## SIMPLE DESIGN APPROACHES FOR PILED FOUNDATIONS

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### ABSTRACT

*Although piled foundations are generally adopted in order to reduce settlement, pile design is still dominated by calculations of ultimate capacity, generally of single piles. There is a need to shift the emphasis of design more towards allowable settlements, particularly differential settlements. However, in order to achieve this, it must be possible to make reasonable assessment of the load-settlement response of the foundation, preferably through simple hand-calculations, rather than sophisticated numerical techniques.*

*This paper summarises a range of simple methods that are available to estimate the load-settlement response of single piles, pile groups, shallow (raft) foundations and piled raft foundations. The methods are then used as a basis for developing an optimum design approach for piled foundations, where appropriate allowance is made for the load-carrying capacity of the pile cap (or raft). The approaches are illustrated by model and field-scale measurements of piled raft performance, and also by case studies based on actual designs.*

## 1. INTRODUCTION

There are a number of compelling arguments for implementing settlement-based design approaches for piled foundations, rather than the current emphasis on the ultimate capacity of individual piles within the foundation, and the use of traditional factors of safety. Among these arguments are:

- the primary purpose of the piles is generally to reduce settlements to an acceptable level;
- the capacity of individual piles is highly sensitive to construction technique, while the pile head stiffness is much less affected;
- pile capacity may vary significantly across a site, perhaps due to different installation depths arising from application of driving formulae to determine acceptability of piles, and yet pile group performance will tend to average out these variations;
- the pile cap, or raft, may carry a significant proportion of the total load, and yet will generally have only a small effect on the overall settlement of the foundation.

The last argument, in particular, emphasises how traditional approaches to pile design may be largely irrelevant, since the actual load on each pile may be reduced due to the contribution of the pile cap, and also the pile cap itself may well lead to an enhanced capacity of each pile (Horikoshi and Randolph, 1995).

Another misconception is that the capacity of a pile is easier to estimate than the settlement under a given load. On the contrary, a wide variety of analytical and numerical techniques now exist for calculating the stiffness of individual piles, pile groups and piled raft foundations. A number of these will be reviewed in the present paper, with emphasis on simplicity, rather than sophisticated numerical analysis. The main limitation at present is often inadequate site investigation data from which to assess the in situ stiffness of the soil (in terms of elastic shear modulus, G, or Young's modulus, $E_s$). However, even here, recent advances through seismic techniques (Robertson et al, 1986; Baldi et al, 1989) and resulting improved correlations with SPT and cone resistance, now allow reasonably accurate assessment of in situ shear modulus values. Recent studies undertaken by Mandolini and Viggiani (1996) show that the small strain shear modulus obtained from seismic techniques is particularly applicable for estimating the settlement of pile groups.

This paper starts with a brief review of available solutions for assessing the settlement of single piles, pile groups, shallow (raft) foundations and piled raft foundations. These solutions are then applied to the results of recent centrifuge modelling of piled raft foundations, where central pile support was used to eliminate differential settlements. This theme is extended to show how optimum design of piled raft foundations may be achieved, minimising differential settlements with minimal pile support. The methods are illustrated through two case histories involving piled raft foundations for tall buildings.

## 2. SINGLE PILE AND PIER STIFFNESS

Elastic solutions for the axial response of a single pile have been presented by a number of authors, starting with the boundary element solutions of Poulos and Davis (1968), Poulos (1968) and Butterfield and Banerjee (1971). Randolph and Wroth (1978) presented an approximate solution, based on separate treatment of the pile shaft and the pile base. For the shaft, the relationship between local shear stress, $\tau_o$, and displacement, w, is given by

$$w = \frac{\zeta d}{2} \frac{\tau_o}{G} \qquad (1)$$

where d is the pile diameter, G is the local shear modulus of the soil and $\zeta$ is a parameter that is related to the maximum radius of influence of the pile. Randolph and Wroth (1978) proposed an expression for $\zeta$ for friction piles of

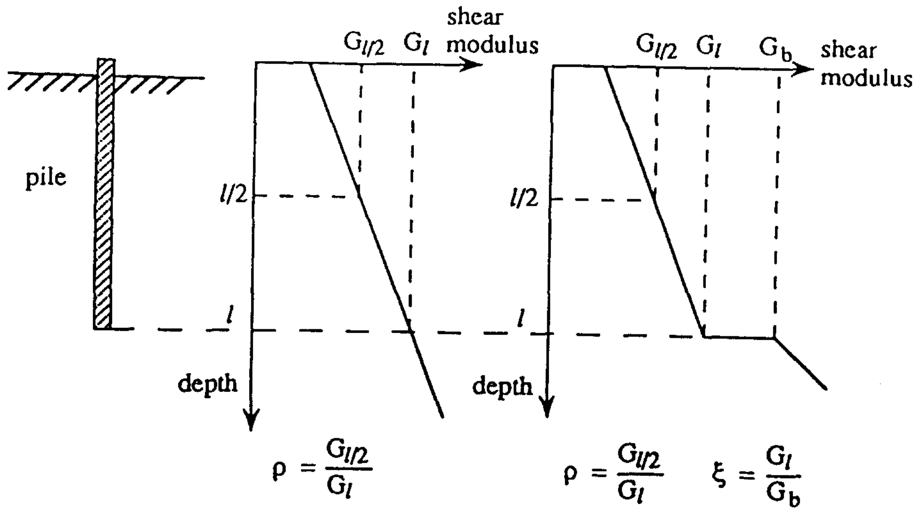
$$\zeta \approx \ln\left[5\rho(1-\nu)\ell / d\right] \qquad (2)$$

where $\nu$ is Poisson's ratio for the soil, $\ell$ is the embedded pile length, and $\rho$ is the ratio of the average shear modulus over the embedded pile length, to the value near (but above) the pile base. Thus the value of $\rho$ will lie in the range 0.5 (soil modulus proportional to depth) to 1 (homogeneous soil).

The full expression for the pile head response, in soil where the shear modulus distribution may be idealised as shown in Figure 1, is given by

$$\frac{P_t}{G_\ell d w_t} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu\ell}{\mu\ell}\dfrac{\ell}{d}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu\ell}{\mu\ell}\dfrac{\ell}{d}} \qquad (3)$$

where $P_t$ and $w_t$ are the load and displacement at the top of the pile, and the other parameters are given below, with the subscript b referring to conditions at or below the




(a) Floating pile (b) End-bearing pile

Figure 1 Idealised profiles for soil shear modulus

pile base:

| | |
|---|---|
| $\eta = d_b/d$ | (under-reamed piles) |
| $\xi = G_\ell/G_b$ | (end-bearing piles) |
| $\rho = G_{avg}/G_\ell$ | (heterogeneity of soil modulus) |
| $\lambda = E_p/G_\ell$ | (pile-soil stiffness ratio) |
| $\zeta = \ell n(2r_m/d)$ | |
| $r_m = \{0.25+\xi[2.5\rho(1-\nu) - 0.25]\}\ell$ | (maximum radius of influence) |
| $= 2.5\rho(1-\nu)\ell$ for $\xi = 1$ | (friction pile) |

and

$\mu\ell = \sqrt{2/\zeta\lambda}(2\ell/d)$ (pile compressibility).

