Analytical Model of EEG

by Statistical Mechanics of Neocortical Interaction

*J.M.Choi, **M.C.Whang, *B.H.Bae, *S.Y.Kim, and **C.J.Kim
*Department of Physics, KAIST
**Ergonomics Lab., KRISS

Abstract

Brain potential is described by using Euler Lagrange equation derived from Lagrangian based on SMNI(Statistical Mechanics of Neocortical Interaction). It is assumed that excitatory neuron firing is amplitude-modulated dominantly by the sum of two modes of frequency ω and 2ω. Time series of this neuron firing is numerically calculated. I_L related to low frequency distribution of power spectrum, I_H high frequency, and S(standard deviation) are introduced for the effective extraction of the dynamic property in this simulated brain potential. I_L, I_H, and S are obtained from EEG of 4 persons in rest state and are compared with thoretical results.

It is of importance in various fields related to human well-being such as comfort-pursued industrial design, psychology, medicine to characterize human emotional states by EEG analysis. The pleasant and unpleasant sensation among various emotional states would be demonstrated to be determined in terms of ε and γ parameters estimated by the simulated I_L- I_H - S relations .

I. Introduction

Statistical mechanical approach of neocortical interaction(SMNI)[1-7] allows us to identify models of EEG whose variables and parameters are reasonably identified by ensembles of synaptic and neuronal interaction. SMNI has demonstrated its capability in describing large scale properties of STM(short term memory) and EEG phenomena[8]. The explicit algebraic form of the probability distribution for mesoscopic columnar interactions is driven by a nonlinear threshold factor of the same form used to describe microscopic neuronal interactions, where electrical-chemical synaptic and neuronal parameters lie within their experimentally observed ranges. SMNI also provides strong quantitative support for an accurate intuitive perspective, portraying neocortical interactions as having physical mechanisms that span disparate spatial scales and functional or behavioral phenomena.

The purpose of this letter is twofold: First, we derive a nonlinear partial differential equation from the Lagrangian for mesocolumnar neocortex interaction. This field equation governs the dynamics of the macroscopic quantities measured by EEG. Second, we solve the obtained field equation analytically and numerically and prove that its results are reasonably consistent with experimentally observed phenomena.

II. Model and Simulation

Brain EEG potential can be modelled by [8]

$$\Phi(t) = f(aM^E(t) + bM^I(t)) \tag{1}$$

where M^E and M^I stand for the mesocolumnar averaged excitatory and inhibitory neuron firings. Constants a and b are contribution factors of excitatory and inhibitory neurons respectively. Using Taylor's expansion and a negligibly small trispectrum derived from EEG signal, $\Phi(t)$ may be written by

$$\Phi(t) = (aM^E + bM^I) + \varepsilon(aM^E + bM^I)^2 \tag{2}$$

and let $M^E(t) = cM^I(t)$, where c represents the ratio of excitatory and inhibitory neurons and can be determined for each electrode site. Then we have the following model equation

$$\Phi(t) = \alpha M^E(t) + \varepsilon\alpha^2 (M^E(t))^2 \tag{3}$$

where $\alpha = a + \dfrac{b}{c}$.

The simplest form of Lagrangian L of EEG dynamics may be given by [8]

$$L = \frac{1}{2\sigma^2}(\dot{\Phi} - m)^2 \tag{4}$$

where m and σ are an averaged value and standard deviation of $\Phi(t)$ respectively, and $\dot{\Phi}(t)$ is a first time derivative of $\Phi(t)$. Lagrangian L in Eq.(4) can be rewritten by

$$L(M^E, \dot{M}^E, t) = \frac{1}{2\sigma^2}(\alpha \dot{M}^E + 2\varepsilon\alpha^2 M^E \dot{M}^E - m)^2 \tag{5}$$

Euler-Lagrange equation becomes

$$\ddot{M}^E + 2\varepsilon\alpha(\dot{M}^E)^2 + 4\varepsilon\alpha M^E \ddot{M}^E + (2\varepsilon\alpha \dot{M}^E)^2 M^E + (2\varepsilon\alpha M^E)^2 \ddot{M}^E = 0 \tag{6}$$

