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AVERAGE LIQUID LEVEL AND PRESSURE DROP FOR COUNTERCURRENT STRATIFIED TWO-PHASE FLOW

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Abstract

To predict the average liquid level under the condition of the countercurrent stratified two-phase flow in a pipe, an analytical model has been suggested. This is made by introducing the interfacial level gradient into the liquid-phase and the gas-phase momentum equations. The analytical method for the gas-phase pressure drop calculation with $f_i \neq f_G$ has also been described using the liquid level prediction model developed in the present study.

I. INTRODUCTION

Because the liquid level (or liquid holdup) is an important parameter for prediction of two-phase pressure drop and flow pattern transition, numerous studies have been carried out in the past including analytical models for predicting the liquid holdup and pressure drop in well-developed stratified flows. Few of these studies display any interfacial level gradient (ILG), though the liquid holdup, the flow pattern transition, and the assumption of an equal axial pressure gradient in each phase can be affected by the presence of the ILG which is a special type of non-uniform stratified flow. Bishop and Deshpande [1] reported that many of the cocurrent stratified flow data exhibited a non-uniform behavior in varying degrees. Taitel and Dukler [2] considered the three different liquid levels such as the equilibrium level, the stability level and the critical level, in the analysis of the transition boundaries for high-viscosity liquids. Recently, Sadatomi et al. [3] formulated a one-dimensional two-fluid model for horizontal cocurrent stratified flows with ILG, considering the balance of all the forces in a comprehensive manner. Although their final equations were the same as those reported by Taitel and Dukler [2], they calculated the liquid level distribution in the channel using the critical liquid level at the channel exit as one of the boundary conditions. Nam [4] proposed the empirical correlations for the prediction of the liquid level gradient and the liquid level at the outlet for the countercurrent stratified flow in a duct and a pipe. From these correlations, he calculated the liquid level distribution in the form of the dimensionless liquid level along the channel. Recently, Bertodano [5] solved the one-dimensional two-fluid model equations to obtain a liquid level profile along the pipe with an inclined riser, including a jump condition to account for the possibility of a hydraulic jump. As the boundary condition, he used the equation obtained from the mathematical critical flow condition. The jump condition was obtained from the consideration of a single-phase flow: for a single-phase flow, the momentum is minimum at the critical depth, and there is a range where there are two liquid levels that have the same momentum. Table 1 shows some existing studies where the ILG was considered.

In the present study, an analytical model has been suggested to predict the average liquid level for the countercurrent stratified two-phase flow in a pipe. This is made by introducing the interfacial level gradient into the liquid-phase and the gas-phase momentum equations. The analytical method for the gas-phase pressure drop calculation is also presented here.

Table 1 Liquid Holdup Prediction Models with ILG Term

Author	Models or Correlations	Flow Condition*	ILG Term	Interfacial Friction Model
Bishop and Deshpande [1] (1986)	1-D Two-Fluid Model	HI, CO	Yes	$f_i \neq f_G$
Taitel and Dukler [2] (1987)	1-D Two-Fluid Model	HI, CO	Yes	$f_i = f_G$
Sadatomi et al. [3] (1993)	1-D Two-Fluid Model	H, CO	Yes	$f_i = f_G$
Nam [4] (1993)	Empirical Correlation	H, CN	Yes	f_i = constant
Bertodano [5] (1994)	1-D Two-Fluid Model	HI, CN	Yes	$f_i \neq f_G$

H: Horizontal, HI: Horizontal and Inclined, CO: Cocurrent, CN: Countercurrent

II. ANALYSIS

Average Liquid Level Calculation

The analysis is made by considering the countercurrent stratified flow in a pipe as shown in figure 1. Assuming a steady incompressible flow and neglecting mass transfer between the phases, the one-dimensional mass and momentum conservation equations for the liquid-phase and the gas-phase [6, 7] can be written, respectively, as

liquid-phase:

$$\frac{\partial}{\partial x}(\alpha_L u_L) = 0 \tag{1}$$

$$\alpha_L \rho_L u_L \frac{\partial u_L}{\partial x} + \alpha_L \frac{\partial P_{iL}}{\partial x} + \frac{A}{S_i}\alpha_L \rho_L g \cos\beta \frac{\partial \alpha_L}{\partial x} = -\tau_L \frac{S_L}{A} - \tau_i \frac{S_i}{A} + \alpha_L \rho_L g \sin\beta \tag{2}$$

