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Boundary Element Method for the Homogenization of the Baffle/Reflector

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ABATRACT

In this study, the boundary element method (BEM) is used to treat the homogenization of the baffle and reflector. First, the multigroup diffusion equations (MDE) within the core are solved using BEM as a source problem under the assumption that the core material is uniformly distributed. Then, the solution to MDE for the water reflector which should be extended infinite can be attained from a boundary source problem also via BEM. Finally, these two solutions are coupled through albedos of the slab-shaped baffle so that one could obtain heterogeneous interface currents and fluxes between the core and the baffle/reflector resulting in the location-dependent equivalent parameters for the baffle/reflector.

I. Introduction

As is well-known to all, the treatment of the baffle/reflector is very important to the core analysis for PWR. So far, two main approaches to handling the baffle/reflector are currently popular in the routine design of reactor physics. The first is to replace the baffle/reflector with albedo boundary conditions, which can analytical be derived for the slab-shaped and L-shaped baffle/reflector[1], or obtained from a numerical point of view via the finite difference method so that the space-dependent albedos could be given[2]. However, the albedo method seems not so accurate in most cases (maximum error in power probably is up to 5.0%)[1-3] from the results publicly issued although it can significantly reduce CPU time and save computer memory storage. The second method is to homogenize the baffle/reflector of all types according to the nodal equivalence theory (NET) by using an identical one-dimensional spectral geometry calculation[4-5]. Nowadays, the latter method is dominantly utilized in the realistic design of core physics because of its both high accuracy and simplicity. However, it as well suffers from a somewhat big inadequacy that it is not able to model the L-shaped and inverse L-shpaed baffle/reflector. So, one takes efforts to apply the Boundary Element Method (BEM)[6] to realize the homogenization of the baffle and reflector in order that space-dependent discontinuity factors could be determined along the interface between the core and the baffle.

II. Boundary Integral Equations for Neutron Diffusion Problems

We are seeking an approximate solution to the simplified neutron diffusion equation for any energy group g on the two-dimensional homogeneous domain Ω as shown in Fig.1 governed by

$$\nabla^2\phi_g - k_g^2\phi_g = -S_g / D_g \qquad (k_g^2 = \Sigma_g / D_g) \tag{1}$$

with mixed boundary conditions

$$\phi = \bar{\phi} \text{ on } \Gamma_1$$
$$q = \partial\phi/\partial\mathbf{n} = \bar{q} \text{ on } \Gamma_2$$

where $\mathbf{n}$ is the outward normal current from Ω. The group subscript g is omitted for simplicity in the following discussion. The left side of Eq. (1) is the modified Helmholz operator.

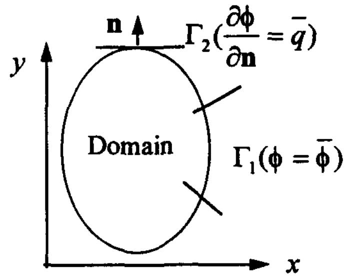


Fig. 1. Definitions of domain and boundary.

The error introduced by replacing ϕ and q by an approximate solution can be minimized by writing the following weighted residual statement:

$$\int_\Omega (\nabla^2\phi - k_g^2\phi)\phi^* d\Omega = \int_{\Gamma_2} (\frac{\partial\phi}{\partial\mathbf{n}} - \bar{q})\phi^* d\Gamma - \int_{\Gamma_1} (\phi - \bar{\phi})\frac{\partial\phi^*}{\partial\mathbf{n}} d\Gamma - \frac{1}{D}\int_\Omega S\phi^* d\Omega, \quad (2)$$

where ϕ^* is interpreted as a weighting function and

$$J^* = -D(\partial\phi^*/\partial\mathbf{n}) \quad (3)$$

The integration of Eq. (2) by parts gives

$$-D\int_\Omega (\nabla^2\phi^* - k_g^2\phi^*)\phi d\Omega = -\int_\Gamma J\phi^* d\Gamma - \int_\Gamma \phi J^* d\Gamma + \int_\Omega S\phi^* d\Omega, \quad (4)$$

Let ϕ^* be the fundamental solution for an infinite system which satisfies the subsidiary equation with a Dirac delta function:

$$\nabla^2\phi_i^* - k^2\phi_i^* + \delta_i = 0. \quad (5)$$

If the weighting function ϕ^* in Eq. (4) is replaced by the fundamental solution ϕ_i^*, then Eq. (4) can be transformed into the following boundary integral equation:

$$D\bar{c}_i\phi_i = \int_\Gamma J_i^*\phi d\Gamma - \int_\Gamma J\phi_i^* d\Gamma + \int_\Omega S\phi_i^* d\Omega, \quad (6)$$

where c_i is a constant that depends on the type of points under consideration: $c_i = 0$ for external points, $c_i = 1$ for internal points, $c_i = 1/2$ on the Lyapunov boundary, or is otherwise proportional to the internal solid angle.

