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Fuzzy ordered filter of implicative commutative semigroups

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Abstract. We introduce fuzzy ordered filter, fuzzy weakly implicative ordered filter and fuzzy implicative ordered filter of implicative commutative semigroups and prove and some results.

1. Introduction.

Chan and shum[1] were investigated the notion of an ordered filter of implicative semigroups and various properties. Jie Meng[10] proved that implicative commutative semigroups are equivalent to BCK-algebras with condition(s). In this paper, using the notion of negatively partially ordered implicative semigroups, we prove that a fuzzy implicative ordered filter of implicative semigroup is a fuzzy ordered filter.

2. Preliminaries.

Definition2.1 An algebraic system $< X, \leq, \cdot, *, 1 >$ where $\leq$ is a binary relation on X, $\cdot$ and $*$ are binary operation (that is constant element) is called a negatively partially ordered implicative semigroup, if it satisfies the following:

(1) $< X, \leq >$ is a partially ordered set

(2) $< X, \cdot >$ is a semigroup

(3) $x \leq y$ implies $x \cdot z \leq y \cdot z$ and $z \cdot x \leq z \cdot y$ for all x,y,z $\in$ X

(4) $x \cdot y \leq x$ and $x \cdot y \leq y$ for all $x, y \in X$

(5) $z \leq x * y$ iff $z \cdot x \leq y$ for all $x, y, z \in X$

From now on a negatively partially ordered implicative semigroup is simply called an implicative semigroup.

Definition2.2 An implicative semigroup $< X; \leq, \cdot, *, 1 >$ is said to be commutative, if it satisfies $x \cdot y = y \cdot x$ for all $x, y \in X$. That is $< X, \cdot >$ is a commutative semigroup.

Proposition2.3 Let $< X, \leq, \cdot, *, 1 >$ be an implicative semigroup, then for $x, y, z \in X$ we have

(6) $x * x = 1$

(7) $x = 1 * x$

(8) $x \leq y * (x \cdot y)$

(9) $x \leq y$ implies $x * z \geq y * z$ and $z * x \leq z * y$

(10) $x \leq 1$

(11) $x \leq y$ if and only if $x * y = 1$

(12) $x * (y * z) = (x \cdot y) * z$

(13) if $< X, \cdot >$ is commutative, then $x * y \leq (z \cdot x) * (z \cdot y)$

Theorem 2.4 Suppose that $< X, \leq, \cdot, *, 1 >$ is an implicative commutative semigroup. Then the following hold; for all x, y, z in X

(14) $x * (y * z) = y * (x * z)$

(15) $y * ((y * x) * x) = 1$

(16) $(y * z) * ((z * x) * (y * x)) = 1$

(17) $(y * z) * ((x * y) * (x * z)) = 1$

Definition 2.5 Let $< X, \leq, \cdot, *, 1 >$ be an implicative semigroup, F is a nonempty subset of X. F is called an ordered filter of X, if for any x, y $\in X$,

(F_1) $x \cdot y \in F$ whenever x,y $\in F$ that is, F is a subsemigroup of X,

(F_2) $x \in F$ and $x \leq y$ imply $y \in F$.

Theorem 2.6 Let $< X, \leq, \cdot, *, 1 >$ be an implicative semigroup F a nonempty subset of X. Then F is an ordered filter of X if and only if it satisfies

(a) $1 \in F$

(b) for all $x, y \in X$, $x * y \in F$ and $x \in F$ imply $y \in F$.

3. Fuzzy ordered filter

Throughout this paper, X denotes an implicative commutative semigroup.

Definition 3.1 A function $\mu : X \longrightarrow [0, 1]$ is called a fuzzy ordered filter of X, if for any x,y $\in$ X, we have

a) $\mu(1) \geq \mu(x)$

b) $\mu(y) \geq \mu(x * y) \wedge \mu(x)$

Theorem 3.2 A fuzzy subset μ of X is a fuzzy ordered filter of X if and only if for every $t \in [0,1]$, $\mu_t = \{x | x \in X, \mu(x) \geq t\}$ is ordered filter of X, when $\mu_t \neq \emptyset$

Theorem 3.3 If a fuzzy subset μ is an arbitrary fuzzy ordered filter for any $x, y \in X$

(18) If $x \leq y$ then $\mu(x) \leq \mu(y)$

(19) $\mu(z * y) \geq \mu(x * y) \wedge \mu(z * x)$

(20) If $x \leq y * z$ then $\mu(z) \geq \mu(x) \wedge \mu(y)$

(21) $\mu(z * (z * y)) \geq \mu(z * (x * y)) \wedge \mu(z * x)$

Definition 3.4 A function $\mu : X \longrightarrow [0,1]$ is called a fuzzy weakly implicative ordered filter of X, if for any $x, y \in X$, $\mu(z * (z * y)) \geq \mu(z * (x * y)) \wedge \mu(z * x)$

Theorem 3.5 A fuzzy subset μ of X is a fuzzy weakly implicative ordered filter of X if and only if μ is a fuzzy ordered filter.

Definition 3.6 A function $\mu : X \longrightarrow [0,1]$ is called a fuzzy implicative ordered filter of X, if for any $x, y \in X$, $\mu(z * y) \geq \mu(z * (x * y)) \wedge \mu(z * x)$.

Definition 3.7 If the equality (22) $(z * x) * (z * y) = z * (x * y)$ holds, then it is called a positive implicative.

Theorem 3.8 A fuzzy implicative ordered filter μ of X is a fuzzy ordered filter.

Theorem 3.9 If X is positive implicative, then a fuzzy ordered filter is a fuzzy implicative ordered filter.

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