길이가 16인 Z_4 위의 Preparata 부호는 연쇄조건을 만족하지 않는다.

양 경 철,○ 임 두 루
한양대학교 전자통신공학과

The Preparata Code of length 16 over Z_4 Does Not Satisfy the Chain Condition[1]

Kyeongcheol Yang and Dooroo Lim
Department of Electronic Communication Engineering
Hanyang University, Sungdong-Ku, Seoul 133-791, Korea

ABSTRACT

In a remarkable paper [3], Hammons et al. showed that, when properly defined, the binary nonlinear Preparata code can be considered as the Gray map of a linear code over Z_4, the so-called Preparata code over Z_4. Recently, Yang and Helleseth [12] considered the generalized Hamming weights $d_r(m)$ for Preparata codes of length 2^m over Z_4 and exactly determined d_r for $r = 0.5, 1.0, 1.5, 2, 2.5$ and 3.0. In particular, they completely determined $d_r(m)$ for any r in the case of $m \leq 6$. In this paper we show that the Preparata code of length 16 over Z_4 does not satisfy the chain condition.

Keywords: linear codes over Z_4, Preparata codes, Kerdock codes, nonlinear codes, weight hierarchy, generalized Hamming weights, minimum support size

I. INTRODUCTION

Let Z_l be the ring of integers modulo l. A linear code $\mathcal{C}$ over Z_4 with blocklength n is an additive subgroup of Z_4^n. The Lee weights of the elements $0, 1, 2, 3$ in Z_4 are $0, 1, 2, 1$, respectively. The Lee weight of a vector $\boldsymbol{a} \in Z_4^n$ is defined to be the sum of the Lee weights of its components. The Gray map $\phi : Z_4 \to Z_2^2$ is defined by $\phi(0) = 00$, $\phi(1) = 01$, $\phi(2) = 11$, and $\phi(3) = 10$. In general, the binary code defined by $C = \phi(\mathcal{C})$ is a nonlinear binary code of length $2n$. Hammons, Kumar, Calderbank, Sloane and Solé [3] have shown that efficient nonlinear codes

[1]This work was supported in part by the Korean Ministry of Information and Communications.

such as Kerdock, Preparata, etc., can be very simply constructed as binary images under the Gray map of linear codes over Z_4.

Let $\mu : Z_4 \to Z_2$ denote the modulo 2 reduction map. We extend μ to $Z_4[x]$ in the natural way. A monic polynomial $g(x) \in Z_4[x]$ is said to be *monic basic irreducible* if $\mu(g(x))$ is a monic irreducible polynomial in $Z_2[x]$. A Galois ring $R_m = GR(4, m)$ of characteristic 4 with 4^m elements is isomorphic to the ring $Z_4[x]/(f(x))$, where $f(x)$ is a monic basic irreducible of degree m. In other words, R_m is an extension of Z_4 of degree m. R_m is a local ring having a unique maximal ideal $M = 2R_m$. Clearly, μ has a natural extension to $R[x]$ and the quotient ring $\mu(R_m) = R_m/M$ is isomorphic to F_{2^m} where F_{2^m} is a finite field with 2^m elements (see [3], [6] for details).

As a multiplicative group, the set R_m^* of units of R_m has the following structure

$$R_m^* \cong Z_{2^m-1} \times \underbrace{Z_2 \times Z_2 \times \cdots \times Z_2}_{m \text{ times}}.$$

Let $\beta \in R_m^*$ be a generator for the multiplicative cyclic subgroup $\cong Z_{2^m-1}$ contained within R_m^*. Let $\mathcal{T}_m = \{0, 1, \beta, \ldots, \beta^{2^m-2}\}$. It can be shown that every element $z \in R_m$ can be expressed uniquely as

$$z = A + 2B, \qquad A, B \in \mathcal{T}_m. \tag{1}$$

It can be also shown that $\alpha = \beta \pmod 2$ is a primitive element in F_{2^m}, thus $\mu(\mathcal{T}_m) = F_{2^m}$.

