FUZZY LESS STRONGLY IRRESOLUTE MAPPINGS AND FUZZY LESS STRONGLY SEMI-CONNECTED SETS

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Abstract: In this paper, we first introduce fuzzy less strongly irresolute, fuzzy pre-less strongly semiopen and fuzzy pre-less strongly semiclosed mappings on fuzzy topological space, and establish their various characteristic properties. Finally, we introduce and study fuzzy less strongly semi-connectedness with the help of fuzzy less strongly semiopen sets and fuzzy less strongly semi-q-neighborhoods.

1. INTRODUCTION AND PRELIMINARIES

Fuzzy topological spaces and fuzzy continuity were introduced in Chang [4] early in 1968. The concepts of fuzzy continuity play an important role in fuzzy topological space. Along this line, many workers [1-3, 5-8, 10-14, 17] have introduced and studied fuzzy non-continuity. Recently, [6] introduced and studied fuzzy less strongly semiopen set, as weaker form of fuzzy strongly semiopen set [2,3], and showed that fuzzy preopen set [3] and fuzzy less strongly semiopen set are independent concepts. Using this concept, Fang [6] also introduced fuzzy less strongly semicontinuous mapping as weaker form of fuzzy continuous mapping. Here in section 2 of this paper we introduce the fuzzy less strongly irresolute, fuzzy pre-less strongly semiopen and fuzzy pre-less strongly semiclosed mappings and obtain some of their characteristic properties. Finally, in section 3, we introduce and study fuzzy less strongly semi-connectedness of a fuzzy set.

Throughout the paper by (X, τ) or simply by X, we mean a fuzzy topological space (for short, fts) due to Chang [4]. A fuzzy point [15] with support $x \in X$ and value α $(0 < \alpha \leq 1)$ is denoted by x_α. For a fuzzy set A in X, $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and A' will respectively denote the closure, interior and complement of A, whereas the constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X, respectively. A fuzzy set A in X is said to be q-coincident with a fuzzy set B, denoted by AqB, if there exists a $x \in X$ such that $A(x) + B(x) > 1$ [15]. It is known [15] that $A \leq B$ if and only if A and B' are not q-coincident, denoted by $A\bar{q}B$. For definitions and results not explained in this paper, the reader is referred [1,2,6,15] in the assumption they are well known.

Definition 1.1. Let A be a fuzzy set in a fts (X, τ). Then A is called

(a) a fuzzy semiopen if there exists a fuzzy open set B in X such that $B \leq A \leq \mathrm{Cl}(B)$ [1],

(b) a fuzzy semiclosed if there exists a fuzzy closed set B in X such that $\mathrm{Int}(B) \leq A \leq B$ [1],

(c) a fuzzy preopen if $A \leq \mathrm{Int}(\mathrm{Cl}(A))$ [3],

(d) a fuzzy preclosed if $\mathrm{Cl}(\mathrm{Int}(A)) \leq A$ [3],

(e) a fuzzy strongly semiopen if there exists a fuzzy open set B in X such that $B \leq A \leq \mathrm{Int}(\mathrm{Cl}(B))$ [2,3],

(f) a fuzzy strongly semiclosed if there exists a fuzzy closed set B in X such that $\mathrm{Cl}(\mathrm{Int}(B)) \leq A \leq B$ [2,3],

(g) a fuzzy less strongly semiopen if there exists a fuzzy open set B in X such that $B \leq A \leq \mathrm{sInt}(\mathrm{sCl}(B))$ [6],

(h) a fuzzy less strongly semiclosed if there exists a fuzzy closed set B in X such that $\mathrm{sCl}(\mathrm{sInt}(B)) \leq A \leq B$ [6].

Obviously, every fuzzy open set is a fuzzy strongly semiopen set, and every fuzzy strongly semiopen set is a fuzzy less strongly semiopen set, and every fuzzy less strongly semiopen set is a fuzzy semiopen set. That none of converses need be true [2,6]. Also, fuzzy less strongly semiopen set and fuzzy preopen set are independent [6].

