폐-루우프 피이드백에 기준한 SM-MF 제어기를 이용한 다기 전력계통안정기 설계 : Part 3 이 상 성 이 박종근 서울대학교 전기공학부

# Design of Multimachine Power System Stabilizer using CLF-based SM-MF Controller : Part 3

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## [Abstract]

In this paper, the sliding mode-model following(SM-MF) power system stabilizer(PSS) including closed-loop feedback(CLF) for single machine system is extended to multimachine system. Simulation results show that the SM-MF multimachine stabilizer is able to achieve asymptotic tracking error between the reference model state and the controlled plant state at different initial conditions.

Keywords: Sliding Mode-Model Following, Closed Loop Feedback, Power System Stabilizer

### 1. Introduction

To design the PSS with better performance, the sliding mode-model following(SM-MF) by K. K. D. Young[1] has been applied to the PSS for an uncertain synchronous generator system[2]. And a sliding mode-model following(SM-MF) power system stabilizer(PSS) including closed-loop feedback(CLF) has been proposed for an uncertain generator system with voltage regulator and exciter for a single machine to the infinite bus system[3]. In this paper, a power system stabilizer(PSS) for single machine system is extended to multimachine system by using a SM-MF with CLF.

#### 2. Multimachine model

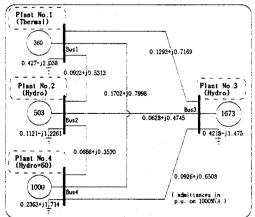


Fig. 1 Three-machine/infinite busbar system.

For three-machine/infinite busbar system, each plant in Fig. 1 is represented by a single equivalent machine with machines

1, 2 and 3, rated 360MVA(Thermal), 503MVA(Hydro) and 1673MVA(Hydro), respectively. And Plant 4 effectively represents an infinite busbar system[4,5].

The 12-th order state equation for a reference model can be expressed as

$$x_{n} = \left[\Delta \delta_{n1}, \Delta \omega_{n1}, \Delta e_{qn1}, \Delta e_{pon1}, \\ \Delta \delta_{n2}, \Delta \omega_{n2}, \Delta e_{qn2}, \Delta e_{pon2}, \\ \Delta \delta_{n3}, \Delta \omega_{n3}, \Delta e_{n3}, \Delta e_{pon3}, \Delta e_{pon3}\right]^{T}$$

$$(1)$$

where  $\Delta \delta_{\bullet}(t)$  is the torque angle for model,  $\Delta \omega_{\bullet}(t)$  the angular velocity for model,  $\Delta e'_{\bullet \bullet}(t)$  the q-axis component of voltage behind transient reactance for model and  $\Delta e_{r_{D\bullet}}(t)$  the equivalent excitation voltage for model.

The 12-th order state equation for the controlled plant can be expressed as

$$x_{p} = \left[\Delta \delta_{p_{1}}, \Delta \omega_{p_{1}}, \Delta e_{\varphi_{1}}^{\prime}, \Delta e_{\varphi_{2}}^{\prime}, \Delta e_{r_{D_{p}}}, \right.$$

$$\Delta \delta_{p_{2}}, \Delta \omega_{p_{1}}, \Delta e_{\varphi_{2}}^{\prime}, \Delta e_{r_{D_{p}}}^{\prime}, \left. \Delta e_{r_{D_{p}}}^{\prime}, \Delta e_{r_{D_{p}}}^{\prime}, \Delta e_{r_{D_{p}}}^{\prime}, \Delta e_{r_{D_{p}}}^{\prime}, \Delta e_{r_{D_{p}}}^{\prime} \right]^{T}$$

$$(2)$$

where  $\Delta \delta_{r}(t)$  is the torque angle for plant,  $\Delta \omega_{r}(t)$  the angular velocity for plant,  $\Delta e_{r}'(t)$  the q-axis component of voltage behind transient reactance for plant and  $\Delta e_{rp_{r}}(t)$  the equivalent excitation voltage for plant.