Let $\mathcal{P}_m$ be the code over Z_4, whose parity-check matrix is given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & \beta & \beta^2 & \cdots & \beta^{2^m-2} \end{bmatrix}. \tag{2}$$

The quaternary code $\mathcal{P}_m$ is called the Preparata code of length 2^m over Z_4. In Hammons, Kumar, Calderbank, Sloane and Solé [3], it is shown that if m is odd, then $\mathcal{P}_m$ has minimum Lee weight 6 and its Gray map $P_m = \phi(\mathcal{P}_m)$ gives a $(2^{m+1}, 2^{2^{m+1}-2m-2}, 6)$ binary nonlinear code and so it is optimal. If m is even, then $\mathcal{P}_m$ has minimum Lee weight 4 and its Gray map $P_m = \phi(\mathcal{P}_m)$ gives a $(2^{m+1}, 2^{2^{m+1}-2m-2}, 4)$ binary nonlinear code. Note that the dual of the Preparata code $\mathcal{P}_m$ over Z_4 is the Kerdock code $\mathcal{K}_m$ over Z_4.

In this paper, we consider the chain condition for the Preparata code $\mathcal{P}_4$ of length 16 over Z_4. The weight hierarchy of $\mathcal{P}_3$ is completely known in [1], and it is well-known that $\mathcal{P}_3$ satisfies the chain condition. The weight hierarchy of $\mathcal{P}_4$ is also completely known in [12]. In this paper, we show that $\mathcal{P}_4$ does not satisfy the chain condition.

The paper is organized as follows. In Section II, preliminaries are given including several lemmas. The concepts of generalized Hamming weights, weight hierarchy, and chain condition are introduced in Section III. In Section IV, we show that $\mathcal{P}_3$ satisfy the chain condition, but $\mathcal{P}_4$ does not satisfy the chain condition. Finally, we give concluding remarks in Section V.

II. Preliminaries and Notation

From now on, we will normally use the capital letters X, Y, A, B, etc., to denote elements in $\mathcal{T}_m$, and the small letters x, y, a, b to represent their corresponding projections modulo 2 in F_{2^m}. For example, we use $a = \mu(A)$, $x_i = \mu(X_i) \in F_{2^m}$ for $A, X_i \in \mathcal{T}_m$, respectively. For a vector $\mathbf{c} \in Z_4^n$, denote its Hamming weight and Lee weight by $w_H(\mathbf{c})$ and $w_L(\mathbf{c})$, respectively.

The module $\mathcal{D}$ over Z_4, generated by $\mathbf{c}_1, \mathbf{c}_2, \ldots \mathbf{c}_s \in Z_4^n$, is denoted by $\mathcal{D} = <\mathbf{c}_1, \mathbf{c}_2, \ldots \mathbf{c}_s>$. Two codes $\mathcal{C}_1$ and $\mathcal{C}_2$ of length n are *equivalent* if there is a permutation σ on $\{1, 2, \ldots, n\}$ such that

$$\mathcal{C}_2 = \{\sigma(\mathbf{c}) \mid \mathbf{c} \in \mathcal{C}_1\}$$

where $\sigma(c_1, c_2, \ldots, c_n) = (c_{\sigma(1)}, c_{\sigma(2)}, \ldots, c_{\sigma(n)})$.

Let $(c_X)_{X \in \mathcal{T}_m}$ be a codeword of the Preparata code $\mathcal{P}_m$. Then it must satisfy

$$\sum_{X \in \mathcal{T}_m} c_X = 0$$

and

$$\sum_{X \in \mathcal{T}_m} c_X X = 0.$$

These relations give an invariant property to $\mathcal{P}_m$.

Lemma 1 ([3]) *The Preparata code defined in (2) is invariant under the doubly transitive group of "affine" permutations of the form*

$$X \to (AX + B)^{2^m}$$

where $A, B \in \mathcal{T}_m$ and $A \neq 0$.

Since any element in R_m can be uniquely represented as in (1), it is very natural to consider addition of elements in $\mathcal{T}_m$. A very useful lemma for adding the elements in $\mathcal{T}_m$ is in the following.

Lemma 2 ([4]) *Let $X_1, X_2, \ldots, X_k \in \mathcal{T}_m$. Then*

$$\sum_{i=1}^{k} X_i = A + 2B, \quad A, B \in \mathcal{T}_m$$

where

$$A = \sum_{i=1}^{k} X_i + 2 \sum_{1 \leq i < j \leq k} \sqrt{X_i X_j}$$

and

$$B = \sum_{1 \leq i < j \leq k} \sqrt{X_i X_j} \ (mod\, 2).$$

The next lemma shows how an equation over R_m can be represented as two equivalent equations over F_{2^m}.

