# 자기학습 퍼지제어기를 이용한 원형 역진자 시스템의 안정화 및 위치 제어

# Balancing and Position Control of an Circular Inverted Pendulum System Using Self-Learning Fuzzy Controller

김 용 태 변 증 남
한국과학기술원 전기 및 전자공학과

Yong-Tae Kim and Zeungnam Bien
Dept. of Electrical Engineering,
Korea Advanced Institute of Science and Technology,
373-1, Kusong-dong, Yusong-Ku, Taejeon, 305-701, KOREA

**Abstract :** In the paper is proposed a hierarchical self-learning fuzzy controller for balancing and position control of an circular inverted pendulum system. To stabilize the pendulum at a specified position, the hierarchical fuzzy controller consists of a supervisory controller, a self-learning fuzzy controller, and a forced disturbance generator. Simulation example shows the effectiveness of the proposed method.

**Keywords** – Inverted Pendulum, Fuzzy Control, Self-Learning Fuzzy Control, Multi-Objective

## 1. Introduction

The inverted pendulum system is a typical example of an unstable nonlinear control system which is difficult to control. In fact, the inverted pendulum problem has been widely used as an example to test new control concepts as well as demonstrate the effectiveness of modern control theory. Control objectives of the inverted pendulum system can be swinging-up the pendulum, balancing of the pendulum at the upright position, and regulation of the cart at an arbitrary specified position. There have been a large amount of research efforts : stabilization of double inverted pendulum on an inclined rail [1], swing up/stabilization of the pendulum with scheduled control input [2], attitude control of a triple-inverted pendulum [3], control of the pendulum with a angular motion type cart [4], swing up control of the pendulum with tree search technique [6], and linear control of the pendulum with a random search [5].

Also, many researchers have used stabilizing problem of the inverted pendulum for demonstrating the success of their learning control methods. After Barto *et.al.*[7] proposed the neural network-based balancing controller, a number of neural network-based learning controllers have been developed [8, 9, 10]. Also, Ha [11] proposed a fuzzy control scheme with multiple rule bases and rule supervisors for swing-up, balancing, and position control. However, while taking nonlinear nature of the pendulum system into account, it is difficult to design rule supervisors and arbitrating rule bases which are implemented to achieve above three objectives.

In this paper, balancing and position control of an circular inverted pendulum system are considered. To stabilize the pendulum at an arbitrary specified position, a fuzzy control system with a robust self-learning algorithm, a forced disturbance generator, and a supervisory decision maker is developed. Simulation example is provided to verify the proposed method. In Section 2, we describe the circular inverted pendulum problem. In Section 3, to solve the problem, a new fuzzy logic-based learning control system is proposed. In Section 4, simulation results are given to verify the proposed controller. Section 5 concludes this paper.

## 2. Problem Description

we consider a circular inverted pendulum shown in Fig.1. The dynamics of the pendulum system is [11] :

$$(J_1 + M_e l_1^2 + ML_e^2 sin\theta_2)\ddot{\theta}_1 - l_1 ML_e cos\theta_2 \ddot{\theta}_2 + (B_1 + ML_e^2 sin2\theta_2\dot{\theta}_2)\dot{\theta}_1 + l_1 ML_e sin\theta_2 \dot{\theta}_2^2 = \tau \quad (1)$$

$$l_1 ML_e cos\theta_2 \ddot{\theta}_1 - (J_2 + ML_e^2)\ddot{\theta}_2 + ML_e g sin\theta_2 + ML_e^2 sin2\theta_2 cos\theta_2 \dot{\theta}_1^2 - B_2\dot{\theta}_2 = 0 \quad (2)$$

where $M_e = m_2 + m_\tau$, $ML_e = m_2 l_{c2} + m_\tau l_2$, $ML_e^2 =$

$m_2l_{c2}^2 + m_r l_2^2$. $\tau$ is the motor torque, $\theta_i$ is the angular position of the $i-th$ link, $l_{c2}$ is the distance from the rotating axis of Link 2 to its center of mass, and $J_i$, $m_i$, $l_i$ and $B_i$ are the mass moment of inertia, mass, length, and rotational damping of the $i$th link, respectively.

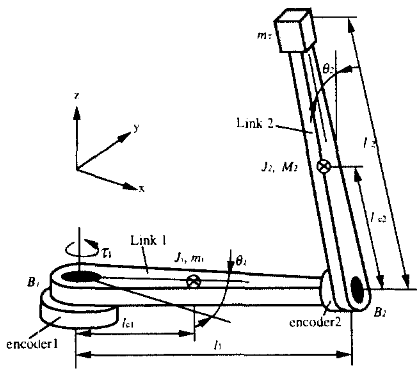



Fig 1. Circular inverted pendulum system.