For long piles, or where the stiffness ratio is low (as in a rock-socketed pile), very little load will reach the base of the pile, and the pile response becomes independent of the pile length. Thus, for piles longer than $\ell/d = 1.5\sqrt{E_p/G_\ell}$, where $E_p$ is the Young's modulus of an equivalent solid pile, the pile head stiffness may be approximated as

$$\frac{P_t}{G_\ell d w_t} \approx \rho\pi\sqrt{\frac{\lambda}{2\zeta}} \tag{4}$$

with $G_\ell$ taken as the shear modulus at a depth of $z = 1.5d\sqrt{E_p/G_\ell}$.

For very stubby piles, the above expression tends to overestimate the stiffness, and it is necessary to correct the value of $\zeta$. Randolph (1994) has proposed that the value of $\zeta$ should be increased by adding a constant of 5 inside the square bracket of equation (2). This correction makes little difference for typical pile geometries, where $\ell/d > 10$, but becomes significant for piers or rock-sockets where $\ell/d$ may be in the range 1 to 5. Horikoshi (1995) has presented a parametric study which confirms the validity of this approach. For non-homogeneous conditions, the extended expression for $\zeta$ then becomes

$$\zeta = \ln\left\{5 + \left[0.5 + \xi\left(5\rho(1-\nu) - 0.5\right)\right]\ell / d\right\} \quad (5)$$

where $\xi = G_\ell/G_b$ with $G_b$ being evaluated at a depth of one diameter below the base of the pile (or pier).

Figure 2 shows design charts based on equation (3), for straight-shafted friction piles ($\eta = \xi = 1$), for three different values of the homogeneity factor, $\rho$. The curves have been terminated where the pile length no longer influences the pile head stiffness, that is for $L/d > 1.5\sqrt{E_p / G_\ell}$.

## 3. PILE GROUPS AND EQUIVALENT PIERS

A number of different approaches may be used to estimate the stiffness of pile groups, most of which are based on the concept of superposition of the displacement field of each individual pile. Poulos (1968) introduced the concept of an interaction factor, $\alpha$, giving the proportion of the displacement of one foundation that occurs at the location of an adjacent foundation. The application of interaction factors is illustrated for the general case of two foundation 'units' (two piles, two shallow foundations, or one pile and one shallow foundation) in Figure 3.

### 3.1 Interaction Between Piles

In the general case, where a foundation system comprises n independent foundation units, the displacement of the $i^{th}$ foundation unit may be written

$$w_i = \sum_{j=1}^{n} \alpha_{ij} \frac{P_j}{k_j} \quad (6)$$

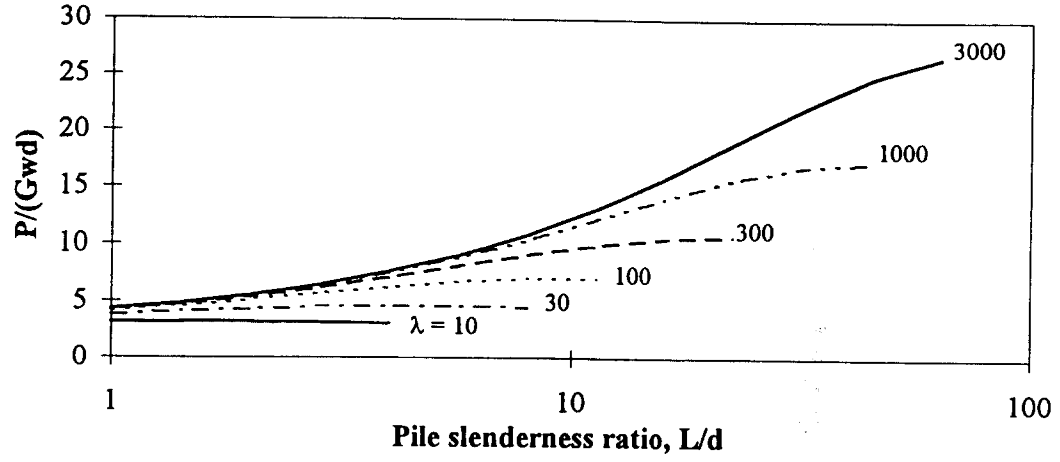
where $P_j$ and $k_j$ are respectively the applied load and stiffness of the $j^{th}$ foundation, and where $\alpha_{ii}$ is taken as unity.

Interaction effects should be applied only to the linear elastic portion of the displacement of any given foundation unit. Non-linear effects may be allowed for simply, following the approach suggested by Mandolin and Viggiani (1996), by taking the foundation response as a hyperbolic relationship, with an ultimate load of $P_{i,ult}$, and replacing the diagonal terms, $\alpha_{ii}$, by

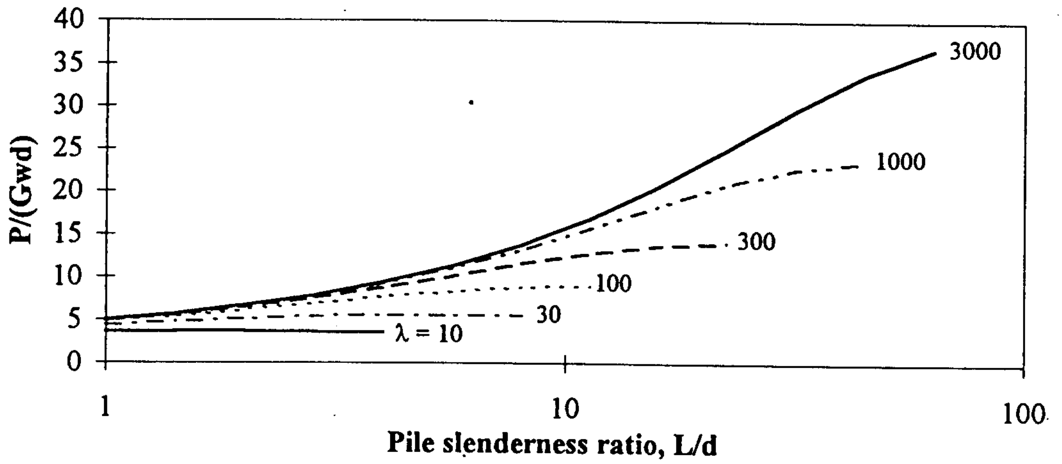
$$\alpha_{ii} = \frac{1}{1 - P_i / P_{i,ult}} \quad (7)$$

One of the limitations of the simple interaction factor approach for groups of piles is that it does not take account of an important facet of pile group behaviour, namely the transfer of an increasingly high proportion of the total applied load to the base of the group, as the size of the group increases. This aspect may be quantified using a modified form of the solution for a single pile.