# 3. Multimachine SM-MF controller including CLF

The state equation for a reference model can be expressed as

$$\dot{x}_{n}(t) = A_{n} \cdot x_{n}(t) + B_{n} \cdot u_{n}(t) \tag{3}$$

where  $A_{\underline{a}}$  is a  $n \times n$  system matrix for model,  $B_{\underline{a}}$  a  $n \times m$  control vector for model,  $x_{\underline{a}} \in R^*$  a state vector for model and  $u_{\underline{a}} \in R^*$  a control input for model.

The control input of a reference model with reference input vector  $r_{\bullet}(t)$  can be expressed as

$$u_{-}(t) = -K_{-} \cdot x_{-}(t) + r_{-}(t) \tag{4}$$

where 
$$K_{-} = R^{-1} \cdot B_{-}^{T} \cdot P_{-}$$
 (5)

is a  $m \times n$  feedback gain vector for model.

P is the symmetric positive definite solution of

$$P_{n} \cdot A_{n} + A_{n}^{T} \cdot P_{n} - P_{n} \cdot B_{n} \cdot R_{n}^{-1} \cdot B_{n}^{T} \cdot P_{n} + Q_{n} = 0$$
(6)

 $Q_n$  and  $R_n$  are positive definite matrices chosen by the designer for model. And  $r_n \in \mathbb{R}^n$  is a reference input vector for model.

By substituting (4) into (3), the closed-loop feedback system for a reference model is

$$\dot{x}_{-}(t) = (A_{-} - B_{-} \cdot K_{-}) \cdot x_{-}(t) + B_{-} \cdot r_{-}(t) \tag{7}$$

Let 
$$A_{\alpha} = A_{\alpha} - B_{\alpha} \cdot K_{\alpha}$$
 (8)

The proposed state equation for a reference model including CLF can be reformed as

$$\dot{x}_{\alpha}(t) = A_{\alpha} \cdot x_{\alpha}(t) + B_{\alpha} \cdot r_{\alpha}(t) \tag{9}$$

where  $A_{in}$  is a  $n \times n$  system matrix including feedback gain for model.

The state equation for the controlled plant with internal parameter variations can be formed as

$$\dot{x}_{p}(t) = (A_{p} + \Delta A_{p}) \cdot x_{p}(t) + (B_{p} + \Delta B_{p}) \cdot u_{p}(t)$$

$$= \widetilde{A}_{p} \cdot x_{p}(t) + \widetilde{B}_{p} \cdot u_{p}(t)$$
(10)

where  $\widetilde{A}_r = A_r + \Delta A_r$ , is a  $n \times n$  system matrix with the parameter variations for plant and  $\widetilde{B}_r = B_r + \Delta B_r$ , a  $n \times m$  control vector with the parameter variations for plant. And  $x_r \in R^*$  is a state vector for plant and  $u_r \in R^*$  a control input for plant.

The control input of the controlled plant with sliding mode control input can be expressed as

$$u_{p}(t) = -K_{p} \cdot x_{p}(t) + u_{x,q}(t) \tag{11}$$

where 
$$K_{r} = R_{r}^{-1} \cdot \widetilde{B}_{r}^{T} \cdot P_{r}$$
 (12)

is a  $m \times n$  feedback gain vector for plant.

P is the symmetric positive definite solution of

$$P_{\bullet} \cdot \widetilde{A}_{\bullet} + \widetilde{A}_{\bullet}^{\mathsf{T}} \cdot P_{\bullet} - P_{\bullet} \cdot \widetilde{B}_{\bullet} \cdot R_{\bullet}^{-1} \cdot \widetilde{B}_{\bullet}^{\mathsf{T}} \cdot P_{\bullet} + Q_{\bullet} = 0$$
 (13)

 $Q_r$ , and  $R_r$  are positive definite matrices chosen by the designer for plant. And  $u_{xx} \in R^{-1}$  is a sliding mode control input vector for plant.