Theorem 1.2 [6]. *For a fuzzy set A in a fts (X, τ), the following are valid:*

(a) $Int(A) \leq A^{\Delta} \leq A_{\Delta} \leq sInt(A) \leq A \leq sCl(A) \leq A_{\sim} \leq A^{\sim} \leq Cl(A)$.

(b) A is fuzzy less strongly semiopen if and only if $A = A_{\Delta}$.

(c) A is fuzzy less strongly semiclosed if and only if $A = A_{\sim}$.

Theorem 1.3. *If A is any fuzzy set and B is a fuzzy less strongly semiopen set in a fts X such that $A\bar{q}B$, then $A_{\sim}\bar{q}B$.*

2. Fuzzy less strongly irresolute, fuzzy pre-less strongly semiopen and fuzzy pre-less strongly semiclosed mappings

Definition 2.1 [6]. A mapping $f : X \to Y$ is said to be fuzzy less strongly semicontinuous (briefly, f.l.s.c.) if $f^{-1}(V)$ is fuzzy less strongly semiopen in X for each fuzzy open set V in Y.

Theorem 2.2. *For a mapping $f : X \to Y$, the following are equivalent:*

(a) f is f.l.s.c.

(b) $sCl(sInt(Cl(f^{-1}(B)))) \leq f^{-1}(Cl(B))$ *for each fuzzy set B in Y.*

(c) $f(sCl(sInt(Cl(A)))) \leq Cl(f(A))$ *for each fuzzy set A in X.*

Definition 2.3. A mapping $f : X \to Y$ is said to be fuzzy less strongly irresolute (briefly, f.l.s.i.) if $f^{-1}(V)$ is fuzzy less strongly semiopen in X for each fuzzy less strongly semiopen set V in Y.

Every f.l.s.i. mapping is f.l.s.c., but the converse is not true by the following Example 2.4. Also, the following examples show that f.l.s.i. mapping and fuzzy continuous mapping are independent.

Example 2.4. Let $X = \{a, b, c\}$ and A, B are fuzzy sets in X defined as follows:

$$A(a) = 0.7, \quad A(b) = 0.3, \quad A(c) = 0.6;$$
$$B(a) = 0.3, \quad B(b) = 0.2, \quad B(c) = 0.4.$$

Let $\tau_1 = \{1_X, 0_X, A, B\}$ and $\tau_2 = \{1_X, 0_X, A\}$. Consider the identity mapping $f : (X, \tau_1) \to (Y, \tau_2)$. Then f is fuzzy continuous and hence f.l.s.c. But f is not f.l.s.i.

Example 2.5. Let $X = \{a, b, c\}$ and A, B are fuzzy sets in X defined as follows:

$$A(a) = 0.7, \quad A(b) = 0.3, \quad A(c) = 0.6;$$
$$B(a) = 0.7, \quad B(b) = 0.8, \quad B(c) = 0.7.$$

Let $\tau_1 = \{1_X, 0_X, A\}$ and $\tau_2 = \{1_X, 0_X, B\}$. Consider the identity mapping $f : (X, \tau_1) \to (Y, \tau_2)$. Then f is f.l.s.i. but not fuzzy continuous.

Theorem 2.6. *For a mapping $f : X \to Y$, the following are equivalent:*

(a) f is f.l.s.i.

(b) For each fuzzy point x_α in X and each fuzzy Lss-nbd V of $f(x_\alpha)$, there exists a fuzzy Lss-nbd U of x_α such that $f(U) \le V$.

(c) For each fuzzy point x_α in X and each fuzzy Lss-q-nbd V of $f(x_\alpha)$, there exists a fuzzy Lss-q-nbd U of x_α such that $f(U) \le V$.

(d) The inverse image of every fuzzy less strongly semiclosed set in Y is fuzzy less strongly semiclosed in X.

(e) $f(A_\sim) \le f(A)_\sim$ for each fuzzy set A in X.

(f) $f^{-1}(B)_\sim \le f^{-1}(B_\sim)$ for each fuzzy set B in Y.

