Proceedings of the Korean Nuclear Society Autumn Meeting
Taejon, Korea, October 1996

H infinity Controller Design for the Reactor Power Control System

Yoon Joon Lee
Cheju National University

Abstract

The robust controller for the nuclear reactor power control system is designed. The reactor model is set up by use of the point kinetics equations and the singly lumped energy balance equations. Since the model is different from the actual plant, the controller which makes the system robust is necessary. The perturbation of the actual plant is investigated with respect to several possible sources of uncertainty. Then the overall system is configured into the two port model and the H_∞ controller is designed. The loop shaping and the permissible control rod speed are considered as the design constraints. The designed H_∞ controller provides the sufficient margins for the robustness, and the system output as well as the control input satisfy their relevant requirements.

1. Introduction

Presently, a great effort has been made to upgrade the control system of the nuclear power plants. This improvements are implemented by use of the digital technologies, which provides the favorable environments for the modern control algorithms. One of typical modern algorithms is the Wiener-Hopf-Kalman (WHK) techniques which provide systematic frame for the design procedure[1]. But it has the presumptions that the process plant be exact with no uncertainties and the properties of noises be known. This is of course impossible in the real world. Because of the limitations of the equations employed in modeling and the changes of the operating conditions, the plant is subject to change. The actual control system should survive all these uncertainties and changes. Hence, it can be said that the purpose of the control system is not the stability but the robustness[2].

In this paper, a robust controller is designed by use of the H_∞ control techniques. The H_∞ control technique provides an efficient and synthetical tool which can deal with the uncertainties and noises. In contrast with the WHK which stresses the performance only, the H_∞ control optimizes both the performance and the robustness, resulting in the more meaningful optimization. Since H_∞ control is an optimization process in the frequency domain, the familiar existing classical techniques can be used.

The reactor is modeled with the kinetics equations and energy balance equations. The uncertainties of this model is investigated. Then a two port model is established and the H_∞ controller is designed. The constraints on the output power as well as on the control rod velocity are considered in the design.

2. Uncertainties of the Plant

The reactor plant is modeled by use of the one group delayed neutron point kinetics equations together with the singly lumped energy balance equations. Then the model is linearized with the assumption of small perturbations. The energy balance equations are used to consider the temperature feedback effects within the reactor. The reactivity acts on the plant is the sum of feedback reactivities and external control effort as below.

$$\frac{d}{dt}\delta\rho = \frac{d}{dt}\delta\rho_{ext} + \alpha_f\frac{d}{dt}\delta T_f + \alpha_c\frac{d}{dt}\delta T_c \qquad (1)$$

It is assumed that the control rod worth is constant through its length. Then the external control effort is written as

$$\frac{d}{dt}\delta\rho_x = v_r\frac{\rho_H}{H} \qquad (2)$$

where v_r is the absolute rod velocity (m/sec), ρ_H is the total rod worth, and H is the rod length. Then the reactor plant can be described by the following linear state equation.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad (3)$$

where $\mathbf{x} = \left(\delta\overline{P} \quad \delta\overline{C} \quad \delta T_f \quad \delta T_c \quad \delta\rho_{ext}\right)^T$.

The dimension of the system matrix **A** is five, and **u** has three elements. By assuming that the coolant inlet temperature and flow rate are constant, and the measured signal is the power only, the multivariable system of above equation is reduced to the SISO system.

It should be understood that the system matrix **A** is the function of the reactor power, and the eigenvalues of the plant vary with the power. It can be found that the plant of Eq. (3) is always stable. But as the power decreases, the most sensitive pole goes to zero and the plant becomes of a double integrator and a lead. This indicates that it is more difficult to control the plant when the power is low. Since the reactor plant of Eq. (3) is of minimum phase, it is possible to build a stable control system only with a unity feedback loop and a feedforward gain. But simple analysis shows that the damping ratio is too large and the control input is unrealistically large. Further, the range of the feedforward gain which makes the system stable is very narrow, which reveals the limitation of the unity feedback configuration.

The reactor model made so far has an error. It comes from the equations themselves which are employed in the modeling as well as the assumptions of the linearization and other simplification. Therefore, although the nominal plant is always stable, the actual perturbed plant may not. However, it is difficult to estimate the errors between the Eq. (3) and the actual system. Hence the uncertainty, or the error of Eq. (3) is investigated with respect to three factors. First is the delay of the control input. In the modeling of the reactor, the differential rod worth is assumed to be constant. But it is described more precisely by

$$\frac{d\rho}{dt} = \frac{\rho_H}{H}\left(1 - \cos\frac{2\pi x}{H}\right)\frac{dx}{dt} \qquad (4)$$

The comparison between Eq. (2) and Eq. (4) shows that there is a control input delay in the modeling.