Note that Euler-Lagrange equation becomes $\ddot{M}^E = 0$ which reduces to a trivial non-periodic case, if the second order term in Eq.(2) is neglected. The stationary states of brain function are focused in this study. We can assume M^E (t) to be as follows;

$$M^E(t) = h^E + \sqrt{2h^{EE}}(\cos(\omega t) + \gamma\cos(2\omega t)) \qquad (7)$$

where $h^E, h^{EE}(t)$ and γ are an average value, variation of M^E (t) and a factor, respectively. Note that $h^{EE}(t)$ is changed very slowly compared to $e^{i\omega t}$. The typical numerical solution of EEG is obtained by the substitution of Eq.(7) into Eq.(6). The comparison between the experimental EEG data in a certain interval and the numerical solution shows that they are matched well with each other, as shown in Fig.1. Substitution of Eq.(7) into Eq.(6) and taking a time-average of the results yield to an averaged Euler-Lagrange equation:

$$\begin{aligned}
&[(F_3^2 + F_4^2) + 2F_4F_7e^{\frac{G}{2}} + (F_7^2 + F_8^2)e^G]\ddot{G} \\
&+[\frac{1}{2}(F_3^2 + F_4^2) + \frac{3}{2}F_4F_7e^{\frac{G}{2}} + (F_7^2 + F_8^2)e^G]\dot{G}^2 \\
&-[\frac{1}{2}(F_1^2 + F_2^2) + \frac{3}{2}F_2F_5e^{\frac{G}{2}} - (F_5^2 + F_6^2)e^G] = 0
\end{aligned} \qquad (8)$$

where

$$F_1 = -\frac{\omega}{2\gamma}B, F_2 = -\omega B, F_3 = \frac{1}{4\gamma}B, F_4 = \frac{1}{4}B,$$

$$F_5 = -\frac{\omega}{2\gamma^2}B, F_6 = -\frac{3\omega}{2\gamma}B, F_7 = \frac{1}{4\gamma^2}B, F_8 = \frac{1}{2\gamma}B, B = \frac{B_1^2}{B_3}$$

$$B_1 = \sqrt{2}\alpha + 2\sqrt{2}\varepsilon\alpha^2h^E, B_2 = \gamma B_1, B_3 = 2\varepsilon\alpha^2, B_4 = 2\gamma B_3,$$

$$\alpha = (a + \frac{b}{c}), G = \ln(\frac{B_4^2}{B_1^2}h^{EE})$$

We solved Eq.(8) with an initial condition of G(0)=2.1 and $\dot{G}(0) = 1.1$. We consider an asymptotic behavior of G(t) after sufficient time evolution. The frequency f is fixed to be 5 Hz, that is $\omega = 2\pi f = 10\pi$, based on the spectral range of about $f \sim 3f$, which corresponds to the experimental dominant range of about 1 ~ 20 Hz. A factor c in Eq.(3) is fixed to be 5 since excitatory neurons exist about 5 times as many as inhibitory neurons in a general neural system. Other fixed variables are determined by a=2.1, b=1.5, $h^E = 10$ and m = 127.

To extract the hidden dynamical property effectively from the simulated and the experimental EEG, I_L, I_H and S are introduced. I_L is defined by

$$I_L = \frac{\log\left(\frac{P(\omega)}{P(2\omega)}\right)}{\log 2} \qquad (9)$$

which means the slope for range between ω and 2ω in the log-log power spectral space $(\log\omega, \log P(\omega))$. I_H is defined by

$$I_H = \frac{\log\left(\frac{P(2\omega)}{P(3\omega)}\right)}{\log 1.5} \qquad (10)$$

which means the slope for range between 2ω and 3ω. S means the standard deviation of brain potential $\Phi(t)$ and is described by

$$S = \frac{\sqrt{\sum_{i=1}^{N}(\Phi_i - m)^2}}{N} \qquad (11)$$

where N and m are the total sampling number and the average of brain potential $\Phi(t)$ over N, respectively. I_L and I_H can provide some information related to spectral distribution of low frequency and of high frequency ranges respectively from EEG [9]. In general, I_L is different from I_H in EEG. S can provide on some information related to the amplitude from EEG. Note that I_L and I_H are inversely proportional to the correlation dimension[10] and inversely proportional to the negative of Shannon entropy[11] approximately in low and high frequency components of EEG respectively.