(g) $f^{-1}(B_\Delta) \le f^{-1}(B)_\Delta$ for each fuzzy set B in Y.

Theorem 2.7. *Let $f : X \to Y$ be one-to-one and onto. Then f is f.l.s.i. if and only if $f(A)_\Delta \le f(A_\Delta)$ for each fuzzy set A in X.*

Definition 2.8. Let $f : X \to Y$ be a mapping from a fts X to another fts Y. Then f is called

(a) fuzzy pre-less strongly semiopen (briefly, f.p.l.s. semiopen) if $f^{-1}(V)$ is fuzzy less strongly semiopen in X for each fuzzy less strongly semiopen set V in Y.

(b) fuzzy pre-less strongly semiclosed (briefly, f.p.l.s. semiclosed) if $f^{-1}(V)$ is fuzzy less strongly semiclosed in X for each fuzzy less strongly semiclosed set V in Y.

Obviously, every f.p.l.s. semiopen (resp. f.p.l.s. semiclosed) mapping is fuzzy less strongly semiopen (briefly, f.l.s. semiopen) [6] (resp. fuzzy less strongly semiclosed (briefly, f.l.s. semiclosed) [6]). But the converses need not be true, in general.

Theorem 2.9. *For a mapping $f : X \to Y$, the following are equivalent:*

(a) f is f.p.l.s. semiopen.

(b) $f(A_\Delta) \leq f(A)_\Delta$ for each fuzzy set A in X.

(c) $f^{-1}(B)_\Delta \leq f^{-1}(B_\Delta)$ for each fuzzy set B in Y.

(d) For each fuzzy set B in Y and each fuzzy less strongly semiclosed set A in X such that $f^{-1}(B) \leq A$, there exists a fuzzy less strongly semiclosed set C in Y such that $B \leq C$ and $f^{-1}(C) \leq A$.

Theorem 2.10. *For a mapping $f : X \to Y$, the following are equivalent:*

(a) f is f.p.l.s. semiclosed.

(b) $f(A)_\sim \leq f(A_\sim)$ for each fuzzy set A in X.

(c) For each fuzzy set B in Y and each fuzzy less strongly semiopen set A in X such that $f^{-1}(B) \leq A$, there exists a fuzzy less strongly semiopen set C in Y such that $B \leq C$ and $f^{-1}(C) \leq A$.

Theorem 2.11. *Let $f : X \to Y$ be one-to-one and onto. Then f is f.p.l.s. semiclosed if and only if $f^{-1}(B_\sim) \leq f^{-1}(B)_\sim$ for each fuzzy set B in Y.*

Theorem 2.12. *Let $f : X \to Y$ and $g : Y \to Z$ be mappings.*

(a) If f and g are f.l.s.i., then $g \circ f$ is f.l.s.i.

(b) If f is f.l.s.i. and g is f.l.s.c., then $g \circ f$ is f.l.s.c.

(c) If f and g are f.p.l.s. semiopen (resp. f.p.l.s. semiclosed), then $g \circ f$ is f.p.l.s. semiopen (resp. f.p.l.s. semiclosed).

(d) If f is f.l.s. semiopen (resp. f.l.s. semiclosed) and g is f.p.l.s. semiopen (resp. f.p.l.s. semiclosed), then $g \circ f$ is f.l.s. semiopen (resp. f.l.s. semiclosed).

3. Fuzzy less strongly semi-connected sets

Definition 3.1. Two non-null fuzzy sets A and B in a fts X are said to be fuzzy less strongly semi-separated (briefly, fuzzy Lss-separated) if $A\bar{q}B_\sim$ and $B\bar{q}A_\sim$.

For any two non-null fuzzy sets A and B in a fts X, the following implications hold: fuzzy separated [7] $\Rightarrow$ fuzzy Lss-separated $\Rightarrow$ fuzzy semi-separated [8].

Theorem 3.2. *Let A and B be non-null fuzzy sets in a fts X.*

(a) If A and B are fuzzy Lss-separated, and A_1 and B_1 are non-null fuzzy sets such that $A_1 \leq A$ and $B_1 \leq B$, then A_1 and B_1 are fuzzy Lss-separated.

