$q=[q_1^T,q_2^T]^T,\ M(q)=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix},\ B=\begin{bmatrix}O_{l\times m}\\ I_{m\times m}\end{bmatrix}$$

$$C=[C_1^T,\ C_2^T]^T,\ G=[G_1^T,\ G_2^T]^T$$

M still preserves the important properties such as symmetry, positive definiteness of the original inertia matrix.

Now consider equation (1). Therefore we may solve for $\ddot{q}_1$ as

$$\ddot{q}_1=-M_{11}^{-1}(M_{12}\ddot{q}_2+C_1+G_1). \quad (4)$$

Substituting (4) into (2) yields

$$\overline{M}_{22}\ddot{q}_2+\overline{C}_2+\overline{G}_2=f \quad (5)$$

where

$$\overline{M}_{22}=M_{22}-M_{21}M_{11}^{-1}M_{12},$$

$$\overline{C}_2=C_2-M_{21}M_{11}^{-1}C_1,$$

$$\overline{G}_2=G_2-M_{21}M_{11}^{-1}G_1.$$

A partial feedback linearizing controller can therefore be defined for equation (5) according to

$$f=\overline{M}_{22}u+\overline{C}_2+\overline{G}_2 \quad (6)$$

where $u\in R^m$ is an additional control input yet to be defined, and the $m\times m$ matrix $\overline{M}_{22}$ is itself symmetric and positive definite (Gu et al., 1993). The complete system up to this point may be written as

$$M_{11}\ddot{q}_1+C_1+G_1=-M_{12}u \quad (7)$$

$$\ddot{q}_2=u \quad (8)$$

2.2 Variable Structure Control

In subsection 2.1 all known nonlinearities are fully incorporated by feedback lineaization in control design. In this subsection a VSC for improving robustness against unmodeled dynamics and disturbances is investigated for equation (7) and (8). The plant dynamics is now in the following form:

$$\dot{x}(t)=F(x,u)=f(x)+B(x)u$$

$$=\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ -M_{11}^{-1}(C_1+G_1)\\ 0_{m\times m}\end{bmatrix}+\begin{bmatrix}0_{l\times m}\\ 0_{m\times m}\\ -M_{11}^{-1}M_{12}\\ I_{m\times m}\end{bmatrix}u \quad (9)$$

where

$$x=\left[q_1^T,q_2^T,\dot{q}_1^T,\dot{q}_2^T\right]^T\in R^{2n},\quad u\in R^m,\ \text{and}\ B(x)\in R^{2n\times m}$$

Associated with the system is a (n-m)-dimensional switching surface (also called a discontinuity or equilibrium manifold),

$$S=\{x\in R^n \mid \sigma(x)=0\} \quad (10)$$

where

$$\sigma(x)=\left[\sigma_1(x),\cdots,\sigma_m(x)\right]^T=0$$

We will often refer to S as $\sigma(x)=0$. These surfaces are designed so that the system state trajectory, restricted to $\sigma(x)=0$, has a desired behavior such as stability or tracking. After proper design of the sliding surface, a control law must guarantee that the sliding surface is reachable, and that the plant's state trajectory is maintained on the sliding surface for all sequential time. We can decompose the general control structure as:

$$u=u_{eq}+u_N \quad (11)$$

where u_{eq} is equivalent control, u_N is reaching control. The reaching control is used in order to guarantee that $S\dot{S}<0$ in a region "close" to the sliding surface. The equivalent control constitutes an equivalent input which produces the motion of the system on the sliding surface whenever the initial state is on the surface. Using the chain rule, we define the equivalent control u_{eq} for system (9) as the input satisfying

$$\dot{\sigma}=\frac{\partial\sigma}{\partial x}\dot{x}=\frac{\partial\sigma}{\partial x}f(x)+\frac{\partial\sigma}{\partial x}B(x)u_{eq}=0$$

Assuming that the matrix product $\frac{\partial\sigma}{\partial x}B(X)$ is nonsingular for all x, one can compute u_{eq} as

$$u_{eq}=-\left[\frac{\partial\sigma}{\partial x}B(x)\right]^{-1}\left(\frac{\partial\sigma}{\partial x}f(x)\right) \quad (12)$$

Therefore, given that $\sigma(x(t_1))=0$, then, for all $t\geq t_1$, the dynamics of the system on the switching surface will satisfy

$$\dot{x}(t)=\left[I-B(x)\left[\frac{\partial\sigma}{\partial x}B(x)\right]^{-1}\frac{\partial\sigma}{\partial x}\right]f(x) \quad (13)$$