The I_L - I_H - S relationship is investigated from the simulated EEG in various conditions of ε and γ parameters and then compared with the experimental EEG. I_L, I_H, and S can be constructed from the simulated EEG as follows.

$$I_L = -\log\left(\frac{B_1}{B_2 + B_3\sqrt{E_1 e^G}}\right)\log 2$$

$$I_H = -\log\left(\frac{\frac{B_2}{\sqrt{E_1 e^G}} + B_3}{B_4}\right)\log 2 \qquad (12)$$

$$S = S_1 e^{G} + S_2 e^{\frac{G}{2}}$$

where $S_1 = \frac{B_3 + B_4}{\sqrt{2}}\sqrt{E_1}, S_2 = \frac{B_1 + B_2}{\sqrt{2}}\sqrt{E_1}, E_1 = \frac{B_1^{\ 2}}{B_4^{\ 2}}$

I_L, I_H, and S are calculated using the solution, G of Eq.(8), and their behaviors of I_L, I_H, and S are investigated with the increment of ε and γ parameters from 0 to 5 by 0.2 unit step. The combined result of I_L - I_H -S relationship is given in Fig. 2.

III. Experiment

Computerized electroencephalograph was used to measure EEG signal from 4 healthy subjects with eye closed in order to eliminate various artifacts and additional unexpected effects. The instrument consists of EEG-amplifier, 8-bit analog to digital converter, and EEG-computer interface, which sampled scalp voltage at 21 electrode at a rate of 204.8Hz. The silver chloride cup electrodes were placed, using a conductive paste, on the 10/20 international electrode system. In order to compare experimental results with simulation, I_L, I_H, and S are calculated from each EEG signal of 21 channels with 1024 sampled data from 4 subjects according to the definition given by Eqs.(9),(10),and (11). The I_L - I_H - S relations from 4 subjects are shown in Fig.3 and very similar to that of the simulation result as shown in Fig.2.

EEG signals from 18 subjects were measured in order to determine emotional states by EEG analysis. Each subject is female or male and has the age within the range of 21 ~27. Four kinds of natural sounds in a woods and four kinds of destructive sounds such as car collision or gun firing were used for both a pleasant and unpleasant feeling, respectively as auditory stimuli. EEG signals were measured before and during each auditory stimulus. The most positive and negative states among all kinds of stimuli were judged by a written questionaire to each subject and only EEG signal in those states was analyzed. I_L, I_H, and S are calculated from EEG signals in four states which are a rest state before pleasant stimulus, a pleasant state, a rest state before unpleasant stimulus and a unpleasant state.

IV. Results

The aspects of I_L, I_H, and S were similar to each other over all electrode sites but here the results at Fp1 were given in Fig. 4. Each figure in (I_L, I_H, S) space of Fig.4 shows 18 points which are calculated from EEG signals of 18 subjects in emotional states by auditory stimuli. The points were widely scattered in the case of no stimulus but they were clustering into a distinctive region marked by ellipse. It was found that the center of the region in the unpleasant state was about (-0.5, -2.7, 19.4) and that in the pleasant state about (3.3, -6.2, 21.6) as shown in Fig.4, which can be characterized by $1<\varepsilon<3$, $0.1<\gamma<0.2$ and $4<\varepsilon<5$, $0.2<\gamma<0.3$ for both the pleasant and the unpleasant state, respectively.

V. Conclusion

The realistic physical modeling of EEG which is applicable into the characterization of emotion was demonstrated in this study. Further developments in this model are needed. This model may be useful in the aspect of possible explanation of the brain function such as emotional state.

Acknowledgment

This research is supported by G7-Project from the Ministry of Science and Technology, Korea Science and Engineering Foundation grant(961-0210-057-2), and Biotech 2000 Project.