A generalized Bessel function[1] can be selected as the fundamental solution for a two-dimensional system:

$$\phi_i^*(\mathbf{r}, \mathbf{r}_i) = K_0(k|\mathbf{r} - \mathbf{r}_i|)/2\pi \quad (7a)$$

and

$$J_i^*(\mathbf{r}, \mathbf{r}_i) = -D[\partial\phi_i^*(\mathbf{r}, \mathbf{r}_i)/\partial\mathbf{n}]. \quad (7b)$$

Note that Eq. (6) still includes the domain integral of the source term:

$$Q_i = \int_\Omega S\phi_i^* d\Omega. \quad (8)$$

III. Numerical Techniques

The integral equation (6) can be discretized into a series of elements. Consider the boundary of a region is divided into N line segments or boundary elements. The points where the unknowns are considered are taken to be in the middle of each segment for the so-called constant elements which are employed in this study. Eq. (6) can then discretized as follows:

$$\sum_{j=1}^{N} H_{ij}\phi_j - \sum_{j=1}^{N} G_{ij}J_j + Q_i = 0. \tag{9a}$$

Eq. (9a) can also be recast in the matrix form:

$$\mathbf{H\phi} - \mathbf{GJ} + \mathbf{Q} = \mathbf{0}. \tag{9b}$$

Rearranging Eq. (9a) so that all unknowns are set up on the left side, we obtain the total linear algebraic system

$$\mathbf{Ax} = \mathbf{f} \tag{10}$$

which can be solved by common numerical methods such as Gaussian elimination. The values of all fluxes and currents on the boundary are determined after solving Eq. (10).

The curvilinear integrals contained in Eq. (9) are as usual calculated using the unidirectional Gauss quadrature formula. In the particular case of $i = j$, the term G_{ij} in the matrix can be calculated analytically. The analytical integral for the generalized Bessel function

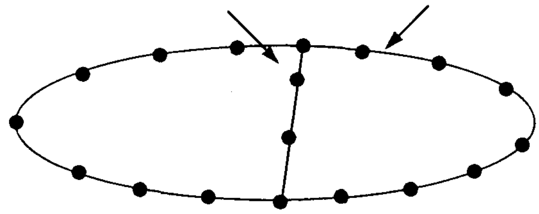
$$\int_0^x K_0(t)dt$$

can be carried out with the help of modified Struve functions. The detailed numerical techniques are in progress.

In addition, if the source term is not uniform in the domain Ω, the domain is required to be divided into meshes for numerical integration. However, this integral can as well be switched into an equivalent boundary integral under some special assumption[6].

IV. Treatment of Core/Reflector Interfaces and Baffle Plates

If the problem under consideration is defined over a region which is only piecewise homogeneous such as a core/reflector domain, the numerical procedures described can be applied to each subregion as they are separated from the others. The final system of equations for the whole region is obtained by adding the set of equations system of Eq. (9) for each subregion together with continuity conditions of fluxes and interface currents. The equations associated with region 1 and 2 shown in Fig. 2 can be written as



$$\phi_I = \phi_I^1 = \phi_I^2$$
$$\mathbf{J}_I = \mathbf{J}_I^1 = -\mathbf{J}_I^2$$

Fig. 2. Schematic of method for two-region problems.

$$\mathbf{H}^1\boldsymbol{\phi}^1 + \mathbf{H}_I^1\boldsymbol{\phi}_I^1 - \mathbf{G}^1\mathbf{J}^1 - \mathbf{G}_I^1\mathbf{J}_I^1 + \mathbf{Q}^1 = \mathbf{0} \quad \text{(region 1)} \tag{10a}$$

and

$$\mathbf{H}^2\boldsymbol{\phi}^2 + \mathbf{H}_I^2\boldsymbol{\phi}_I^2 - \mathbf{G}^2\mathbf{J}^2 - \mathbf{G}_I^2\mathbf{J}_I^2 + \mathbf{Q}^2 = \mathbf{0} \quad \text{(region 2).} \tag{10b}$$

using the same form as Eq. (9b). Here the superscripts 1, 2 and I denote regions 1 and 2 and the interface, respectively. The interface continuity (or compatibility) conditions between the two regions are described as

$$\boldsymbol{\phi}_I = \boldsymbol{\phi}_I^1 = \boldsymbol{\phi}_I^2 \tag{11a}$$