By substituting eq.(11) into eq.(10), the proposed state equation for the controlled plant including CLF can be expressed as

$$\dot{x}_{s}(t) = \left(\tilde{A}_{s} - \tilde{B}_{s} \cdot K_{s}\right) \cdot x_{s}(t) + \tilde{B}_{s} \cdot u_{ss}(t) \tag{14}$$

Let 
$$\widetilde{A}_{b} = \widetilde{A}_{\bullet} - \widetilde{B}_{\bullet} \cdot K_{\bullet}$$
 (15)

The proposed state equation for the controlled plant including CLF can be reformed as

$$\dot{x}_{s}(t) = \widetilde{A}_{bs} \cdot x_{s}(t) + \widetilde{B}_{s} \cdot u_{ss}(t) \tag{16}$$

where  $\tilde{A}_{n}$  is a  $n \times n$  system matrix including feedback gain with the parameter variations for plant.

The error vector and the differential error vector are

$$e(t) = x_{-}(t) - x_{-}(t)$$
 (17)

$$\dot{e}(t) = \dot{x}_{p}(t) - \dot{x}_{p}(t) \tag{18}$$

The limits of the error vector and the differential error vector are

$$\lim e(t) = 0 \tag{19}$$

$$\lim_{t \to \infty} \dot{e}(t) = 0 \tag{20}$$

From eq.(9), eq.(16) and eq.(18), we get

$$\dot{e}(t) = \dot{x}_{\mu}(t) - \dot{x}_{\mu}(t)$$

$$= A_{tm} \cdot x_{\mu}(t) + B_{\mu} \cdot r_{\mu}(t) - \widetilde{A}_{tr} \cdot x_{\mu}(t) - \widetilde{B}_{\mu} \cdot u_{sst}(t)$$
(20)

$$x_{\bullet}(t) = e(t) + x_{\bullet}(t) \tag{21}$$

By substituting eq.(21) into eq.(20), we get

$$\dot{e}(t) = A_{t-} \cdot e(t) + \left[ A_{t-} - \widetilde{A}_{t-} \right] \cdot x_{r}(t) + B_{-} \cdot r_{-}(t) - \widetilde{B}_{r} \cdot u_{SM}(t)$$
 (22)

Suppose the sliding mode exists on all hyperplanes. Then, during sliding, the switching surface vector in the error state space can be expressed as

$$s(e(t)) = G^{\tau} \cdot e(t) \Rightarrow 0 \tag{23}$$

$$\dot{s}(e(t)) = G^{\tau} \cdot \dot{e}(t) \Rightarrow 0 \tag{24}$$

In the above eq.(23), the algorithm of the sliding surface gain  $G^r$  is found in references[3,4]. To determine a control law that keeps the system on  $s(e(r)) \Rightarrow 0$ , we introduce the Lyapunov function

$$V(e(t)) = s^{2}(e(t))/2$$
 (25)

The time derivative of V(e(t)) is given by

$$V(e(t)) = s(e(t)) \cdot \dot{s}(e(t)) \tag{26}$$

$$=G^{\tau} \cdot e(t) \cdot G^{\tau} \cdot \dot{e}(t) \tag{27}$$

$$=G^{\tau}\cdot e(t)\cdot G^{\tau}\cdot \left[A_{km}\cdot e(t)+\left[A_{km}-\widetilde{A}_{kp}\right]\cdot x_{p}(t)\right]$$

$$+B_{\omega}\cdot r_{\omega}(t) - \widetilde{B}_{\rho}\cdot u_{xy}(t) \bigg] \leq 0 \tag{28}$$