Reference

[1] L. Ingber, Physica D, 5, **83**(1982)

[2] L. Ingber, Phys. Rev. A. **28**, 395(1983)

[3] L. Ingber, Phys. Rev. A. **31**, 1183(1985)

[4] L. Ingber, Phys. Rev. A. **42,** 7057(1990)

[5] L. Ingber, Phys. Rev. A. **44,** 4017(1991)

[6] L. Ingber, Phys. Rev. A. **45**, 2183(1992)

[7] L. Ingber, Phys. Rev. E. **49,** 4652(1994)

[8] L. Ingber, *in Neocortical Dynamics and Human EEG Rhythms* edited by P.L.Nunez(Oxford University, New York, 1995),p.628.

[9] J.M.Choi, B.H.Bae and S.Y.Kim, JKPS(accepted) (1997)

[10] A.R.Osborne and A.Provensale, Physica D, **35**,357(1989)

[11] C.Beck and F.Schlogl, *Thermodynamics of chaotic systems,* p.44-64, p.116, p.146-p.157, Cambridge University Press, 1993.

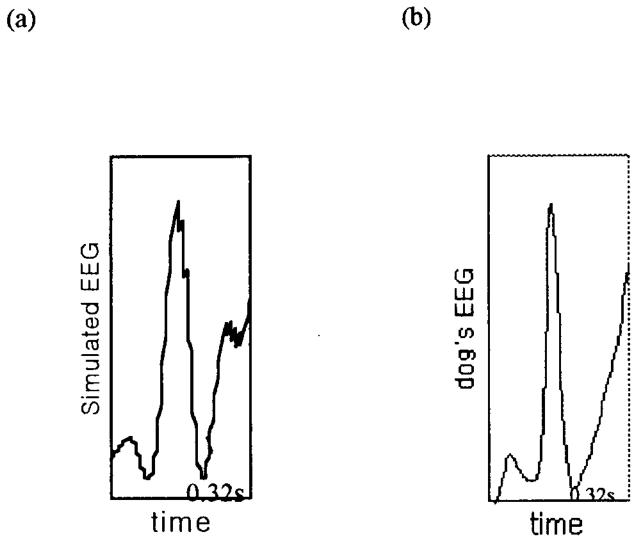

Figure 1. Comparison of the simulated EEG(a) with the experimental EEG from the dog's deep sleep state(b); The unit of vertical axis is arbitrary