gas-phase:

$$\frac{\partial}{\partial x}(\alpha_G u_G) = 0 \tag{3}$$

$$\alpha_G \rho_G u_G \frac{\partial u_G}{\partial x} + \alpha_G \frac{\partial P_{iG}}{\partial x} - \frac{A}{S_i}\alpha_G \rho_G g \cos\beta \frac{\partial \alpha_G}{\partial x} = \tau_G \frac{S_G}{A} + \tau_i \frac{S_i}{A} + \alpha_G \rho_G g \sin\beta \tag{4}$$

where α is the fraction of the cross sectional area occupied by each phase, u is the velocity, ρ is the density, P is the pressure in each phase, A is the cross sectional area of the pipe, S is the peripheral length of each phase, S_i is the interface width, β is the downward inclination angle from the horizontal, τ is the shear stress, and the subscripts, i , G, and L denote interface, gas phase, and liquid phase, respectively. In Eqs. (2) and (4), the pressure over the cross section is considered to vary due only to hydrostatic forces. In Eqs. (2) and (4), $d\alpha_L/dx$ and $d\alpha_G/dx$ can be expressed in terms of the ILG, i.e., dH_L/dx, using the following relationship together with the fact that $\alpha_G = 1 - \alpha_L$:

$$\partial\alpha_L/\partial x = -(\partial\alpha_G/\partial x) = (S_i/A)(dH_L/dx) \tag{5}$$

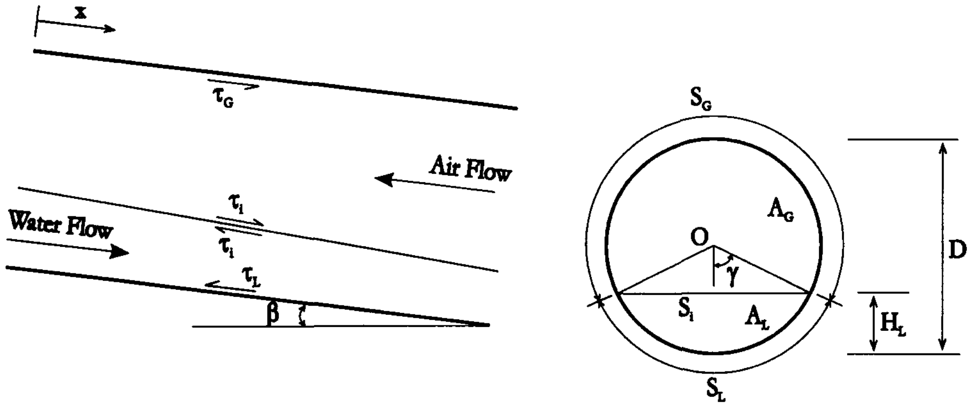

Figure 1
Model Used for Analysis of Countercurrent Stratified Flow with ILG

where H_L is the liquid level. Again, assuming that the surface tension (σ) is negligible, the following approximate relation can be used for the interface pressure of each phase:

$$P_{iG} - P_{iL} = \sigma\left(\partial^2 H_L/\partial x^2\right) = 0 \quad \text{or} \quad P_{iG} = P_{iL} = P_i \tag{6}$$

Substituting Eqs. (5) and (6) into Eqs. (2) and (4) and using Eqs. (1) and (3), the following expressions can be obtained:

$$\left(\rho_L g\cos\beta - \frac{S_i}{\alpha_L A}\rho_L u_L^2\right)\frac{\partial H_L}{\partial x} + \frac{\partial P_i}{\partial x} - \rho_L g\sin\beta + \frac{(S_L\tau_L + S_i\tau_i)}{\alpha_L A} = 0 \tag{7}$$

$$\left(\rho_G g\cos\beta + \frac{S_i}{\alpha_G A}\rho_G u_G^2\right)\frac{\partial H_L}{\partial x} + \frac{\partial P_i}{\partial x} - \rho_G g\sin\beta - \frac{S_G\tau_G + S_i\tau}{\alpha_G A} = 0 \tag{8}$$