The second source of uncertainties are physical parameters of the reactor. For example, the moderator temperature coefficient is subject to change with the boron concentration, and the fuel temperature coefficient is to change with the fuel temperature. The fuel gap heat transfer coefficient has a value ranging widely from 2,000 to 11,000 w/m^2 ℃. The physical data both for the nominal and perturbed plants are summarized in Table 1. These values are quoted from the FSAR of Kori Unit 2[3]. The last one of the uncertainties are the initial power level. As explained above, since the reactor plant is the function of the initial power, the error in the initial power measurement results in the uncertainty. With these uncertainties, the perturbed plant can be written as

$$\tilde{G}(s) = G_p(s) \cdot D(s) \qquad (5)$$

where $G_p(s)$ is the plant with the worst physical data and D(s) is the delay in the second order equation by the Pade relation.

Table 1. Physical Properties of Nominal and Perturbed Plants

Property	Nominal Plant	Perturbed Plant
Moderator Temp. Coeff.	0 pcm/ °C	+14.4 pcm/ °C
Fuel Temp. Coeff.	-3.7 pcm/ °C	-2.28 pcm/ °C
Gap Heat Transfer Coeff.	4,850 w/m^2 °C	2,000 w/m^2 °C
Control Input Delay	None	Yes

In general, the delay has a large effect on the system. All the actual systems have delays because of the initial torque load on the actuator, which may result in instability. The frequency analysis shows that the gain and phase margins decrease as the control input delay time increases. Also the critical delay time, which makes the system unstable, becomes shorter as the initial power is lower. For example, when the initial power is 100%, the critical delay time is about 5.8 sec, but with the initial power of 10%, it decreases to 2.5 sec.

The robustness of the system can be verified by the small gain theorem, which says that following conditions should be satisfied for the robustness.

$$|\Delta a| < \frac{1}{|KS|},\ |\Delta m| < \frac{1}{|T|} \qquad (6)$$

where the additive uncertainty $\Delta a = \tilde{G}(s) - G(s)$, multiplicative uncertainty $\Delta m = \frac{\tilde{G}(s) - G(s)}{G(s)}$, S is the sensitivity and T is the complementary uncertainty.

For the case of the initial power of 90%, and for the delay of 4 seconds, the singular value (SV) plots of uncertainties and sensitivities show that the system is robust. But since there is almost no additive stability margin (ASM) and multiplicative stability margin (MSM), the time response of the system shows the marginal stability.

In addition to the output characteristics, the control input which acts on the plant should be considered for the control system design. The control system which configured in the unity feedback system with the feedforward gain only shows a plausible output characteristics. But that configuration yields the unreasonable control input of a large magnitude. Therefore, it is necessary to design a controller which satisfy the output tracking properties as well as the control input requirements.

3. H_∞ Controller Design

Figure 1 describes the reactor power control system in the unity feedback configuration. The controller K(s) which is to be designed is located on the feedforward loop. The disturbance, d, acts on the plant, and the measurement noise, n, on the feedback loop. The system of Fig. 1 is redrawn into the two port model of Fig. 2. It consists of P(s) and K(s). **P**(s) has the multi inputs of exogenous vector **w** and the control input u, and has the multi outputs comprised of plant output y_p and u. The reasons of including u in the outputs are to impose a limitation on the control input magnitude and to meet the rank conditions for the existence of H_∞ controller.

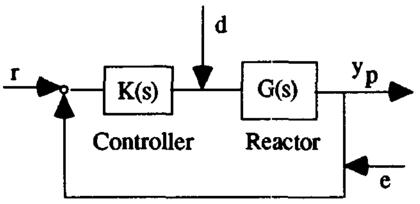

Fig. 1. Reactor Power Control System with Uncertainties

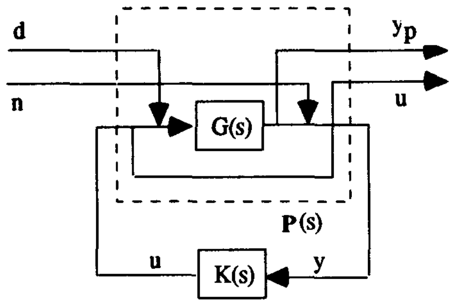

Fig. 2. Reactor Power Control System in Two Port Model

By defining the system input and output vector as **w** and **z**, respectively, the system of Fig. 2 could be written in the following state equations.

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}_1 w + \mathbf{B}_2 u \\ z &= \mathbf{C}_1 x + \mathbf{D}_{11} w + \mathbf{D}_{12} u \\ y &= \mathbf{C}_2 x + \mathbf{D}_{21} w + \mathbf{D}_{22} u \end{aligned} \quad (7)$$

In the above equation, the system matrix **A** and $\mathbf{B}_2$ are obtained from the plant, and other matrices and vectors are determined from Fig. 2, and all the matrices and vectors should satisfy the rank conditions for the existence of the H_∞ controller.

The H_∞ controller design is similar to the LQG design in that both are in the frame of the Riccati equations. However, there is a fundamental difference between them. In the H_∞ control, the controller becomes different depending on how the exogenous signals act on the system. That is, $\mathbf{B}_1$ of Eq. (7) has an effect on the controller for the H_∞ control, while it has an effect on the cost function only without changing the controller for the case of LQG control. What makes the H_∞ control more difficult is the conditions for the existence of the solutions of Eq. (7). Contrary to the LQG, there is no guarantee that the Riccati equations of the H_∞ always have the solutions.