A Lyapunov approach is used for deriving conditions on the control u that will derive the state trajectory to the equilibrium manifold. Let $V(x,\sigma)=\frac{1}{2}\sigma^T(x)\sigma(x)$ be a Lyapunov candidate function. The control u must be chosen so that the time derivative of $V(x,\sigma)$ is negative definite for $\sigma\neq 0$. To this end, consider

$$\dot{V}(x,\sigma)=\frac{1}{2}\dot{\sigma}^T\sigma+\frac{1}{2}\sigma^T\dot{\sigma}=\sigma^T\dot{\sigma}$$

$$=\sigma^T\left\{\frac{\partial\sigma}{\partial x}f+\frac{\partial\sigma}{\partial x}Bu\right\}$$

$$=\sigma^T\left\{\frac{\partial\sigma}{\partial x}f+\frac{\partial\sigma}{\partial x}B(u_{eq}+u_N)\right\}$$

$$=\sigma^T\frac{\partial\sigma}{\partial x}Bu_N \quad (14)$$

where we have suppressed specific x dependency. Choose u_N so that

$$\dot{V}(x,\sigma)=\sigma^T\frac{\partial\sigma}{\partial x}Bu_N$$

$$= -\sigma^T \frac{\partial \sigma}{\partial x} B \left[\frac{\partial \sigma}{\partial x} B \right]^{-1} P\sigma$$

$$= -\sigma P \sigma < 0 \qquad (15)$$

where $P > 0$

3. Control of Container Crane

3.1 Modeling of a Crane

Consider the container crane system shown in Fig. 1.

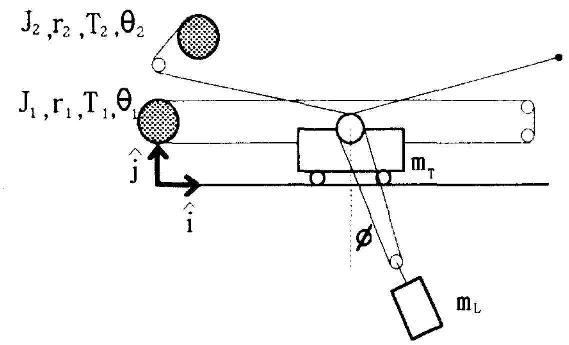

Fig. 1 Schematic of a Container Crane

The following are assumed. (i) The container has a plane motion, i.e. the container motion is restricted to the vertical plane. (ii) The flexibility of the structure is neglected. (iii) All mechanical friction and electrical damping are neglected. (iv) The container is assumed to be a point mass.

Using the Lagrange equation, the equations of motion with the above assumptions are obtained as follows.

$$(J_1 + (m_T + m_L) r_1^2)\ddot{\theta}_1 + \frac{1}{2} m_L r_1 r_2 \ddot{\theta}_2 \sin\phi + \frac{1}{2} m_L r_1 r_2 \theta_2 \ddot{\phi} \cos\phi$$
$$+ m_L r_1 r_2 \dot{\theta}_2 \dot{\phi} \cos\phi - \frac{1}{2} m_L r_1 r_2 \theta_2 \dot{\phi}^2 \sin\phi = T_1 \qquad (16a)$$

$$\frac{1}{2} m_L r_1 r_2 \ddot{\theta}_1 \sin\phi + (J_2 + \frac{1}{4} m_L r_2^2)\ddot{\theta}_2 - \frac{1}{4} m_L r_2^2 \theta_2 \dot{\phi}^2 - \frac{1}{2} m_L g r_2 \cos\phi = T_2 \qquad (16b)$$

$$2 r_1 \ddot{\theta}_1 \cos\phi + r_2 \theta_2 \ddot{\phi} + 2 r_2 \dot{\theta}_2 \dot{\phi} + 2 g \sin\phi = 0 \qquad (16c)$$

Now introduce new variables representing the trolley distance and rope length as

$$x = r_1 \theta_1$$
$$l = \frac{1}{2} r_2 \theta_2 \qquad (17)$$

Define

$$J_1 = m_1 r_1^2, \quad J_2 = m_2 r_2^2, \quad T_1 = F_1 r_1, \quad T_2 = F_2 r_2$$

where m_1 = equivalent mass of the trolley drive, m_2 = equivalent mass of the hoist drive, F_1 = traction force of the trolley motor, F_2 = traction force of the hoist motor. Therefore the following relations hold.

$$\dot{\theta}_1 = \frac{\dot{x}}{r_1}, \quad \ddot{\theta}_1 = \frac{\ddot{x}}{r_1}, \quad \dot{\theta}_2 = 2\frac{\dot{l}}{r_2}, \quad \ddot{\theta}_2 = 2\frac{\ddot{l}}{r_2} \qquad (18)$$