The first terms in the above equations represent the pressure force variations due to the interfacial level gradient, ILG. Equations (7) and (8) can be solved for the interface pressure gradients and equated:

$$\begin{aligned}\left[\Delta\rho g\cos\beta - S_i\left(\rho_L u_L^2/\alpha_L A + \rho_G u_G^2/\alpha_G A\right)\right]\left(dH_L/dx\right) &= \Delta\rho g\sin\beta \\ &- \left(\tau_G S_G/\alpha_G A + \tau_L S_L/\alpha_L A\right) - \tau_i S_i\left(1/\alpha_G A + 1/\alpha_L A\right)\end{aligned} \tag{9}$$

where $\Delta\rho = \rho_L - \rho_G$. In order to calculate the liquid level using Eq. (9), the left hand side term including ILG has to be evaluated. However, it is not simple to evaluate this term. Nam [4] observed that the liquid level gradient increases rapidly with increasing the gas flow rate and increases with increasing the liquid flow rate, even though there is no gas flow over the liquid-phase. He also developed the empirical correlations of the level slope for countercurrent flow in a pipe as shown in Table 2. Table 2 shows correlations selected for the evaluation of ILG, τ_L, τ_G, and τ_i. To obtain the liquid level, combined momentum equation (CME), Eq. (9), can be solved by iterative procedure if appropriate conditions are given.

Figure 2 shows the results of the level prediction by CME as a function of the superficial gas velocity (j_G) when the liquid superficial velocity (j_L) is 0.10186 m/s. As shown in this figure, the interfacial friction factor does not show a large effect on the prediction results of the liquid level when the gas flow rate is low. As the gas flow rate increases, however, the interfacial friction modeling significantly affects the predicted liquid level.

Table 2 Selected Correlations for ILG and Shear Stresses

	Correlation
Interfacial Level Gradient [4]	$\frac{dH_L^*}{dx^*}=\frac{1}{2}\left(j_L/\sqrt{gD}\right)^{1/2}\left[1+(\rho_G/\rho_L)^{0.25}\left(j_G/\sqrt{gD}\right)\right]^{2.25}$
Wall Shear Stresses ($\tau_L = 0.5 f_L \rho_L u_L^2$ and $\tau_G = 0.5 f_G \rho_G u_G^2$)	f_G and f_L : Blasius Equation
Interfacial Shear Stress ($\tau_i = 0.5 f_i \rho_G (u_G + u_L)^2$)	f_i correlation 1* [9, 10] : $(f_{i,h})_{corr} = 0.01(3X)^{(0.8+X)/4}$ $(f_{i,incl})_{corr} = a + (b + cX^3)/\sin\beta$ f_i correlation 2 [10]: $(f_{i,h})_{corr} = 2.126\times10^{-4}\,\mathrm{Re}_L^{0.35}\,\mathrm{Re}_r^{0.102}$ $(f_{i,incl})_{corr} = 2.549\times10^{-9}\,\mathrm{Re}_L^{0.478}\,\mathrm{Re}_r^{0.874}\,\beta^{-0.718}$

* $X = 0.02\left(u_r/\sqrt{gD}\right)^{2/3}\ \mathrm{Re}_r^{1/3}\ \left(1+D_L/D\right)^{1/2}$, $a = 5.66\times10^{-3}$, $b = 1.798\times10^{-5}$, and $c = 3.472\times10^{-3}$

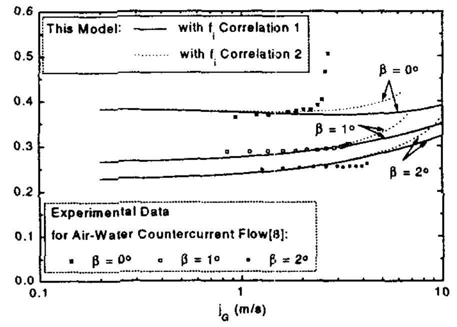

Figure 2
Average Liquid Level by CME with ILG for
$j_L = 0.10186\ \mathrm{m/s}$

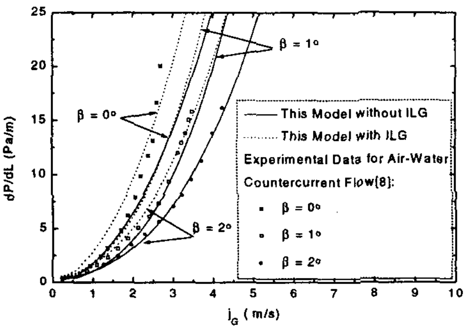

Figure 3
Pressure Drop Calculation for
$j_L = 0.10186\ \mathrm{m/s}$

Two-Phase Pressure Drop Calculation

Two-phase pressure drop in the gas-phase can be calculated from the gas-phase momentum equation, Eq. (8), neglecting the hydrostatic force over the cross section in the gas phase, $(dP_G/dx) \approx (dP_i/dx)$. For nearly horizontal and slightly inclined pipe flow conditions, the liquid levels are obtained from the CME as described previously. Figure 3 shows the results of the pressure drop calculation for $j_L = 0.10186\ \mathrm{m/s}$. It can be found that for nearly horizontal pipe flow, the present method with ILG agrees well with the experimental data, but for slightly inclined pipe flow, predicted values are much higher than the experimental data. For slightly inclined pipe flow, however, the results obtained by the present method without