and

$$\mathbf{J}_I = \mathbf{J}_I^1 = -\mathbf{J}_I^2, \tag{11b}$$

based on the continuity of current and flux. The currents $\mathbf{J}_I^1$ and $\mathbf{J}_I^2$ are defined as having opposite values at the interface because the curvilinear integral along each boundary is performed in a counterclockwise direction. Substituting of Eq. (11) into Eq. (10) gives

$$\begin{bmatrix} \mathbf{H}^1 & -\mathbf{G}^1 & \mathbf{H}_I^1 & -\mathbf{G}_I^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_I^2 & \mathbf{G}_I^1 & \mathbf{H}^2 & -\mathbf{G}^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}^1 \\ \mathbf{J}^1 \\ \boldsymbol{\phi}_I \\ \mathbf{J}_I \\ \boldsymbol{\phi}^2 \\ \mathbf{J}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{Q}^1 \\ \mathbf{Q}^2 \end{bmatrix} = \mathbf{0}. \tag{12}$$

The total system of linear equations derived by moving all unknowns to the left side and the known values to the right side can be solved Gaussian elimination, so that all the values of flux and current become known at the interface. The technique for two-region problems can be applied to the core/reflector interface problem. This problem is unique in that the core is the inner part of the infinite reflector. This leads to the following: (1) Since all the boundary points of the core are adjacent to the reflector, the curvilinear integrals $\mathbf{H}$ and $\mathbf{G}$ along an external boundary do not exist for the core region; (2) It is well known that reflector savings effect is saturated at a water thickness of 20 to 30 cm. Therefore, the actual reflector of the finite thickness can be replaced by an infinite one. In this case, no curvilinear integral is required along the infinite external boundary of the reflector because at this boundary both the flux and current can be assumed to vanish. Both types of fundamental solutions ϕ_i^* and J_i^* defined by Eqs. (7a) and (7b) also reduce to zero. Therefore, the matrix equation, Eq. (12), can be reduced to

$$\begin{bmatrix} \mathbf{H}_I^C & -\mathbf{G}_I^C \\ \mathbf{H}_I^R & \mathbf{G}_I^R \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}_I \\ \mathbf{J}_I \end{bmatrix} + \begin{bmatrix} \mathbf{Q}^C \\ \mathbf{Q}^R \end{bmatrix} = \mathbf{0} \tag{13}$$

for the core/reflector interface problems. In this case, the interface conditions are described as

$$\boldsymbol{\phi}_I = \boldsymbol{\phi}_I^C = \boldsymbol{\phi}_I^R \tag{14a}$$

and

$$\mathbf{J}_I = \mathbf{J}_I^C = -\mathbf{J}_I^R, \tag{14b}$$

where the superscripts C and R denote core and reflector, respectively. The curvilinear integral belonging to the core is computed in the counterclockwise direction while the one belonging to the reflector should be performed in the clockwise direction along the core/reflector interface.

Consider next the problem that consists of three regions: the core, the baffle, and the reflector. The boundary element equations for the core-side and reflector side surfaces of the baffle-plate region have the following forms:

$$\mathbf{H}^C \boldsymbol{\phi}^C - \mathbf{G}^C \mathbf{J}^C + \mathbf{Q}^C = \mathbf{0} \quad \text{(for core side)} \tag{15a}$$

and

$$\mathbf{H}^R \boldsymbol{\phi}^R - \mathbf{G}^R \mathbf{J}^R + \mathbf{Q}^R = \mathbf{0} \quad \text{(for reflector side)} \tag{15b}$$

where the suffixes of the groups are omitted for simplicity.

Another matrix equation is required to describe the relationship between the energy-dependent flux and current at the core and the reflector side surfaces of the baffle-plate region:

$$\begin{bmatrix} \phi_{1,R} \\ J_{1,R} \\ \hline \phi_{2,R} \\ J_{2,R} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{0} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \phi_{1,C} \\ J_{1,C} \\ \hline \phi_{2,C} \\ J_{2,C} \end{bmatrix} \tag{16}$$

Each submatrix in Eq. (16) can be derived analytically from the assumption of one-dimensional infinite slab geometry since a baffle plate is relatively thin in practice[1]. For convenience, they are omitted here.

V. Calculation of Homogenization Equivalence Parameters of the Baffle/Reflector

The procedure for the homogenization of the baffle/reflector is summarized as follows: A simple BEM calculation is carried out to produce heterogeneous reference solutions including few-group flux and normal-current solutions along the core/baffle interface and the baffle/reflector interface while the flux within the reflector can be computed with the currents and fluxes on the interface directly produced by BEM and Eq. (6). Then, we can obtain FVWs of all types nodes of the baffle/reflector generally including L-shaped and inverse L-shaped and flat-slab nodes of the baffle/reflector. Finally, we can utilize FVWs and heterogeneous interface currents between the core and the baffle to determine the location-dependent discontinuity factors with the aid of the one-region above-discussed BEM.