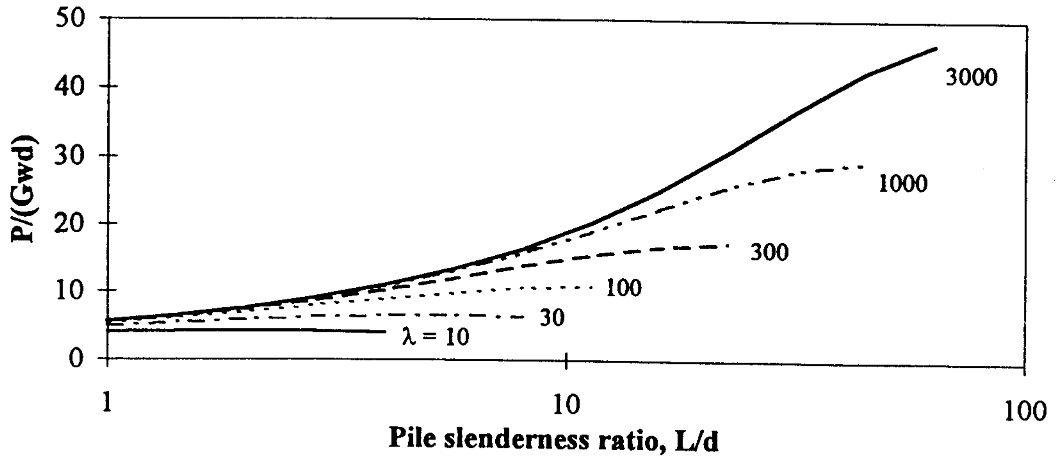
The single pile solution given in equation (3) is based on an assumed logarithmic variation of displacement around the pile shaft (corresponding to 1/r variations of shear strain and shear stress). This means that the displacement at a radius, r, from a given pile may be written as (Randolph and Wroth, 1979)




(a) ρ = 0.5




(b) ρ = 0.75




(c) ρ = 1

**Figure 2 Design charts for normalised pile stiffness**

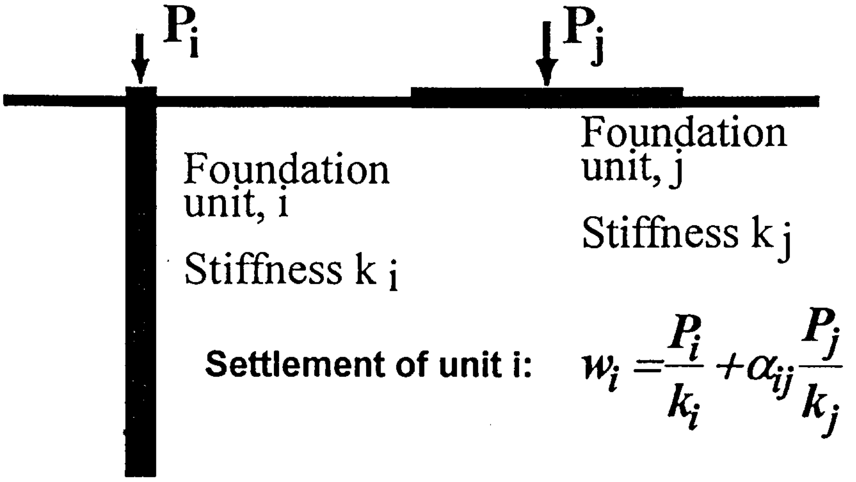



Figure 3 Interaction between foundation units

$$w(r) = w_1\left[1 - \frac{\ln(2r/d)}{\zeta}\right] \tag{8}$$

where $w_1$ is the displacement of the pile in question. A lower limit of zero should be taken for the bracketed term.

The displacement field around the pile base may be taken to decrease inversely with radius, according to the classical solution for a concentrated load. For a group of n piles, the response of any particular pile may be calculated using the approach shown in Figure 4, whereby modified factors, $\zeta^*$ and $\xi^*$ are computed for the shaft and base response, and then these factors are used in equation (3) to calculate the overall pile response.

### 3.2 Pile Group Efficiency

In order to avoid the detailed computations associated with the interaction factor approach outlined above, two alternate methods are available. The first method, described by Fleming et al (1992), is based on the concept of pile group 'efficiency' (in terms of pile group stiffness, not capacity). Thus, the pile group stiffness (total load divided by average settlement) may be written as

$$K_g = \eta n k_1 \quad \text{where } \eta = n^{-e} \tag{9}$$

Here, $k_1$ is the stiffness of a single isolated pile, while $\eta$ is the group efficiency, expressed as the total number of piles, n, to a negative power, -e. The value of e will generally lie in the range 0.4 to 0.6, with the precise values depending on a number of parameters, principally the slenderness ratio of the pile, the relative stiffness, $E_p/G_\ell$, and the spacing ratio, s/d. Charts for estimating e have been given by Fleming et al (1992).

### 3.3 Equivalent Pier Approach

The second method which may be used to estimate the stiffness of a pile group, particularly where the group layout is approximately square in plan, is to replace the group by an equivalent pier, as originally proposed by Poulos and Davis (1980). Figure 5 shows the general approach, with the diameter of the equivalent pier given by

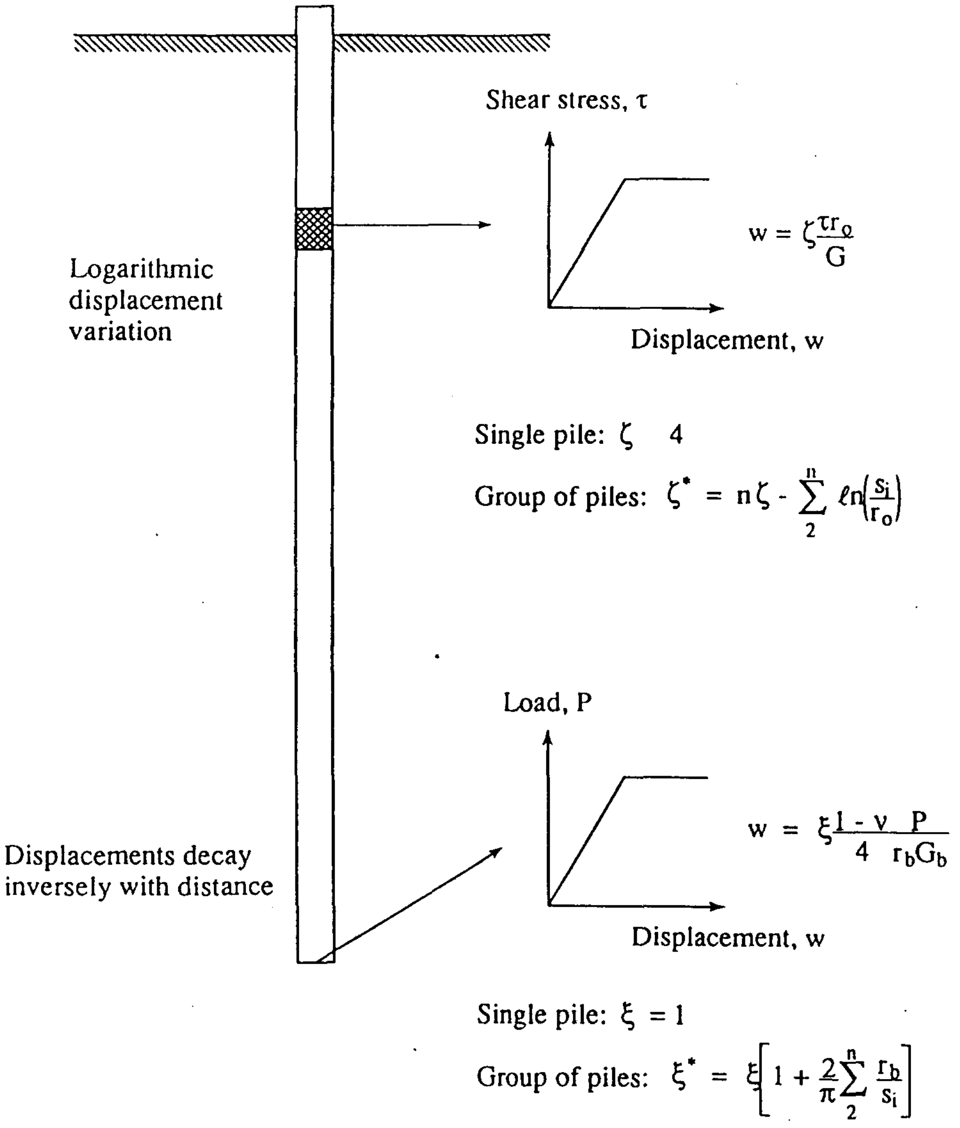



Figure 4 Elements of single pile and group pile response

$$d_{eq} = \sqrt{\frac{4}{\pi} A_g} = 1.13\sqrt{A_g} \tag{10}$$

where $A_g$ is the overall area occupied by the pile group, and the Young's modulus of the pier by