Substituting (18) into (16) the following are obtained

$$(m_1 + m_T + m_L)\ddot{x} + \ddot{l} m_L \sin\phi - m_L l \dot{\phi}^2 \sin\phi + 2 m_L \dot{l} \dot{\phi} + m_L l \ddot{\phi} \cos\phi = F_1 \qquad (19a)$$

$$\frac{1}{2} m_L \ddot{x} \sin\phi + 2(m_2 + \frac{1}{4} m_L)\ddot{l} - \frac{1}{2} m_L l \dot{\phi}^2 - \frac{1}{2} m_L g \cos\phi = F_2 \qquad (19b)$$

$$l \ddot{\phi} + 2 \dot{l} \dot{\phi} + g \sin\phi + \ddot{x} \cos\phi = 0 \qquad (19c)$$

Note that equations (19) are in the form of underactuated mechanical system in Section 2. Specifically the terms in equations (1) and (2) correspond to

$$M_{11} = m_L l^2, \quad M_{12} = M_{21}^T = \begin{bmatrix} m_L l \cos\phi & 0 \end{bmatrix},$$

$$M_{22} = \begin{bmatrix} m_1 + m_T + m_L & m_L \sin\phi \\ \frac{1}{2} m_L \sin\phi & 2(m_2 + \frac{1}{4} m_L) \end{bmatrix},$$

$$C_1 = 2 \dot{l} \dot{\phi} m_L l, \quad C_2 = \begin{bmatrix} -m_L l \dot{\phi}^2 \sin\phi + 2 m_L \dot{l} \dot{\phi} & \frac{1}{2} m_L l \dot{\phi}^2 \end{bmatrix}^T,$$

$$G_1 = m_L g l \sin\phi, \quad G_2 = \begin{bmatrix} 0 & -\frac{1}{2} m_L g \cos\phi \end{bmatrix}^T$$

We now apply the partial feedback linearization. If we take the input to trolley f as

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \overline{M}_{22} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \overline{C}_2 + \overline{G}_2 \qquad (20)$$

where

$$\overline{M}_{22} = \begin{bmatrix} m_1 + m_T + m_L - m_L \cos^2\phi & m_L \sin\phi \\ \frac{1}{2} m_L \sin\phi & 2(m_2 + \frac{1}{4} m_L) \end{bmatrix},$$

$$\overline{C}_2 = \begin{bmatrix} -m_L l \dot{\phi}^2 \sin\phi + 2 m_L \dot{l} \dot{\phi} - 2 \dot{l} \dot{\phi} m_L \cos\phi \\ -\frac{1}{2} m_L l \dot{\phi}^2 \end{bmatrix}, \quad \overline{G}_2 = \begin{bmatrix} -m_L g \cos\phi \sin\phi \\ -\frac{1}{2} m_L g \cos\phi \end{bmatrix}$$

where u_1, u_2 is the additional control input to be decided in the sequel, the resulting partially linearized system is obtained as

$$\ddot{\phi} = -\frac{1}{l}(2 \dot{l} \dot{\phi} + g \sin\phi) - \frac{\cos\phi}{l} u_1$$
$$\ddot{x} = u_1$$
$$\ddot{l} = u_2 \qquad (21)$$

Now we follow the procedures suggested in subsection 2.2. The residual sway at the end of first stage can be assumed to be small. Therefore linearize (21) around the equilibrium point, then the following is obtained.

$$\dot{x} = Ax + Bu$$
$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -g/l(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1/l(t) & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \qquad (22)$$

We define a set of sliding manifold in the state space passing the origin, and recall the assumption that $\frac{\partial \sigma}{\partial x} B(x)$ is nonsingular.

$$\sigma = C\tilde{x} \qquad (23)$$

where $\sigma \in R^2$, $\tilde{x} = x - x_d \in R^6$, $C \in R^{2\times 6}$ is a real-valued gain matrix selected so that $s = 0$ defines a stable manifold in the state space with the desired dynamics.

To complete the controller design, we first compute the equivalent control for the equation (22).

$$u_{eq} = -(CB)^{-1}CA$$

Then, choose the reaching control as following:

$$u_N = (CB)^{-1}\hat{u}_N$$
$$= -(CB)^{-1}P\sigma$$

where $P > 0$

4. Simulations

In this section, we present a series of simulations with the proposed controller: combined partial feedback linearization and VSC. The parameters used in the simulations are as follows: $m_1 = 10kg$, $m_2 = 10kg$, $m_t = 30kg$, $m_L = 100kg$, $g = 9.81m/s^2$. The trolley is assumed to have reached its desired position ($x_d = 5m$). It is assumed that the residual sway angle is $5°$, and the rope length changes from 3m to 5m. It is shown in Fig. 2 that the swing motion dissipates in 3 seconds.