Lemma 3 ([4]) *Let* $\mathbf{e} = (e_X)_{x\in\mathcal{T}_m}$ *and let* $E_j = \{X \mid e_X = j\}$ *for* $j = 0, 1, 2, 3$. *The equation given by*

$$\sum_{X\in\mathcal{T}_m} e_X X = A + 2B, \quad A, B, \in \mathcal{T}_m, e_X \in Z_4$$

is equivalent to the two binary equations

$$a = \sum_{X\in E_1\cup E_3} x$$

$$b^2 = \sum_{X\in E_2\cup E_3} x^2 + \sum_{\substack{X,Y\in E_1\cup E_3 \\ X<Y}} xy$$

where $<$ *is any ordering of the elements in* $\mathcal{T}_m$.

III. Generalized Hamming Weights of a Linear Code over Z_4

Wei [9] introduced the notion of generalized Hamming weights and weight hierarchy for linear codes and showed that the weight hierarchy of a linear code characterizes its performance on the wire-tap channel of type II. It is possible to extend his concepts to linear codes over Z_4. Ashikhmin [1] showed that the weight hierarchy of a linear code over Z_4 also characterizes its performance on the wire-tap channel of type II.

Let $\mathcal{C}$ be an $[n, k]$ linear code over Z_4, and let $\mathcal{B} \subset \mathcal{C}$ be a subcode. Then the support of $\mathcal{B}$ is defined as follows:

$$\chi(\mathcal{B}) = \{i \mid c_i \neq 0 \text{ for some } (c_1, \ldots, c_n) \in \mathcal{B}\}.$$

For any r, where $0 \leq r \leq k$ and $2r$ is an integer, the rth generalized Hamming weight of $\mathcal{C}$ is defined as

$$d_r(\mathcal{C}) := \min\{|\chi(\mathcal{B})| : \mathcal{B} \text{ is a submodule of } \mathcal{C} \text{ with } |\mathcal{B}| = 4^r\}.$$

Conventionally, d_0 is assumed to be 0. The weight hierarchy of $\mathcal{C}$ is the sequence given by $\{d_r(\mathcal{C})\}_{r=0.5}^{k}$, where $2r$ is an integer.

There are numerous recent results on the generalized Hamming weights for codes over a finite field, in particular, for binary codes (see [8] for a survey of recent results). However, for codes over Z_4 very little is known. Ashikhmin [1] presented the duality theorem on the generalized Hamming weight d_r for linear codes over Z_4, and determined $d_{0.5}$ and d_1 for the Kerdock code of length 2^m over Z_4. Yang, Helleseth, Kumar and Shanbhag [11] gave a lower bound on d_r for Kerdock codes over Z_4 and exactly determined d_r for $r = 0.5, 1.0, 1.5, 2$, and 2.5. In case of

Kerdock codes of length 2^{2m}, they also determined d_r for any r, where $0 \le r \le m$ and $2r$ is an integer. Recently, Yang and Helleseth [12] also considered the generalized Hamming weights for Preparata codes of length 2^m over Z_4 and exactly determined d_r for $r = 0.5, 1.0, 1.5, 2, 2.5$ and 3.0.

In order to determine the generalized Hamming weights of a linear code over Z_4, a tighter lower bound is essential. The following lemma is very useful to get a lower bound on the rth generalized Hamming weight $d_r(\mathcal{C})$.

Lemma 4 ([11]) *Let $\mathcal{C}$ be an $[n,k]$ linear code over Z_4, and let $\mathcal{B} \subset \mathcal{C}$ be a submodule which has 4^r codewords and support size s_r. Then we have*

$$s_r = \frac{1}{4^r} \sum_{\mathbf{c} \in \mathcal{B}} w_L(\mathbf{c})$$

where $w_L(\cdot)$ is the Lee weight of $\mathbf{c}$.

The concept of the chain condition was introduced by Wei and Yang [10] in their study of the weight hierarchy of product codes. An $[n,k]$ code $\mathcal{C}$ over Z_4 is said to *satisfy the chain condition* if there is an increasing sequence $\{\mathcal{D}_r\}_{r=0.5}^{k}$ such that $\mathcal{D}_r$ is an r-dimensional submodule of $\mathcal{C}$ achieving $d_r(C)$ and $\mathcal{D}_r \subset \mathcal{D}_{r+0.5}$ for all r. Or equivalently, $\mathcal{C}$ satisfies the chain condition if

(a) there exists a permutation of the coordinates, and

(b) there is an increasing sequence $\{\mathcal{D}_r\}_{r=0.5}^{k}$ of submodules of $\mathcal{C}$ such that $\dim(\mathcal{D}_r) = r$, $\mathcal{D}_r \subset \mathcal{D}_{r+0.5}$ and $\chi(\mathcal{D}_r) = \{1, 2, \ldots, d_r\}$ for all r.