From eq.(28), the control input vector with switching for the controlled plant can be represented by

$$u_{sol}^{r}(t) \ge \left(G^{\tau} \cdot \widetilde{B}_{s}\right)^{-1} \cdot G^{\tau} \cdot \left[A_{to} \cdot c(t) + \left[A_{to} - \widetilde{A}_{to}\right] \cdot x_{s}(t) + B_{s} \cdot r_{s}(t)\right]$$

$$for \quad G^{\tau} \cdot c(t) > 0$$
(29)

$$u_{sv}^{T}(t) \leq \left(G^{T} \cdot \widetilde{B}_{p}\right)^{-1} \cdot G^{T} \cdot \left[A_{tn} \cdot e(t) + \left[A_{tn} - \widetilde{A}_{tp}\right] \cdot x_{p}(t) + B_{n} \cdot r_{n}(t)\right]$$

$$for \quad G^{T} \cdot e(t) < 0 \tag{30}$$

From eq.(29) and eq.(30), the following control input with sign function for the controlled plant can be reformed

$$u_{s,t}^{n_{gn}}(t) = \left[ SE_{gain} \cdot e(t) + SP_{gain} \cdot x_{p}(t) + SU_{gain} \cdot r_{n}(t) \right] \cdot \mu \cdot sign(s(e(t)))$$
(31)

where  $\mu$  is a bias gain.

$$SE_{sain} := \left(G^{\tau} \cdot \widetilde{B}_{r}\right)^{-1} \cdot G^{\tau} \cdot A_{\kappa m} \tag{32}$$

is an equal error feedback gain.

$$SP_{p,sin} := \left(G^{T} \cdot \widetilde{B}_{p}\right)^{-1} \cdot G^{T} \cdot \left(A_{Km} - \widetilde{A}_{sp}\right)$$
(33)

is an equal plant feedback gain.

$$SU_{soin} := \left(G^{\tau} \cdot \widetilde{B}_{r}\right)^{-1} \cdot G^{\tau} \cdot B_{r} \tag{34}$$

is an equal input gain.

## 4. Data analysis and simulation

The data of a model is found in references [4,5]. In this paper, the values of the  $12 \times 12$  system matrix  $A_{-}$  are decomposed into the 4-block form

$$A_{\mathbf{m}} = \begin{bmatrix} A_{\mathbf{m}|1} & A_{\mathbf{m}|2} \\ A_{\mathbf{m}|2} & A_{\mathbf{m}|2} \end{bmatrix}$$

where

The  $12 \times 3$  control matrix  $B_{\perp}$  is given

The values of  $K_*$  are

$$K_{-} = \begin{bmatrix} 42.80 & 2310 & 557 & 1.2 & 9.7 & -1265 & 53 & 0.08 & -1.01 & -3165 & .499 & 0.1 \\ -74.31 & 8064 & -4.65 & 0.4 & 42.8 & 811 & 2.04 & .075 & -68.8 & -94.34 & 4.11 & .04 \\ 5.80 & 1408 & 189 & .02 & 8.8 & -763 & .08 & .0009 & 22.5 & -586.2 & 2.76 & .08 \end{bmatrix}$$

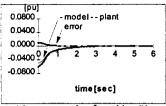
Then for simulations, the controlled plant system matrix and the plant input vector are given by adding the plant parameter uncertainties with reference model.

$$\tilde{A}_{n} = A_{n} + \Delta A_{n} = A_{n} + 10\%$$
 of  $A_{n}$ 

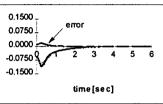
$$\tilde{B}_{a} = B_{a} + \Delta B_{a} = B_{a} + 10\%$$
 of  $B_{a}$ 

The 12 x 3 sliding surface matrix including CLF is obtained as

For the torque angle of machine #1, #2 and #3, the time domain simulations are carried out for 6 sec.



(a) torque angle of machine #1.



(b) torque angle of machine #2.

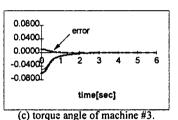


Fig. 2 Torque angle waveforms.

Fig. 2 shows that the proposed multimachine SM-MF PSS is able to achieve asymptotic tracking error between the reference model state and the estimated plant state at different initial conditions.

#### 5. Conclusion

The sliding mode-model following(SM-MF) power system stabilizer(PSS) including closed-loop feedback(CLF) for single-machine power system has been extended to multimachine systems. The multimachine SM-MF PSS has been designed not only to damp out the low frequency oscillations of the power system by including CLF, but also to achieve asymptotic tracking error between the reference model state and the controlled plant state at different initial conditions.

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