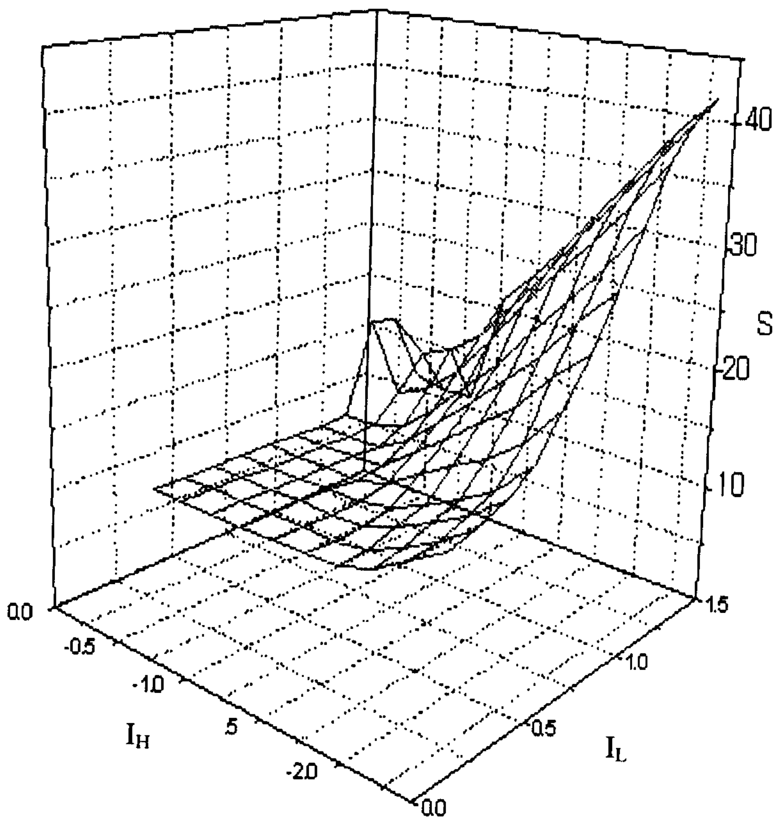

Figure 2. I_L - I_H -S relation for the simulated EEG signal

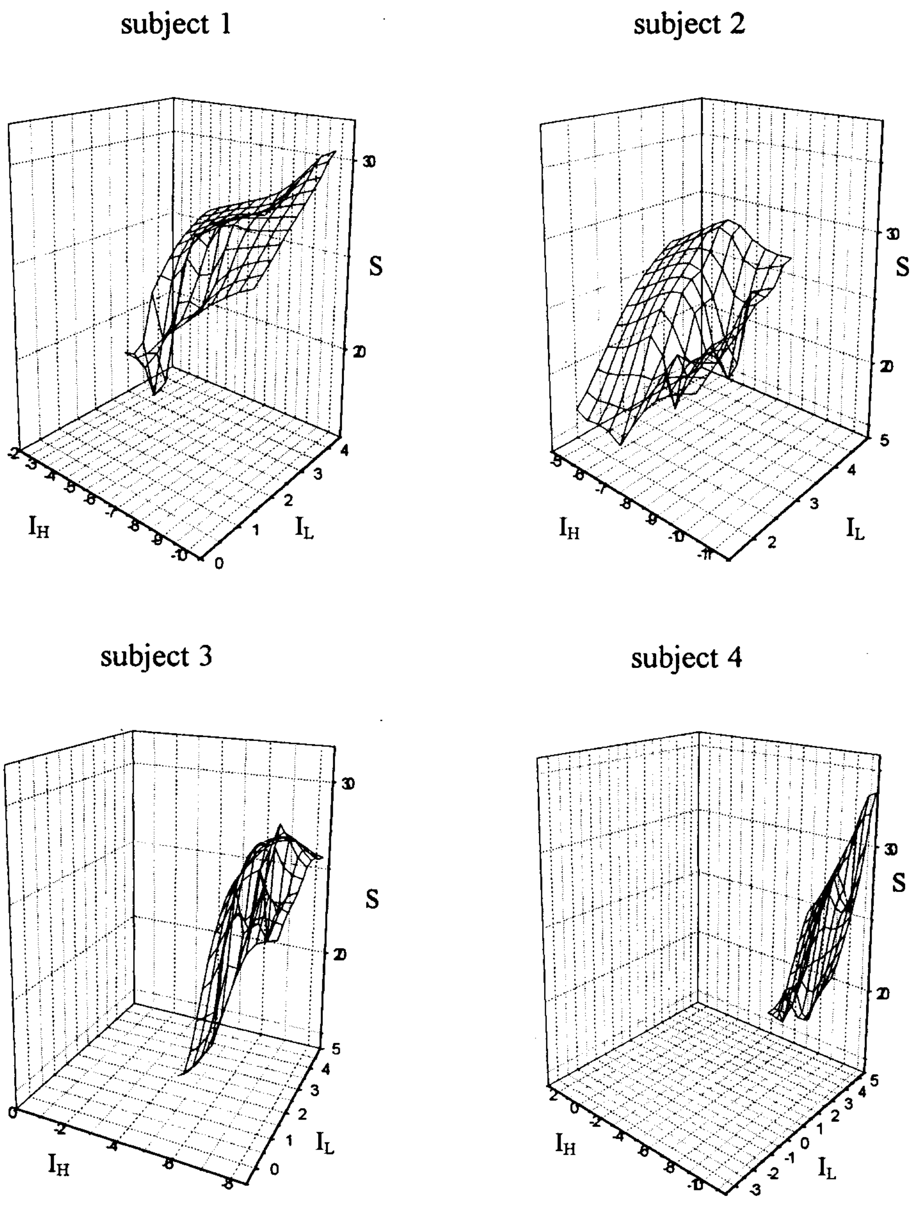

Figure 3. I_L - I_H -S relation for the experimental EEG signals of 4 subjects