IV. The Preparata Code $\mathcal{P}_4$ of Length 16 Does Not Satisfy the Chain Condition

Note that the projection $\mu(\mathcal{P}_m)$ of the Preparata code $\mathcal{P}_m$ is exactly the extended binary Hamming code $\mathcal{H}_m$ of length 2^m, whose parity check matrix is given by

$$\mu(H) = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & \alpha & \alpha^2 & \cdots & \alpha^{2^m-2} \end{bmatrix}.$$

Thus for any codeword $\mathbf{c}$ in $\mathcal{P}_m$ such that $2\mathbf{c} \neq 0$, the codeword $2\mathbf{c}$ has even Hamming weight of at least 4, so has Lee weight of at least 8.

Theorem 5 *Let m, $m \ge 3$, be an integer and let $\mathcal{P}_m$ be the Preparata code of length 2^m over Z_4. Then we have*

$$d_r \ge d_{\lceil r \rceil}(\mathcal{H}_m)$$

where $\mathcal{H}_m$ is the extended binary Hamming code of length 2^m and $\lceil r \rceil$ is the least integer greater than or equal to r.

Proof: We note that every module of dimension r has $\lceil r \rceil$ linearly independent vectors of the form $2\boldsymbol{c}$ over Z_4. The theorem follows from the fact that $2\boldsymbol{c}$ is just a codeword of $\mathcal{H}_m$, multiplied by 2 (mod 4). □

Remark 6 The extended Hamming code $\mathcal{H}_m$ is the $[2^m, 2^m - m - 1, 4]$ binary code. The weight hierarchy of the extended Hamming code $\mathcal{H}_m$ is well-known in the following [9]:

$$\{d_r(\mathcal{H}_m) \,|\, 1 \le r \le 2^m - m - 1\} = \{2, 3, \ldots, 2^m\} \setminus \{2^s + 1 \,|\, 0 \le s \le m - 1\}.$$

It may be needed to systematically check a number of submodules spanned by at least $\lceil r \rceil$ generators in order to determine $d_r(m)$ and the check the chain condition. The following simple tests are very useful to check that a linear combination $\boldsymbol{c}$ of generators is a codeword in $\mathcal{P}_m$:

T.1 For any codeword $\boldsymbol{c} = (c_X)_{X \in \mathcal{T}_m} \in \mathcal{P}_m$, we have $\sum_{X \in \mathcal{T}_m} c_X = 0$ (mod 4).

T.2 For any two distinct codewords $\boldsymbol{c_1}, \boldsymbol{c_2} \in \mathcal{P}_m$, we have $w_H(\boldsymbol{c_1} - \boldsymbol{c_2}) \ge 4$ and $w_H(2\boldsymbol{c_1} - 2\boldsymbol{c_2}) \ge 4$.

T.3 No vector of the form $(1\,1\,3\,3)$ with support size 4 is a codeword in $\mathcal{P}_m$.

T.4 A vector of the form $(1\,1\,1\,1)$ with support size 4 can be a codeword of $\mathcal{P}_m$ for even m, but not a codeword of $\mathcal{P}_m$ for odd m.

T.5 If m is even, $w_L(\boldsymbol{c}) \ge 4$ for any nonzero codeword $\boldsymbol{c} \in \mathcal{P}_m$. If m is odd, $w_L(\boldsymbol{c}) \ge 6$ for any nonzero codeword $\boldsymbol{c} \in \mathcal{P}_m$.

Let β be a generator for R_3^* satisfying $\beta^3 + 2\beta^2 + \beta + 3 = 0$. Then the Preparata code $\mathcal{P}_3$ has the parity-check matrix H given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 & 3 & 3 & 3 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 & 1 & 1 \end{bmatrix}. \tag{3}$$

Note that $HH^t = 0$ (mod 4), where H^t is the transpose of H. This implies that the Preparata code $\mathcal{P}_3$ of length 8 is self-dual, so it is the Kerdock code $\mathcal{K}_3$ of length 4 whose generator matrix is given by H in (3). In particular, $\mathcal{P}_3$ is called the *octacode* in [2]. It is well-known [3] that its image under the Gray map is the Nordstrom-Robinson code.