$$E_{eq} = E_s + \left(E_p - E_s\right)\left(\frac{A_p}{Ag}\right) \tag{11}$$

where $E_p$ is the Young's modulus of each pile (assuming a solid cylinder of diameter, d), $E_s$ is the Young's modulus of the soil, and $A_p$ is the total cross-sectional area of the piles in the group.

The load-settlement response of the equivalent pier may be calculated using the solution for a single pile, equation (3), or the design charts shown in Figure 2. The overall aspect

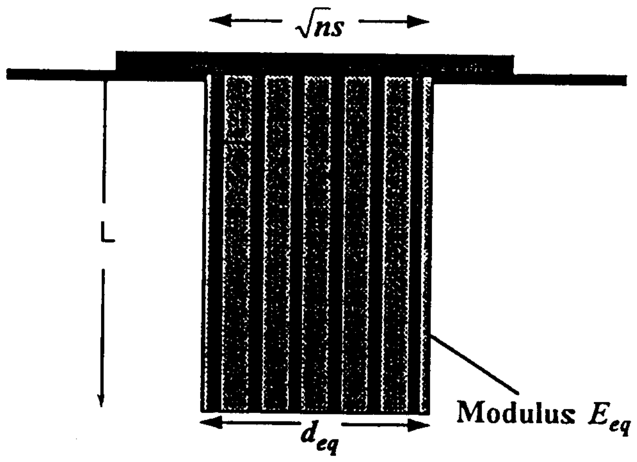



Figure 5 Equivalent pier approach for pile groups

ratio of the pier will generally be rather low, and solutions for rock-socket response (for example, Carter and Kulhawy (1988)) are particularly suitable. The equivalent pier will furnish an estimate only of the average settlement of the pile group, not the differential settlement. However, Randolph and Clancy (1993) have shown that the differential settlement, by comparison with a raft foundation, may be categorised by a parameter, R, given by

$$R = \sqrt{\frac{ns}{\ell}} \tag{12}$$

For values of R which are greater than 4, the pattern of differential settlement (assuming a fully flexible pile cap) is very similar to that for a raft foundation of the same plan dimensions. For smaller values of R, the amount of differential settlement decreases approximately in proportion to the value of R.

### 3.4 Comparison of Different Approaches

An example is given, comparing estimates of stiffness obtained by the various methods, for a 5 x 5 pile group with spacing of $s/d = 4$, slenderness ratio of $\ell/d = 25$, and stiffness ratio of $E_p/G = 1000$. Values of pile head stiffness and load sharing calculated using the numerical approach of Clancy and Randolph (1993) are shown at the left of Table 1. The resulting group pile stiffness (in terms of the load per pile) reduces from 27.1 for a single pile to 3.9 for the pile group - an estimated 'efficiency' of 3.9/27.1 = 0.14. The proportion of load taken by the pile base is estimated to increase from 6.1 % for a single pile, to 13.5 % for the pile group.

For the closed-form solution, modified values of $\zeta$ and $\xi$ are calculated as 43.2 and 2.78 respectively, compared with values of 4.47 and 1 for the single pile. Equation (3) then gives a normalised stiffness for a typical pile in the group of 4.4, with 21.5 % of the load transmitted to the pile base. The approximate method thus gives a higher pile group stiffness, by about 13 %, compared with the numerical analysis, and indicates a significantly higher proportion of load reaching the pile bases. The difference in the estimated stiffnesses is partly due to the tendency for the numerical analysis to underestimate stiffness by comparison with a full boundary element analysis of the group.

**Table 1 Comparison of single pile and pile group response**

| | Numerical analysis | | Equation (3) | | Equivalent pier |
|---|---|---|---|---|---|
| | Single pile | 5 x 5 group | Single pile | 5 x 5 group | 5 x 5 group |
| $\frac{P_t}{Gdw}$ | 27.1 | 3.9 | 27.1 | 4.4 | 4.4 |
| $\frac{P_b}{P_t}$ | 6.1 % | 13.5 % | 6.1 % | 21.5 % | (41.8 %) |

The final column in Table 1 shows the results of treating the pile group as an equivalent pier. This leads to a similar estimate of pile group stiffness to the closed form expression, and is by far the simplest calculation of the three. The estimated proportion of load reaching the base of the equivalent pier includes all load transmitted between the actual piles, and thus is considerably higher than the values obtained from the other analyses. Interestingly, if an equivalent raft approach were to be used, situated at a depth of 2/3 the length of the piles and using a 1:4 load spread, the average pressure on the raft would be 43 % of the actual applied pressure, which is comfortingly close to the figure from the equivalent pier analysis. The estimated non-dimensional stiffness using an equivalent raft approach is 3.5, which is some 10 - 20 % lower than the other values.

## 4. RAFT PERFORMANCE

Before discussing piled raft foundations, it is helpful to review some aspects of the performance of raft foundations. A full analysis of raft response may be obtained from purpose-written computer packages, such as FOCALS (Wardle and Fraser, 1975), or FLEA (Small, 1994). A simpler approach for estimating the settlement response of a raft by hand calculations is to consider average and differential settlements separately. Firstly, an estimate of the average settlement may be made, assuming that the raft is effectively rigid (the average settlement is largely independent of the relative rigidity of the raft); secondly, differential settlements may be estimated, taking account of the relative stiffness of raft and soil.

The normalised stiffnesses for a circular raft of diameter, D, may be expressed as (Poulos and Davis, 1974)

$$\frac{P}{GwD} = \frac{2}{1-\nu} \tag{13}$$

For a rectangular raft of dimensions B x L, the corresponding expression is

$$\frac{P_t}{GwB} = \frac{2}{1-\nu}\sqrt{\frac{L}{B}}I_r \tag{14}$$

where $I_r$ is an influence coefficient that varies (approximately linearly) from about 1.1 for L/B = 1, to 1.4 for L/B = 10 (Poulos and Davis, 1974).

In both of these expressions, the shear modulus must be chosen taking due account of the soil stratigraphy. A simple guideline, for soils where the modulus profile is relatively uniform, is to take the value of shear modulus at a depth of approximately half the diameter of a circular raft of the same area as the actual foundation.

### 4.1 Differential Settlements and Bending Moments

Attention in the present paper will be restricted to the differential settlement along the longer line of symmetry of the raft. The differential settlement, $\Delta w$, may be expressed as a proportion of the average settlement, $w_{avg}$, as a function of a raft-soil stiffness ratio. For circular rafts, Clancy (1993) has shown that the most appropriate stiffness ratio is

$$K_{rs} = \frac{E_r\left(1-v_s^2\right)}{E_s\left(1-v_r^2\right)}\left(\frac{2t}{D}\right)^3 \tag{15}$$

where t is the raft thickness, v is Poisson's ratio, and the subscripts 'r' and 's' stand for raft and soil respectively.