In this paper, balancing of the pendulum(Link 2) and position control of the arm(Link 1) are considered. Swing up controllers of pendulum were proposed by some researchers [2, 6, 11]. It can be easily designed by using fuzzy rule tables which is constructed by examining its mathematical model and understanding its dynamic behavior [11]. Also, a fuzzy controller for balancing of the pendulum and a fuzzy controller for regulation of the arm can be easily designed [11]. However, it is difficult to design fuzzy controller that satisfy two control objectives, that is, balancing of the pendulum and regulation of the arm at arbitrary position. To solve the problem, Ha [11] uses another rule base to arbitrate both objectives. Because control rules for balancing of the pendulum and control rules for position control of the arm can be opposite, it is very difficult to obtain the arbitration rules. In addition, it is more difficult to design a learning control algorithm that satisfy both objectives. In the paper, to solve this problem, a hierarchical self-learning fuzzy controller is developed.

## 3. Design of Hierarchical Self-Learning Fuzzy Controller

We consider two control objectives of the circular inverted pendulum system as follows.

**Objective I :** Position control of arm at its target position from an arbitrary initial position.

**Objective II :** Balancing of pendulum at the upright position.

To achieve this two objectives, a hierarchical fuzzy control system with three rule bases and a robust learning algorithm is developed considering its dynamic behavior as well as human's control strategies to stabilize the pendulum at a specified position. Human's control strategy is as follows.

**Step 1.** Stabilize the pendulum at an arbitrary position.

**Step 2.** If arm is not at the specified position, move arm toward the opposite direction of the specified position until angle error of pendulum reaches to certain value that is determined according to the distance from the desired position(Fig. 2).

**Step 3.** Stabilize the pendulum as fast as possible.

**Step 4.** If arm is at the specified position, stop. Otherwise, continue with Step 2.

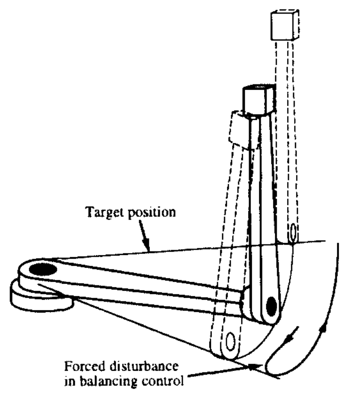



Fig 2. Graphical description of human's control strategy.

Fuzzy control can be used as an effective mean to capture the human's expertised knowledge and achieve multiple control objectives of nonlinear systems. Therefore, the proposed controller is designed based on the above human's control strategy by using the fuzzy control theory. Overall structure of the proposed controller is shown in Fig.3.

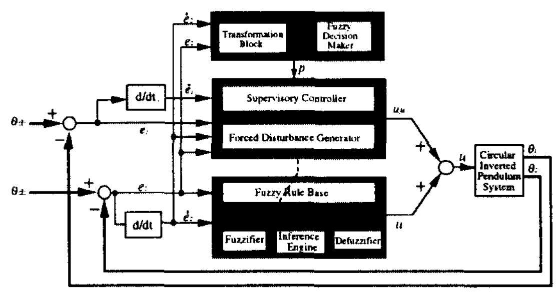



Fig 3. Overall structure of the proposed controller.

It includes a supervisory controller, three fuzzy rule tables for balancing of pendulum, position control of arm, and decision of control performance. Supervisory controller decides whether the system is in the balancing phase or forced disturbance phase(Step 2) by monitoring the plant states. Rule table in the forced disturbance phase can be obtained from human's experience and insight(Fig. 4). Balancing rules can be learned using fuzzy logic-based learning algorithms, independently. However, the control action in the forced disturbance phase generate similar situation(Fig. 5) produced by external disturbance and the balancing rules will be modified according to the performance decision. It should be noted that the balancing rules will be continuously modified according to the performance evaluation as long as the states are disturbed by the control action in the forced disturbance phase even though the states may move satisfactorily toward the band. This implies that when there exist frequent control actions by the forced disturbance, the control performance can be eventually deteriorated by the repeated modification of balancing rules [13].

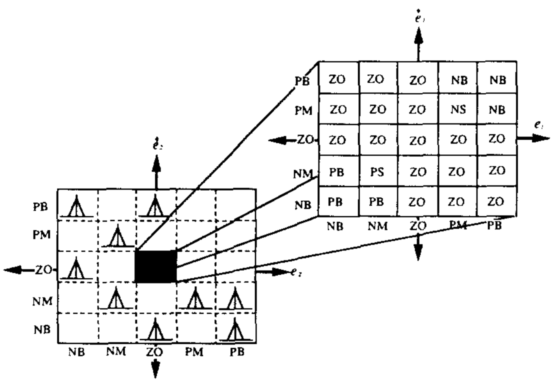



Fig 4. Rule tables of the proposed controller.