Proposition 7 ([1]) *The weight hierarchy of the Preparata code $\mathcal{P}_3$ of length 8 over Z_4 is given by*

$$\{4,\ 5,\ 6,\ 6,\ 7,\ 7,\ 8,\ 8\}.$$

Proposition 8 *The Preparata code $\mathcal{P}_3$ of length 8 over Z_4 satisfies the chain condition.*

Proof: Consider the generator matrix (or parity-check matrix) H of $\mathcal{P}_3$, given in (3). Let c_i be the $(4-i)$th row of H. Define an r-dimensional submodule $\mathcal{D}_r$ of $\mathcal{P}_3$ as follows:

$$\mathcal{D}_r = \begin{cases} < c_1, \ldots, c_r >, & \text{if } 2r \text{ is even} \\ < c_1, \ldots, c_{r-0.5}, 2c_{r+0.5} >, & \text{if } 2r \text{ is odd.} \end{cases}$$

Then $\mathcal{D}_r$ achieves $d_r(3)$ given by Proposition 7 and $\mathcal{D}_i \subset \mathcal{D}_j$ for $i < j$ $(0.5 \le i < j \le 4)$. Hence, $\mathcal{P}_3$ satisfies the chain condition. □

Proposition 9 ([12], Theorem 31) *The weight hierarchy of the Preparata code $\mathcal{P}_4$ of length 16 over Z_4 is given by*

$$\{4,\ 4,\ 6,\ 6,\ 7,\ 8,\ 8,\ 9,\ 10,\ 10,\ 11,\ 11,\ 12,\ 12,\ 13,\ 13,\ 14,\ 14,\ 15,\ 15,\ 16,\ 16\}.$$

Lemma 10 ([12]) *Let $\mathcal{D}$ be a 2-dimensional submodule and let $\chi = \{X_1, X_2, \ldots, X_6\}$ be its support. Let G be the generator matrix for $\mathcal{D}$ with restriction to χ, given by*

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 1 \end{bmatrix}.$$

Then $\mathcal{D}$ is a submodule of $\mathcal{P}_m$ if and only if $4\,|\,m$.

Lemma 11 ([12]) *Let $\mathcal{D}$ be a 2-dimensional submodule and let $\chi = \{X_1, X_2, \ldots, X_6\}$ be its support. Let G be the generator matrix for $\mathcal{D}$ with restriction to χ, given by*

$$\begin{bmatrix} 1 & 1 & 1 & 3 & 2 & 0 \\ 1 & 3 & 2 & 0 & 1 & 1 \end{bmatrix}.$$

Then $\mathcal{D}$ is a submodule of $\mathcal{P}_m$ if and only if $3\,|\,m$.

To show the non-existence of chain for the Preparata code $\mathcal{P}_4$, we need the following lemma.

Lemma 12 *Let C be a [8,4,4] binary code whose codewords have even weight. Let A_i be the number of codewords of Hamming weight i. Then C has the weight distribution given by*

$$A_0 = 1, \quad A_4 = 14, \quad A_8 = 1$$

and is equivalent to the binary code with generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Proof: Clearly, we have

$$A_4 + A_6 + A_8 = 15,$$
$$4A_4 + 6A_6 + 8A_8 = 64.$$

It is easy to check that $A_4 = 14$, $A_6 = 0$, and $A_8 = 1$. Since $A_4 > 7$, so there are three linearly independent codewords of weight 4. Hence, these codewords can be used as the first second, and third rows of the generator matrix, respectively. The fourth row can be chosen the codeword of weight 8, which can be also replaced by a proper codeword of weight 4, as shown in (4). □

Theorem 13 *The Preparata code $\mathcal{P}_4$ of length 16 over Z_4 does not satisfy the chain condition.*

Proof: Note that $d_{0.5}(4) = 4$, $d_{1.0}(4) = 4$, $d_{1.5}(4) = 6$, $d_{2.0}(0) = 6$, $d_{2.5}(4) = 7$, $d_{3.0}(4) = 8$, $d_{3.5}(4) = 8$, and $d_{4.0}(4) = 9$ by Proposition 9. Suppose the chain condition holds for the Preparata code $\mathcal{P}_4$. Let $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$, and $2\mathbf{c}_4$ be codewords of $\mathcal{P}_m$ such that the module $\mathcal{D}_r$ defined by

$$\mathcal{D}_r = \begin{cases} <\mathbf{c}_1, \ldots, \mathbf{c}_r>, & \text{if } 2r \text{ is even} \\ <\mathbf{c}_1, \ldots, \mathbf{c}_{r-0.5}, 2\mathbf{c}_{r+0.5}>, & \text{if } 2r \text{ is odd} \end{cases}$$

is an r-dimensional submodule of $\mathcal{P}_4$, achieving $d_r(4)$ for $0.5 \leq r \leq 3.5$.