For rectangular rafts, an appropriate definition of raft-soil stiffness should (a) be consistent with equation (15) for the case of a square raft of the same area as the circular raft, and (b) give approximately the same normalised differential settlement for a given value of raft-soil stiffness, over a wide range of aspect ratios, B/L. It may be shown that a suitable definition for general raft shapes is

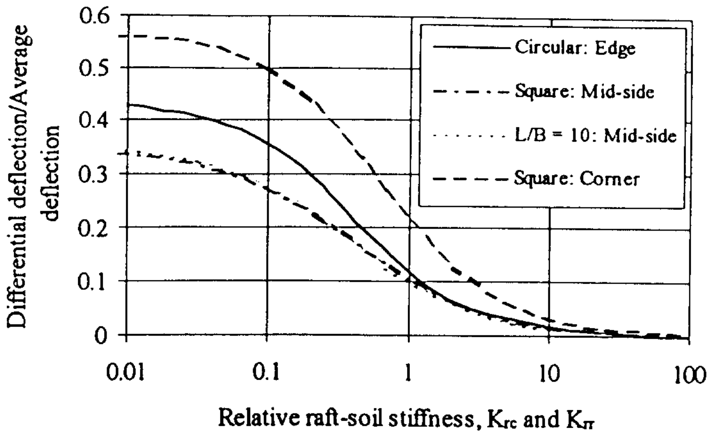
$$K_{rs} = 5.57\frac{E_r\left(1-v_s^2\right)}{E_s\left(1-v_r^2\right)}\left(\frac{B}{L}\right)^{0.5}\left(\frac{t}{L}\right)^3 \tag{16}$$

For B = L, an equal area circular raft would have $D = 2B\sqrt{\pi}$, and the above expression reverts to the definition of $K_{rc}$ in equation (15).

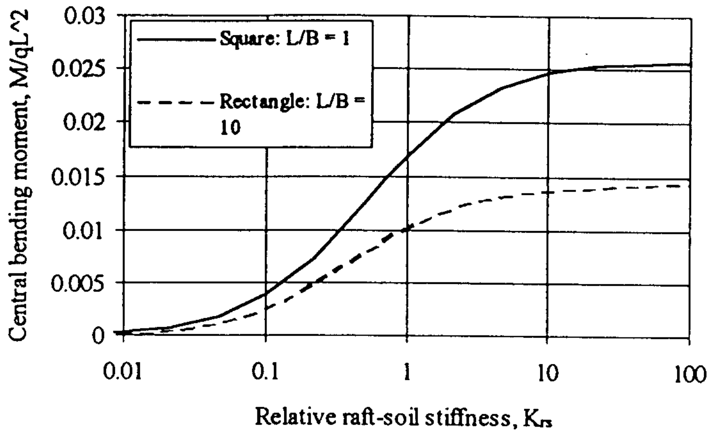
Figure 6(a) shows the variation of differential settlement for a wide range of raft-soil stiffness ratios. As expected, the centre-edge differential settlement for the circular raft falls between the centre-midside, and centre-corner, differential settlement for the square raft. The centre-midside settlement for the rectangular raft with L/B = 10 lies close to the corresponding curve for the square raft. Figure 6(b) shows the variation of bending moment at the centre of the raft, which rises from zero for a fully flexible raft, to a maximum for a rigid raft. The maximum value of normalised bending moment (per unit length), expressed as $M/qL^2$ where q is the uniform applied load, is greater for the square raft than for the rectangular raft. However, the shapes of the two curves are very similar, using the definitions of raft-soil stiffness given earlier.

## 5. PILED RAFT ANALYSIS

The form of the curves for differential settlement and bending moments in Figure 6 illustrates the choice in design, whereby suppression of differential settlements by the adoption of a thick raft will lead to correspondingly large bending moments. The aim should be to achieve minimal differential settlements and bending moments simultaneously, by incorporating pile support over the central area of the raft.

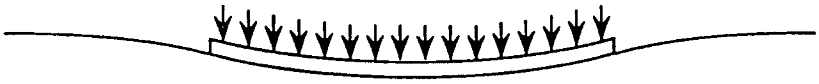



(a) Differential settlements

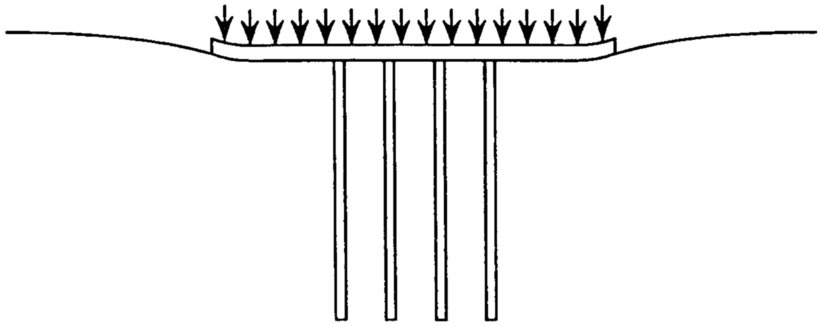



(b) Bending moments

Figure 6 Effect of raft stiffness on differential settlements and bending moments



Uniformly loaded raft foundation



Raft plus settlement reducing piles

Figure 7 Principle behind settlement reducing piles

Figure 7 illustrates the principles behind the design of central pile support. The central 'settlement reducing' piles (Burland et al, 1977) provide additional support for the central region of the raft, preventing 'dishing' of the raft. At the same time, the piles lead to a distribution of contact pressure between raft and soil which is similar to that under a rigid raft, with very low contact stress in the centre, and asymptotically high stresses at the edges of the raft. If the number and size of piles is judged correctly, the maximum bending moment in the raft will also be low. This form of pile support is consistent with suggestions by Padfield and Sharrock (1983).

## 5.1 Average Settlement and Load Sharing

Methods of analysis for piled rafts, of varying degrees of sophistication, have been presented over recent years (Clancy and Randolph, 1993; 1995; Poulos, 1994). Computer codes such as HyPR (Clancy, 1993) allow full treatment of pile-soil-raft interaction, allowing accurate estimates of settlement patterns, bending moments in the raft, and load distribution down individual piles to be made, with due allowance for non-linear effects such as pile-soil slip. That code has been used as the basis for a parametric study, to be presented later.

Perhaps the simplest approach for estimating the average settlement of a piled raft, and the load sharing between raft and piles, is that outlined by Randolph (1983), whereby the separate stiffnesses of the pile group and of the raft are combined through the use of an interaction factor. Adopting subscripts of p for the pile group, and r for the raft (or pile cap), the settlement of each component may be expressed as (Randolph, 1983)

$$\begin{Bmatrix} w_p \\ w_r \end{Bmatrix} = \begin{bmatrix} 1/k_p & \alpha_{pr}/k_r \\ \alpha_{rp}/k_p & 1/k_r \end{bmatrix} \begin{Bmatrix} P_p \\ P_r \end{Bmatrix} \tag{17}$$

where $\alpha_{pr}$ and $\alpha_{rp}$ are interaction factors, and P and k are the loads and stiffnesses relating to each component. From the reciprocal theorem, the terms on the trailing diagonal of the flexibility matrix must be equal, so that the interaction factors are related by $\alpha_{pr} = \alpha_{rp}(k_r/k_p)$.