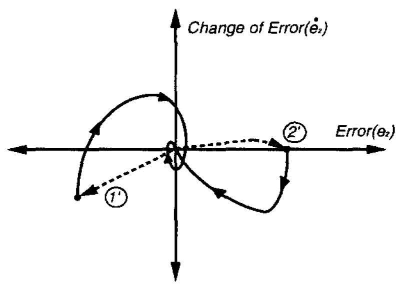



Fig 5. State trajectories in the forced disturbance phase.

To overcome the problem, the robust learning algorithm proposed by Kim and Bien [13] is used to learn the balancing rules of the pendulum. We choose a sliding line as

$$s = \dot{e}_2 + \lambda e_2 , \quad (3)$$

where $\lambda$ is positive constant, and the reaching condition as

$$V_r = \frac{1}{2} \frac{d}{dt} s^2 = s \dot{s} \leq -\eta s^2 \quad (4)$$

where $\eta$ is positive constant. Then, a performance decision table can be designed as shown in Fig. 6.

| $\dot{s}$ \ $s$ | NB | NS | ZO | PS | PB |
|---|---|---|---|---|---|
| PB | ZO | ZO | ZO | NB | NB |
| PS | PS | ZO | ZO | NS | NB |
| ZO | PS | PS | ZO | NS | NS |
| NS | PB | PS | ZO | ZO | NS |
| NB | PB | PB | ZO | ZO | ZO |

Fig 6. Fuzzy performance decision maker

Let us consider that a fuzzy logic controller has singleton fuzzifier, product-sum inference, center of average defuzzifier [12] as follows :

$$u = \frac{\sum_{k=1}^{N} \Pi_{i=1}^{M} \mu_{A_{i,k}}(x_i)\, v_k}{\sum_{k=1}^{N} \Pi_{i=1}^{M} \mu_{A_{i,k}}(x_i)}, \quad (5)$$

where $M$ and $N$ represent the numbers of input variables and total rules, respectively, and $\mu_{A_{i,k}}$ denotes the membership function of the $k$th input fuzzy set for the $i$th input variable. $v_k$ denotes the point at which the output membership function of the $k$th rule attains its maximum.

To simplify our notation, we can define the normalized firing strength of each control rule as $\theta_k = \Pi_{i=1}^{M} \mu_{A_{i,k}}(x_i) / \sum_{k=1}^{N} \Pi_{i=1}^{M} \mu_{A_{i,k}}(x_i)$. Then, the fuzzy controller can be written as

$$u = \sum_{k=1}^{N} \theta_k \times v_k . \quad (6)$$

The proposed learning algorithm is as follows :

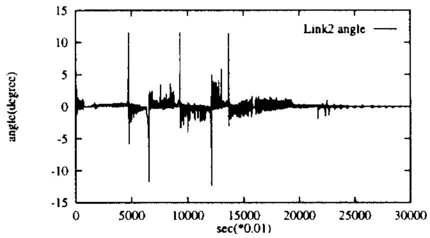
$$\dot{v}_k = -k_v \cdot \theta_k \cdot p, \quad k = 1, 2, \cdots, N \quad (7)$$

where $k_v$ is the learning rate, $\theta_k$ is the normalized firing strength, and $p$ is the rule modification index. The fuzzy singleton output is updated by both the normalized firing strength that denotes the contribution of the rule to the control input of the plant and the rule modification index that is calculated by the fuzzy performance decision maker.

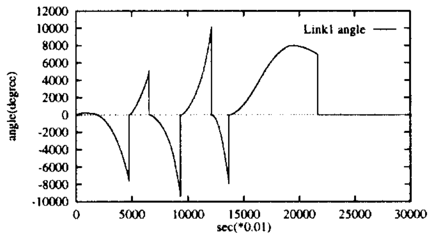
## 4. Simulation Results

we apply the developed hierarchical self-learning fuzzy controller to the balancing and the position control of the circular inverted pendulum system. Parameters of the system are as follows : $l_1 = 0.22\ m$, $l_2 = 0.12\ m$, $m_2 = 0.055\ Kg$, $J_1 = 0.0175\ Kg \cdot m^2$, $J_2 = 1.98 \times 10^{-4}\ Kg \cdot m^2$, $B_1 = 0.118\ N \cdot m \cdot s$, and $B_2 = 8.3 \times 10^{-5}\ N \cdot m \cdot s$. We choose the design parameters as : $\lambda = 10.0$, $\eta = 30.0$. The simulation results are shown in Fig. 7. These results show that the proposed self-learning fuzzy controller can be successfully applied to the balancing and the regulation of the circular inverted pendulum. Fig. 8 shows the simulation results after the learning process is completed.