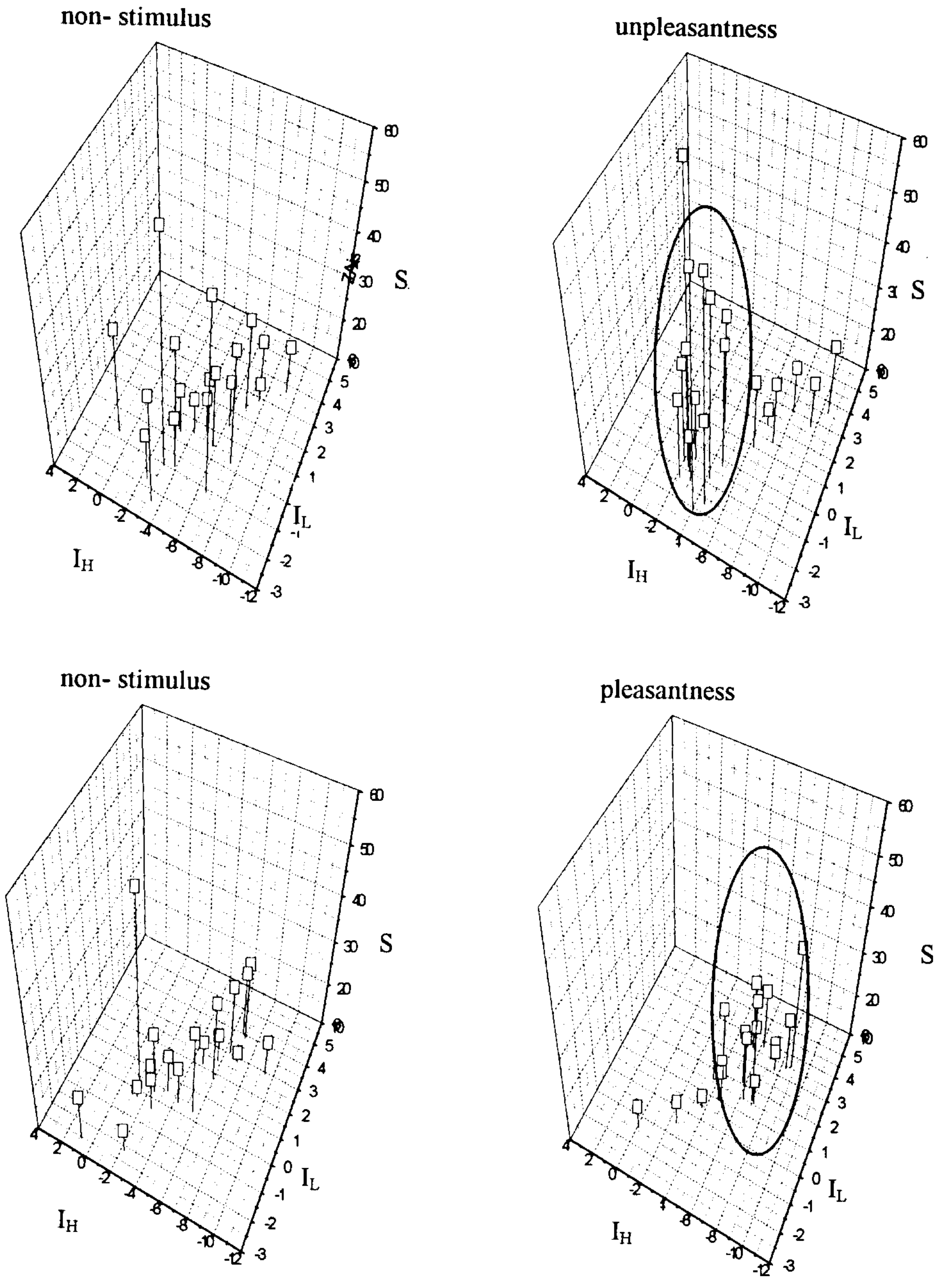

Figure 4. I_L - I_H -S relationship for the experimental EEG signals of 18 subjects at each electrode site of Fp1 in both unpleasant and pleasant states during auditory stimuli