Since the (average) settlement of piles and raft are identical, these two expressions allow the overall stiffness, $k_{pr}$, and the proportion of load taken by the raft to be calculated as:

$$k_{pr} = \frac{k_p + (1-2\alpha_{rp})k_r}{1-\alpha_{rp}^2(k_r/k_p)} \quad \text{and} \quad \frac{P_r}{P_p} = \frac{(1-\alpha_{rp})k_r}{k_p + (1-2\alpha_{rp})k_r} \tag{18}$$

For single piles (or equivalent piers) with circular caps of diameter, D, Randolph (1983) has shown that the interaction factor, $\alpha_{rp}$, may be approximated by $\alpha_{rp} = 1 - \ell n(D/d)/\zeta$.

Clancy and Randolph (1993) have shown that, as the group size increases, the value of $\alpha_{rp}$ tends towards a constant value of about 0.8, independent of the pile spacing, slenderness ratio or stiffness ratio. The piled raft stiffness is then given by

$$k_{pr} = \frac{1-0.6(k_r/k_p)}{1-0.64(k_r/k_p)} k_p \quad \text{and} \quad \frac{P_r}{P_p} = \frac{0.2}{1-0.6(k_r/k_p)} \frac{k_r}{k_p} \tag{19}$$

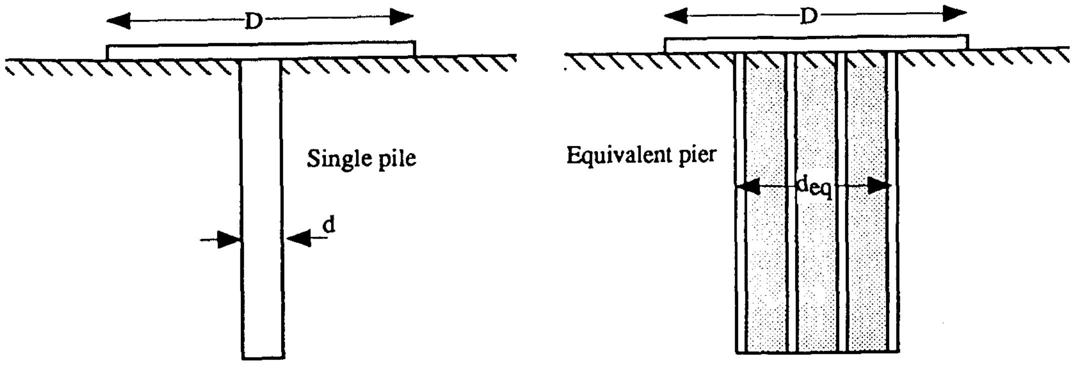



Figure 8 Single pile or equivalent pier with extended pile cap

from which it may be seen that the combined stiffness of the foundation will be close to that of the pile group alone and the ratio of loads carried by the pile cap (or raft) and the pile group will typically lie in the range 0.3 - 0.5 times ($k_r/k_p$).

It should be noted that pile cap contact with the ground has relatively little effect on the overall stiffness of the foundation, as might be anticipated from the accuracy of the equivalent pier approach. Essentially, the only modification needed for piled rafts is to extend the area of the equivalent pier to match the full area of the pile cap, rather than the area occupied by the piles themselves. However, the load carried by the pile cap is important, conceptually, in deciding the number and size of the pile support.

The original approach of Randolph (1983) may be combined with the equivalent pier concept in order to assess the performance of a raft with central pile group support. It will be shown later that the optimum arrangement is for the stiffness of the pile group (alone) to be comparable with that of the raft, that is $k_p \approx k_r$, which leads to approximately equal load sharing between raft and pile group (at least for elastic conditions). Also, the pile group should be distributed over the central 16 to 25 % of the raft, giving $d_c/d \approx 2$ - 2.5. These conditions lead to $\alpha_{rp} \approx 0.6$ - 0.7, resulting in an overall stiffness which is approximately 20 % higher than that of the pile group alone.

### 5.2 Example Analysis

In order to illustrate the principles discussed above, an example foundation is analysed, consisting of a 36 m square raft subjected to an average loading of 600 kPa (total load of 780 MN). Figure 9 shows the foundation layout, with two different piling arrangements. The first consists of a 9 x 9 grid of piles at 4 m spacing, with other properties as given in Table 2 below.

**Table 2 Soil, pile and raft properties for example analysis**

| Soil | | | Pile | | | Raft | | |
|---|---|---|---|---|---|---|---|---|
| G | 100 | MPa | $E_p$ | 35000 | MPa | L | 36 | m |
| ν | 0.4 | | $\ell$ | 20 | m | B | 36 | m |
| | | | d | 0.8 | m | $K_{rs}$ | 0.1 | |

The raft stiffness (assuming a rigid raft) may be calculated from equation (14) as $k_r$ = 13.5 MN/mm. The stiffness of the pile group may be estimated from an equivalent pier of diameter 36 m (equivalent in area to the 32 m wide pile group) and Young's modulus

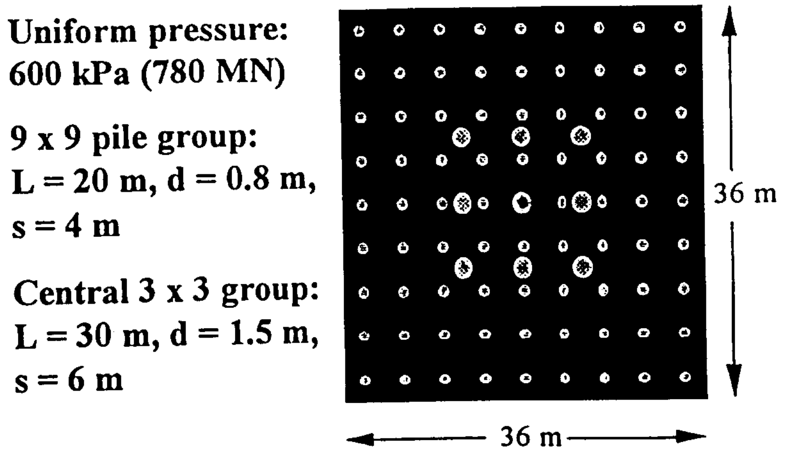



Figure 9 Example analysis of piled raft

1660 MPa (equation (11)), giving $k_p$ = 16.2 MN/mm. Taking $\alpha_{rp}$ equal to 0.8, the overall stiffness of the piled raft may be estimated as 17.4 MN/mm, with 67 % of the load carried by the pile group. A direct calculation of the piled raft stiffness using an equivalent pier of diameter 40.6 m, and Young's modulus of 1370 MPa leads to a stiffness of 17.5 MN/mm, which is consistent with the estimated value.

The ratio R for the pile group is 4, and hence differential settlements may be estimated with adequate accuracy from those for a purely raft foundation. For the relative raft stiffness of $K_{rs}$ = 0.1, normalised differential settlements are 27 % (mid-side to centre) and 50 % (corner to centre). For an applied (average) vertical stress of 600 kPa, this would lead to an average settlement of the piled raft of 44 mm, and differential settlements of 12 mm between mid-side and centre, and 22 mm between corner and centre. A full numerical analysis yields identical differential settlements, but an average settlement some 18 % higher, at 52 mm.