The results show that the proposed controller fails to balance the pendulum and to regulate the arm for several trials and eventually the controller can balance the pendulum near the specified position.

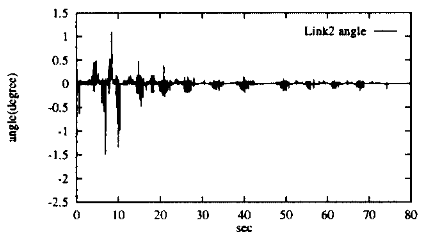



(a) Link 2 angle $\theta_2$.

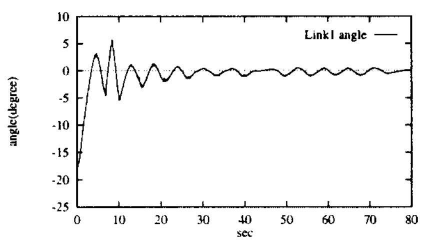



(b) Link 1 angle $\theta_1$.

Fig 7. Simulation results for balancing and regulation.




(a) Link 2 angle $\theta_2$.




(b) Link 1 angle $\theta_1$.

Fig 8. Simulation results after leaning.

## 5. Conclusions

In the paper, a hierarchical fuzzy controller for balancing and position control of the circular inverted pendulum is proposed. To satisfy two control objectives, the hierarchical fuzzy controller consists of a self-learning fuzzy controller for balancing, a forced disturbance generator which emulates human's control strategy, and a supervisory controller for the arbitration of two control objectives. Simulation example shows the effectiveness of the proposed method.

## References


[1] K. Furuta, K. Hiroyuki and K. Kosuge, "Digital Control of a Double Inverted Pendulum on a Inclined Rail," *Int. Journal of Control*, Vol. 32, No. 5, pp. 907-924, 1980.

[2] S. Mori, H. Nishihara and K. Furuta, " Control of Unstable Mechanical System - Control of Pendulum," *Int. Journal of Control*, Vol. 23, No. 5, pp. 673-692, 1976.

[3] K. Furuta, T. Ochiai and N. Ono, "Attitude Control of a Triple Inverted Pendulum," *Int. Journal of Control*, Vol. 39, No. 6, pp. 1351-1365, 1984.

[4] K. Furuta, M. Yamakita and S. Kobayashi, "Swing Up Control of Inverted Pendulum ," *Porc. of IEEE Int. Conf. on Industrial Electronics*, pp. 2193-2198, 1991.

[5] S. Geva and J. Sitte, "A Cartpole Experiment Benchmark for Trainable Controllers," *IEEE Control System Magazine*, October, pp. 40-51, 1993.

[6] Y. Xia and H. Inooka, "Application of Tree Search to the Switching Control of a Pendulum," *IEEE Tr. on Systems, Man, and Cybernetics*, Vol. 22, No. 5, pp. 1193-1198, 1992.

[7] A.G. Barto, R.S. Sutton and C.H. Anderson, "Neuronlike Adaptive Elements That Can Solve Difficult Learning Control Problems," *IEEE Trans. Sys. Man and Cyb.*, Vol. 13, No. 5, pp. 834-846, 1983.

[8] B. Widrow, "The Original Adaptive Neural Net Broom-Balancer," *Proc. IEEE Int. Symp. Circuits and Systems*, pp. 351-357, 1987.

[9] R. Hecht-Nielsen, *Neurocomputing*, Reading, MA: Addison-Wesley , pp. 342-345, 1990.

[10] C.C. Lee and H.R. Berenji, "An Intelligent Controller Based on Approximate Reasoning and Reinforcement Learning," *Proc. of IEEE Int. Sysmp. on Intelligent Control*, pp. 200-205, 1989.

[11] Y.K. Ha and A. Tomizuka, "Fuzzy Global and Local Motion Control of an Inverted Pendulum Using Multiple Rule Bases," *Proc. of Asian Control Conf.*, July, pp. 27-30, 1994.

[12] L.X. Wang, "Stable Adaptive Fuzzy Control of Nonlinear Systems," *IEEE Tr. on Fuzzy Systems*, Vol. 1, No. 2, pp. 146-155, 1993.

[13] Y.T. Kim and Z. Bien, "Robust Self-Leaning Fuzzy Logic Controller," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 1172-1177, 1995.