(b) If $A\bar{q}B$ and either both fuzzy less strongly semiopen or both fuzzy less strongly semi-closed, then A and B are fuzzy Lss-separated.

(c) If A and B are either both fuzzy less strongly semiopen or fuzzy less strongly semi-closed, and if $C_A(B) = A \wedge B'$ and $C_B(A) = B \wedge A'$, then $C_A(B)$ and $C_B(A)$ are fuzzy Lss-separated.

Theorem 3.3. *Two non-null fuzzy sets A and B are fuzzy Lss-separated if and only if there exist fuzzy less strongly semiopen sets U and V such that $A \leq U$, $B \leq V$, $A\bar{q}V$ and $B\bar{q}U$.*

Definition 3.4. A fuzzy set which can not be expressed as the union of two fuzzy Lss-separated sets is said to be a fuzzy less strongly semi-connected (briefly, fuzzy Lss-connected).

For any non-null fuzzy set A in a fts X, the following implications hold: fuzzy semi-connected [8] $\Rightarrow$ fuzzy Lss-connected $\Rightarrow$ fuzzy connected [7].

Example 3.5. Let $X = \{a,b,c\}$ and A a fuzzy set of X defined by $A(a) = 0.4$, $A(b) = 0.2$, $A(c) = 0.2$. Then $\tau = \{1_X, 0_X, A\}$ is a fts. Consider fuzzy sets B and C of X defined as follows:

$$B(a) = 0.5, \quad B(b) = 0.3, \quad B(c) = 0.3;$$
$$C(a) = 0.4, \quad C(b) = 0.3, \quad C(c) = 0.3.$$

Since $\mathrm{sCl}(B) = B$ and $\mathrm{sCl}(C) = C$, $\mathrm{sCl}(B)\bar{q}C$ and $\mathrm{sCl}(C)\bar{q}B$. Hence B and C are fuzzy semi-separated. But $B_\sim = C_\sim = A'$. Then $B_\sim \mathrm{q}C$ and $C_\sim \mathrm{q}B$ and hence B and C are not fuzzy Lss-separated. Thus $B = B \vee C$ is fuzzy Lss-connected but not fuzzy semi-connected.

Example 3.6. Let $X = \{a,b,c\}$ and A a fuzzy set of X defined by $A(a) = 0.7$, $A(b) = 0.3$, $A(c) = 0.6$. Then $\tau = \{1_X, 0_X, A\}$ is a fts. Consider fuzzy sets B and C of X defined as follows:

$$B(a) = 0.2, \quad B(b) = 0.6, \quad B(c) = 0.4;$$
$$C(a) = 0.2, \quad C(b) = 0.4, \quad C(c) = 0.4.$$

Since $B_\sim = B$ and $C_\sim = C$, $B_\sim \bar{q}C$ and $C_\sim \bar{q}B$. Hence B and C are fuzzy Lss-separated. But $\mathrm{Cl}(B) = \mathrm{Cl}(C) = A'$. Then $\mathrm{Cl}(B)\mathrm{q}C$ and $\mathrm{Cl}(C)\mathrm{q}B$ and hence B and C are not fuzzy separated. Thus $B = B \vee C$ is fuzzy connected but not fuzzy Lss-connected.

Theorem 3.7. *Let A be a non-null fuzzy Lss-connected set in a fts X. If A is contained in the union of two fuzzy Lss-separated sets B and C, then exactly one of the following condition (a) and (b) holds:*

(a) $A \leq B$ and $A \wedge C = 0_X$.

(b) $A \leq C$ and $A \wedge B = 0_X$.

Theorem 3.8. *Let $f : X \to Y$ be onto.*

(a) If f is f.l.s.i. and A is fuzzy Lss-connected set in X, then $f(A)$ is fuzzy Lss-connected set in Y.

(b) If f is f.l.s.c. and A is fuzzy Lss-connected set in X, then $f(A)$ is fuzzy connected set in Y.

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