Consider the submodule $\mathcal{D}_{3.5}$, whose support is given by $\{X_1, X_2, \ldots, X_8\}$. Note that the submodule $< 2\mathbf{c}_1, 2\mathbf{c}_2, 2\mathbf{c}_3, 2\mathbf{c}_4 >$ restricted to $\{X_1, X_2, \ldots, X_8\}$ is a [8, 4, 4] binary code, multiplied by 2 (mod 4). Thus, by Lemma 10 and Lemma 12, we can assume without loss of generality that

$$\begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \\ 2\mathbf{c}_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 1 & 0 & 0 \\ * & \Delta & * & 0 & * & 0 & 1 & 2 \\ 0 & 2 & 2 & 0 & 2 & 0 & 0 & 2 \end{bmatrix}$$

where $* \in \{1,3\}$ and $\Delta \in \{0,2\}$. If $\Delta = 0$, then $w_H(\mathbf{c}_3 + 2\mathbf{c}_4) = 5$. If $\Delta = 2$, then $w_H(\mathbf{c}_3 + 2\mathbf{c}_4) = 4$, so we have $\mathbf{c}_3 + 2\mathbf{c}_4 = (1\,0\,1\,0\,1\,0\,1\,0)$. Note that $< \mathbf{c}_2, \mathbf{c}_3 + 2\mathbf{c}_4 >$ is a 2-dimensional submodule of $\mathcal{P}_4$ with support size 6, so it achieves $d_2(4)$. But, this is impossible by Lemma 10 and Lemma 11, so we get a contradiction. □

V. Concluding Remarks

The Preparata code of length 8 over Z_4 (the *octacode*) is self-dual and satisfies the chain condition. In this paper, we show that the Preparata code of length 16 does not satisfy the chain condition. Therefore, an interesting and natural problem arises: Does the Preparata code of length 32 satisfy the chain condition ?

References

[1] A. E. Ashikhmin, "Generalized Hamming weights for Z_4-linear codes," *Proc. IEEE Int. Symp. Inform. Theory*, p. 306, Trondheim, Norway, June 1994.

[2] G. D. Forney, "Dimension/length profiles and trellis complexity of linear block codes," *IEEE Trans. Inform. Theory*, vol. 40, pp. 1741-1752, Nov. 1994.

[3] A. R. Hammons, P. V. Kumar, A. R. Calderbank, N. J. A. Sloane, and P. Sole, "The Z_4-linearity of Kerdock, Preparata, Goethals, and related codes," *IEEE Trans. Inform. Theory*, vol. 40, pp. 301-319, Mar. 1994.

[4] T. Helleseth and P. V. Kumar, "The algebraic decoding of the $Z_4 - linear$ Goethals code," *IEEE Trans. Inform. Theory*, vol. 41, pp. 2040-2048, Nov. 1995.

[5] T. Helleseth, P. V. Kumar, and A. Shanbhag, "New codes with the same weight distributions as the Goethals codes and the Delsarte-Goethals codes," To appear in *Designs, Codes and Cryptography*, vol. ??, pp. ??, 199?.

[6] P. V. Kumar, T. Helleseth, and A. R. Calderbank, "An upper bound for Weil exponential sums over Galois rings and its applications," *IEEE Trans. Inform. Theory*, vol. 41, pp. 456-468, Mar. 1995.

[7] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes.* North Holland, New York, 1977.

[8] M. Tsfasman and Vladut, "Geometric approach to higher weights," *IEEE Trans. Inform. Theory*, vol. IT-, pp. 0-0, Nov. 1995.

[9] V. K. Wei, "Generalized Hamming weights for linear codes," *IEEE Trans. Inform. Theory*, vol. 37, pp. 1412-1418, Sept. 1991.

[10] V. K. Wei and K. Yang, "On the generalized Hamming weights of product codes," *IEEE Trans. Inform. Theory*, vol. 39, pp. 1709-1713, Sept. 1993.

[11] K. Yang, T. Helleseth, P. V. Kumar, and A. Shanbhag, "On the weight hierarchy of Kerdock codes over Z_4," *IEEE Trans. Inform. Theory*, vol. 42, pp. 1587-1593, Sept. 1996.

[12] K. Yang and T. Helleseth, "On the weight hierarchy of Preparata codes over Z_4," submitted to *IEEE Trans. Inform. Theory*, September 1996.