VI. Numerical Results ans Conclusions

The mathematical model for the homegenization of the baffle/reflector were examined with the Zion-1 core benchmark problem[5]. The location-dependent discontinuity factors and the flux-volume-weighted cross sections for all types of the baffle/reflector nodes are given in Table I. Taking advantage of the data given in Table I, homogeneous global calculations are then carried out under two cases. Case I is that the equivalent parameters for all types of the baffle/reflector nodes are replaced by those of node (9,1) (as in the conventional one-dimensional spectral geometry calculation for the baffle/reflector homogenization) while Case II is that the location-dependent equivalent parameters are used for different types of the baffle/reflector nodes. The power distributions for these two cases are shown in Fig. 3. It is clear that the the results for the latter case is better than those for the first case. Note that the reference caculation in Fig. 3 was performed explicitly modeling the baffle and reflector.

Table I
Location-dependent equivalent parameters for Zion baffle/reflector

Node	Discontinuity factor		D		Σ_a		Σ_{21}
	Group 1	Group 2	Group 1	Group 2	Group 1	Group 2	
(9,1)	1.1598	0.1952	1.29127	0.29291	0.00151	0.01837	0.01805
(9,2)	1.1579	0.1952	1.29117	0.29291	0.00151	0.01838	0.01805
(9,3)	1.1590	0.1958	1.29086	0.29292	0.00151	0.01841	0.01803
(9,4)	1.1372	0.1997	1.28902	0.29230	0.00152	0.01865	0.01790
(8,5) x-direction	1.1632	0.2030	1.25440	0.29441	0.00174	0.02289	0.01588
(8,5) y-direction	1.1314	0.1826	1.25440	0.29441	0.00174	0.02289	0.01588
(8,6) x-direction	1.1481	0.1920	1.30225	0.29245	0.00144	0.01700	0.01879
(7,7)	1.1630	0.2061	1.24805	0.29481	0.00178	0.02409	0.01516

1.6346 -0.95 -0.56	1.7807 -0.94 -0.55	1.5373 -0.85 -0.49	1.5678 -0.75 -0.43	1.2548 -0.60 -0.32	1.1652 -0.37 -0.15	0.7972 0.01 0.15	0.5049 0.69 0.83	(9,1)
	1.5864 -0.90 -0.54	1.6758 -0.86 -0.51	1.3977 -0.73 -0.42	1.3667 -0.55 -0.30	1.0338 -0.29 -0.10	1.9177 0.13 0.26	0.4890 0.87 0.94	(9,2)
		1.4483 -0.72 -0.41	1.4801 -0.61 -0.35	1.1820 -0.41 -0.23	1.0829 -0.22 -0.15	0.7177 -0.39 0.39	0.4890 1.74 1.63	(9,3)
k_{eff} 1.27496 1.27501 1.27495			1.2448 -1.06 -0.27	1.2177 -0.13 -0.06	0.8931 0.28 0.26	0.7175 0.92 0.67	0.3143 2.80 0.95	(9,4)
				1.0782 0.16 0.06	0.8457 1.06 0.82	0.5219 2.47 1.67	(8,5)	
			Reference------ Case I---------- Case II---------		0.6614 1.72 1.10	2.3176 2.83 1.54	(8,6)	
						(7,7)		

Fig. 3. Results for the Zion-1 core benthmark problem.

References

1. P.C. KALAMBOKAS and A.F. HENRY, "The Representation of Light-Water Reflectors by Boundary Conditions," *Nucl. Sci. Eng.*, **61**, 181 (1976).
2. M. SEGEV, "Application of Light Water Reactor Core/Reflector Boundary Conditions," *Nucl. Sci. Eng.*, **90**, 221 (1985).
3. YUNG-AN CHAO and C.A. SUO, "A Two-Dimensional Two-Group Albedo Model for Pressed Water Reactor Reflector, *Nucl. Sci. Eng.*, **88**, 103 (1984).
4. K. KOEBKE, H. HAASE, L. HETZELT and H. WINTER, "Application and Verification of the Simplified Equivalence Theory for Burnup States, *Nucl. Sci. Eng.*, **92**, 56 (1986).
5. K. SMITH, "Assembly Homogenization Techniques for Light Water Reactor Analysis," *Progress in Nuclear Energy*, **17**, 303, 1986.
6. MASAFUMI ITAGAKI and CARLOS A. BREBBIA, "Space-Dependent Core/Reflector Boundary Conditions Generated by the Boundary Element Method for Pressurized Water Reactors," *Nucl. Sci. Eng.*, **107**, 246 (1991).