The purpose of the H_∞ control is to design the admissible controller[4], that is well posed and has finite order, which makes the infinity norm of the overall closed loop system satisfy the following objective function.

$$\|R_{zw}\| = F_l(\mathbf{P},\mathbf{K}) < \gamma \quad (8)$$

where $F_l(\mathbf{P},\mathbf{K})$ is the linear fractional transformation (LFT).

The existence of the admissible controller, with the rank conditions satisfied means there is a positive definite solution for the Riccati equation derived from Eq, (7). Further, the requirement of internal stability sets forth another Riccati equation whose solution should be positive definite also. From these equations, it can be shown that the H_∞ controller is represented by

$$K = F_l(\mathbf{K}_a, U) \quad (9)$$

where the U is the feedbacked variable, and $\mathbf{K}_a$ is the matrix whose elements are expressed in terms of the solutions of the Riccati equations and system matrices.

For the reactor control system of Eq. (7), the above algorithms are applied. However, the controllers designed are different each other depending on the variable on which the disturbance acts. When the disturbance acts on x_1, x_3, and x_4 (for the case of x_2, there is no solution), the norm of the controller is very small, which indicates the excessive robustness and poor performance. Contrary to this, when the disturbance is input to x_5, the norm of the controller is very large. Hence the controller can be obtained by controlling the norm of the controller. However, the singular value plots show that the case of x_5 is more desirable because it has a typical loop shaping characteristics.

The another factor that should be considered is the magnitude of the control input, that is, the control rod speed. The maximum rod speed is about 2 cm/sec. With this constraint, the controller is determined finally as

$$K(s) = \frac{22s^4 + 8939s^4 + 31856s^2 + 20738s + 1080}{s^5 + 564.2s^4 + 7.718 \cdot 10^4 s^3 + 2.351 \cdot 10^5 s^2 + 7.577 \cdot 10^4 s + 4544} \quad (10)$$

With this controller, the gain margin of the system is 78 dB, and the phase margin is 89°, which are sufficient for the robustness. Figures 3 and 4 show the time response of the output power and the control input, respectively, when the power is step increased from 90 to 100%. The output follows the command signal within about 60 seconds, and the maximum control rod speed does not exceed the permissible speed. Further there is no overshooting which should be less than 3 % as specified in the FSAR.

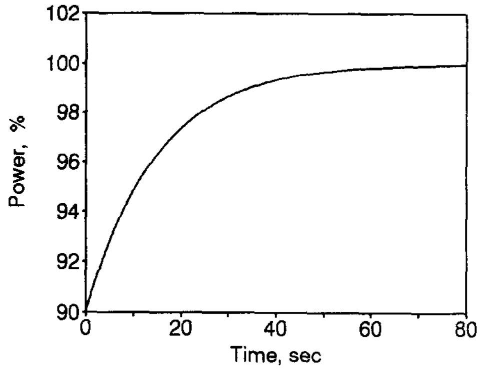

Fig. 3. Output Responses of the Designed System

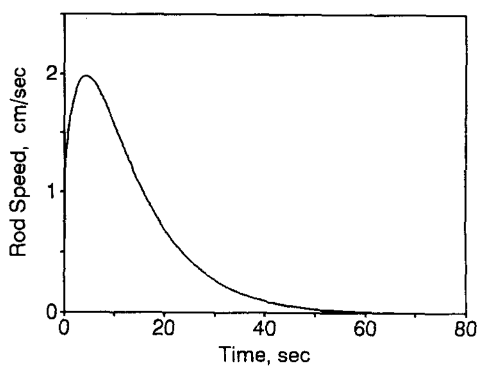

Fig. 4. Control Rod Speeds of the Designed System

4. Conclusion

The real plants always have uncertainties which arise from the mathematical modeling and the change of operating conditions. Hence the actual control system should be robust to endure these uncertainties. The H_∞ control technique provides the synthetical tool which considers both the robustness and the performance. To design the robust controller for the reactor power control system, the reactor is modeled by use of the point kinetics equations and the singly lumped energy balance equations. This model has uncertainties, and the possible sources of the uncertainties are investigated.

The overall system of the unity feedback configuration is converted to the two port model and the

H_∞ technique is applied. Since the H_∞ controller depends on the way by which the exogenous signals act on the system, it is designed by changing the state variable which interacts with the disturbance. Then by controlling the norm of the controller, together with the loop shaping, the H_∞ controller is designed. The constraint on the maximum rod speed and the requirement on the output transient are considered also. The designed controller gives the system sufficient margins for the robustness and meets all the requirements set forth by the FSAR.

References

1. Y. J. Lee, "The Control Rod Speed Design for the Nuclear Reactor Power Using Optimal Control Theory," *J. of KNS*, **2 6**(4), 536-547 (1994)
2. P. Dorato (Ed.), *Robust Control*, IEEE Press (1987)
3. *Final Safety Analysis Report of Kori Unit 2, 2nd Ed.*, Korea Electric Corp. (1989)
4. M. Green, D. J. N. Limbeer, *Linear Robust Control*, Prentice Hall (1995)