The second arrangement of piles consists of a 3 x 3 group, at a spacing of 6 m, situated over the central region of the raft. The pile length is increased to 30 m, and the diameter to 1.5 m. The equivalent pier parameters for the central pile group are $d_{eq}$ = 15.2 m, $E_{eq}$ = 3310 MPa. Equation (3) then gives the pier stiffness as $k_p$ = 9.85 MN/mm. The overall stiffness estimated from equation (18), taking $\alpha_{rp}$ = 0.6, is 14.1 MN/mm, with about 75 % of the load carried by the raft. The total load on the pile group is therefore estimated as 191 MN.

For a limiting shaft friction of 125 kPa, and end-bearing of 2 MPa, the ultimate pile capacity is 21.2 MN, or group capacity of 191 MN. Thus, from the above calculation, nearly full mobilisation of the pile capacity is expected. Allowing for non-linear effects, the real piled raft stiffness is likely to be marginally lower, with a slightly higher fraction of load carried by the raft. Assuming that some 80 % of the pile capacity is actually mobilised under working conditions, the net pressure carried by the piles over the central region is then about 470 kPa, which correspond to 80 % of the applied pressure.

A full analysis of the piled raft has been carried out using the program, HyPR. Settlement profiles are shown in Figure 10, under the uniformly distributed working load of 780 MN, for the three cases of:

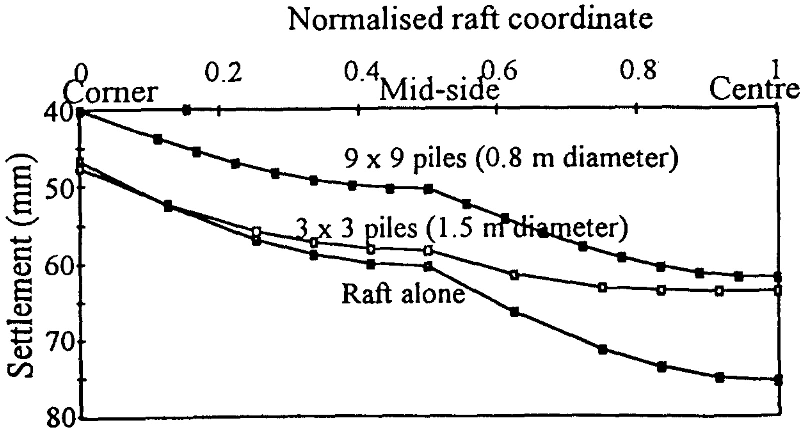



Figure 10 Settlement profiles for 36 m square piled raft

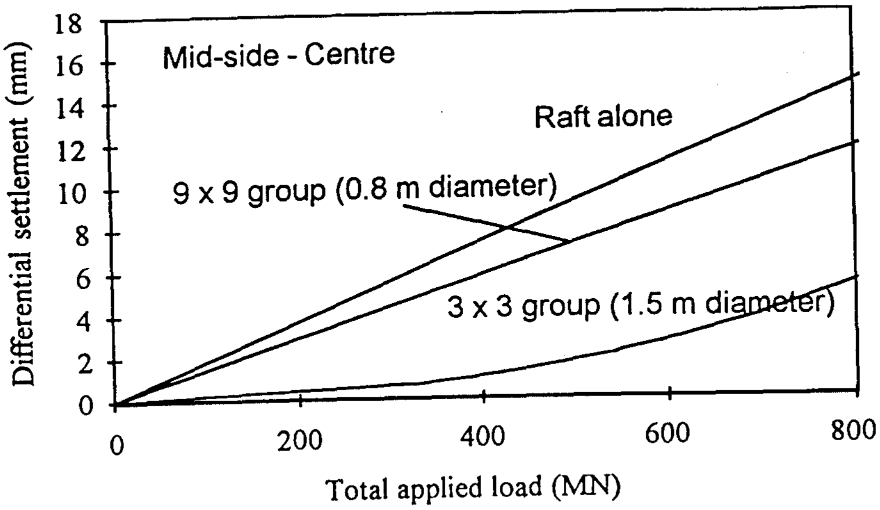



Figure 11 Development of differential settlement for 36 m square piled raft

(1) raft alone (fully flexible raft);
(2) fully piled (9 x 9) raft, 0.8 m diameter piles, with raft-soil stiffness of $K_{rs} = 0.1$;
(3) central 3 x 3 pile group, 1.5 m diameter piles, fully flexible pile cap.

As anticipated, the central piles have little effect on the corner and edge settlements, and indeed reduce the average settlement by less than 5 % (piled raft stiffness of 14.2 MN). However, their effectiveness in reducing differential settlements is evident. By contrast, the fully piled raft reduces the average settlement by 20 % (piled raft stiffness of 16.5 MN/mm), but shows virtually no change in differential settlements compared with the unpiled raft, apart from a slight reduction due to the finite pile cap stiffness.

The effectiveness of the central piles in reducing differential settlements is seen clearly in Figure 11. At low load levels, the larger diameter central piles practically eliminate mid-side to centre differential settlements. Even at the working load of 780 MN, when the capacity of the central piles is nearly fully mobilised, the differential settlement is reduced to about 30 % of that for the unpiled raft, and less than 50 % of that for the fully piled raft.

Differential settlements between the corner and the centre of the raft are still reduced by the central piles, although only by about 50 % compared with the unpiled raft. In

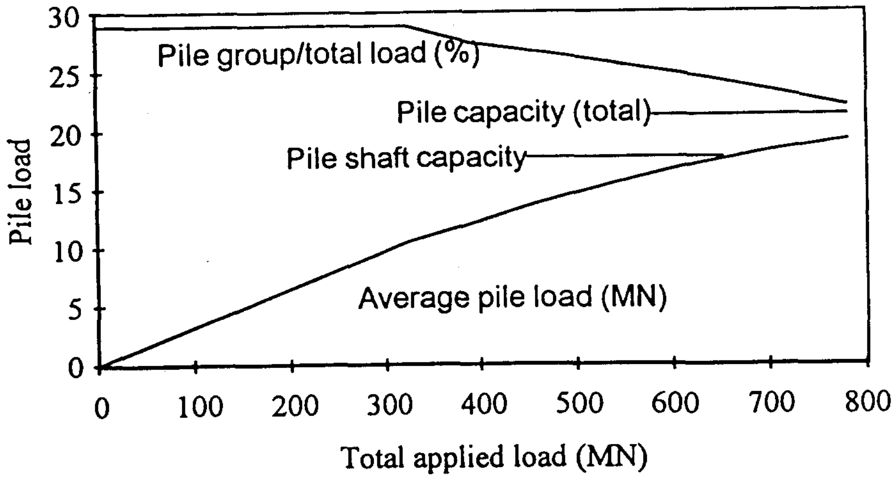



Figure 12 Development of pile loads for central 3 x 3 group

practice, stiffening of the raft towards the corners due to the superstructure will help reduce such differential settlements further.

For the case of the central 3 x 3 pile group, it is instructive to consider the distribution of load between the piles and the raft. As the total load on the raft is increased, the piles start to develop their limiting shaft capacity, with slip between the piles and the surrounding soil starting from the pile tips. This contrasts with the case of a single pile, where slip will generally start at the ground surface.