ILG agree more closely with data as shown in figure 3. For nearly horizontal pipe flow, on the other hand, this method predicts much lower values than the experimental data.

III. RESULTS AND DISCUSSION

Figure 4 shows the comparisons between the predicted liquid levels and the experimental data. The predicted values of the liquid level were obtained from CME with ILG. The results show that the present method predicts experimental data within -25% and +10%. Since the present method does not account for the hydraulic jump, the predicted values are much lower than the experimental data obtained in the presence of hydraulic jump for nearly horizontal pipe flow [8]. However, for slightly inclined pipe flow, predicted values with the present method are in good agreement with the experimental data.

As described in the previous section, in order to calculate the two-phase pressure drop in the gas-phase for horizontal pipe flow, it is reasonable to consider the interfacial level gradient. For slightly inclined pipe flow, on the other hand, the results of the prediction of the pressure drop without ILG are much better than those with ILG. The predicted results of the two-phase pressure drop obtained from the gas-phase momentum equation can be found in figure 5, in which ILG is considered for nearly horizontal pipe flow but is not for slightly inclined pipe flow. For most of the test range except at high superficial gas velocity, the calculation error is within ±15% . The large error at high superficial gas velocity is mainly due to the wave effect which is not incorporated in the calculation of the pressure drop. These results may also be attributed to the fact that the CME underestimates the liquid level as shown in figure 4.

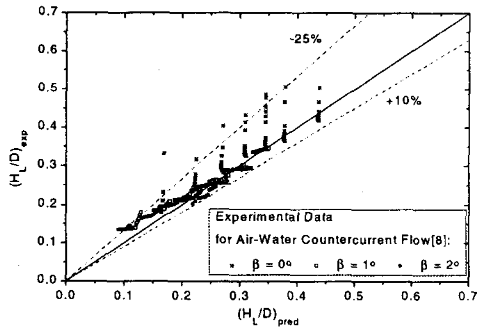

Figure 4
Comparisons between the Predicted Liquid Level by CME and the Experimental Data [8]

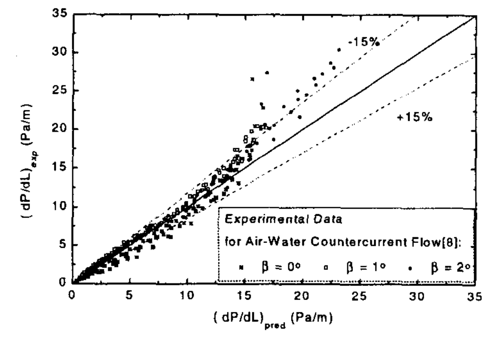

Figure 5
Comparisons of the Predicted Pressure Drop with the Experimental Data [8]

IV. CONCLUSIONS

The average liquid level for countercurrent stratified gas-liquid flow has been calculated from CME with ILG while the pressure drop in the gas-phase has been predicted from the gas momentum equation with or without ILG. From these calculations, following conclusions can be made:

(1) Since the present method does not take the hydraulic jump into account, the predicted values are much lower than the measured data for nearly horizontal pipe flow, but for

slightly inclined pipe flow, the present method is in good agreement with the measured data.

(2) Although the prediction of the liquid level for stratified countercurrent gas-liquid flow depends mainly upon the prediction model or method, interfacial friction modeling may be of great importance in the analysis of flow pattern transition criteria or flow instability.

(3) While it is reasonable to account for ILG for horizontal pipe flow under the experimental conditions, the results of the prediction of the pressure drop without ILG are much better than those with ILG for slightly inclined pipe flow.

In view of the present work, following recommendations are made: (1) Experimental work be performed in which the pressure drops for both phases and interfacial level gradient are measured simultaneously, and (2) a model or a correlation be developed to predict the liquid level near the point of instability such as CCFL or flooding.

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