5. Conclusions

A combined nonlinear control techniques, feedback lineaization and variable structure control, for underactuated mechanical systems was investigated. The partial feedback linearization decouples the closed loop system, and linearizes the active joint variables thus brings the whole system to a partially linear one. The second stage VSC performs robustness in the presence of disturbances. The developed algorithm was applied to the residual sway control of a container crane system.

후기

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References

[1] F. Boustany and B. d'Andrea-Novel, "Adaptive Control of Noncompletely Controlled Mechanical Systems Using Dynamic Feedback Linearization and Estimation Design," *Int. J. Adaptive Control and Signal Processing*. Vol. 6, pp.589-610, 1992.

[2] K. S. Hong, S. C. Sohn and M. H. Lee, "Sway Control of a Container Crane (Part Ⅰ): Modeling, Control Strategy, Error Feedback Control via Reference Velocity Profiles," *J. of Control, Automation and Systems Engineering*, Korea, Vol. 3, No. 1, pp. 23-31, 1997a.

[3] K. S. Hong, S. C. Sohn and M. H. Lee, "Sway Control of a Container Crane (Part Ⅱ): Regulation of the Pendulum Motion through Patterning the Trolley Moving Velocity," *J. of Control, Automation and Systems Engineering*, Korea, Vol. 3, No. 2, pp. 132-138, 1997b.

[4] M. W. Spong,, "Energy Based Control of a Class of Underactuated Mechanical Systems," *Proc. 13th IFAC World Congress*, San Francisco, USA, pp. 431-435, 1996.

[5] M. Wiklund, A. Kristenson and K. J. Astrom, "A New Strategy for Swinging Up an Inverted Pendulum," *Proc. IFAC Symposium*, Sydney, Australia, pp. 151-154, 1993.

[6] Y. Sakawa and Y. Shindo, "Optimal Control of Container Cranes," *Automatica*, Vol. 18, No. 3, pp. 257-266, 1982.

[7] Y. L. Gu and Y. Xu, "A Normal Form Augmentation Approach to Adaptive Control of Space Robot Systems," *Proc. IEEE Int. Conf. on Robotics and Automation*, Atlanta, GA, Vol. 2, pp. 731-737, 1993.

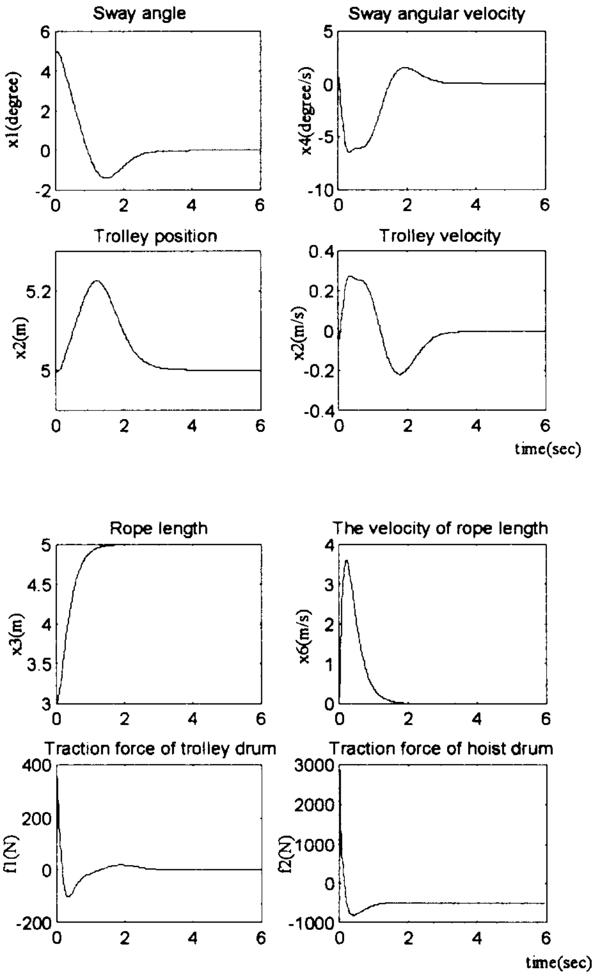

Fig. 2 Sway-Control of a Container Crane