Figure 12 shows the development of total load taken by the piles, which gradually rises towards the limiting capacity of 191 MN. The proportion of the total load carried by the piles reduces from 28 % for elastic conditions, to 22 % as the piles fail. These ratios are consistent with the simple estimate of 25 % for elastic conditions, as discussed earlier.

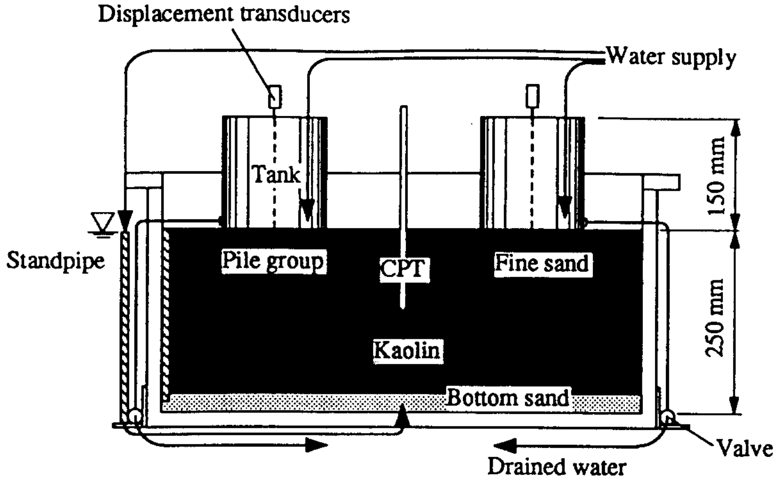
## 6. CENTRIFUGE MODEL TESTS

Centrifuge model tests of flexible raft and piled raft structures have recently been reported by Horikoshi and Randolph (1995). The tests consisted of 1:100 scale models of 14 m diameter tanks, with one tank supported by different combinations of piles, ranging from a 3 x 3 group over the central part of the raft, to larger groups of 21 and 69 piles. The general arrangement of the model tests is illustrated in Figure 13(a), while the different pile groups are detailed in Figure 13(b).

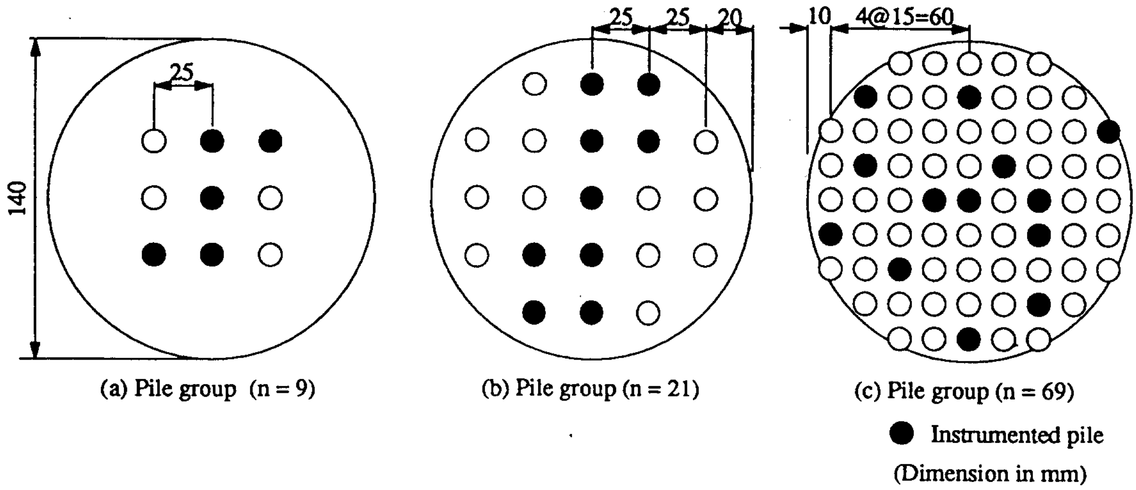
The load from the model tanks was applied in two stages. During reconsolidation of the clay (after placement of the models), the average loading was 42 kPa, with some 40 % applied through the walls of the tanks. After reconsolidaton was complete, the water level in the tanks was raised, to give an additional loading of up to 95 kPa. The maximum (total) load applied to the foundations was therefore 2.16 kN at model scale, or 21.6 MN at prototype scale.

Tests on single piles showed that the ultimate capacity was about 40 N (400 kN at prototype scale). Thus the different pile group configurations have an overall capacity of 17 %, 39 % and 128 % of the applied load, for the 9, 21 and 69 pile groups.

Figure 14 shows the average settlements for the additional 95 kPa of loading, for the four foundation configurations. The settlements were similar between the 9-piled and




**(a) General section of centrifuge package**




**(b) Arrangement for each pile group**

Figure 13 Centrifuge model tests (Horikoshi and Randolph, 1995)

unpiled rafts, with average settlements of about 0.7 mm (70 mm prototype). Lower settlements of 0.5 mm and 0.3 mm were measured for the 21 and 69-piled rafts.

The differential settlements are shown in Figure 15. It may be seen that the central 9-pile group virtually eliminated differential settlements, reducing them from 0.5 mm for the unpiled raft, to less than 0.1 mm (10 mm prototype). The consistency between different tests is also remarkable, given the small magnitude of the displacements. Similarly small differential settlements were measured for the 21 and 69-piled rafts.

The distribution of load between raft and pile group is illustrated in Figure 16 for the 9 and 21-pile groups. (Essentially 100 % of the load was carried by the piles for the 69-pile group, see Horikoshi and Randolph (1995).) At high load levels, the piles are loaded to close to their ultimate capacity, taking respectively 15 and 40 % of the total load. At lower load levels, the proportion of the load taken by the piles is higher, as expected, with maximum proportions of 25 % for the 9-pile group, and 80 % for the 21 pile group.

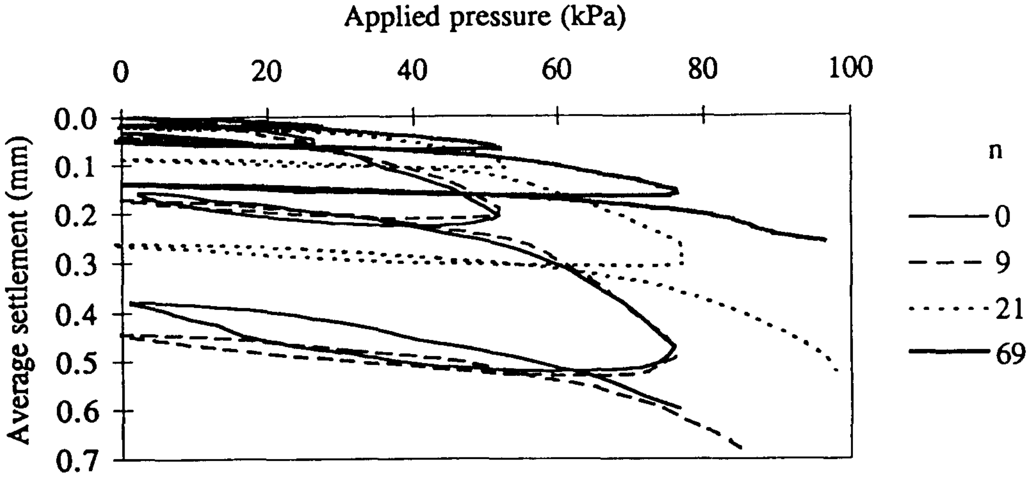
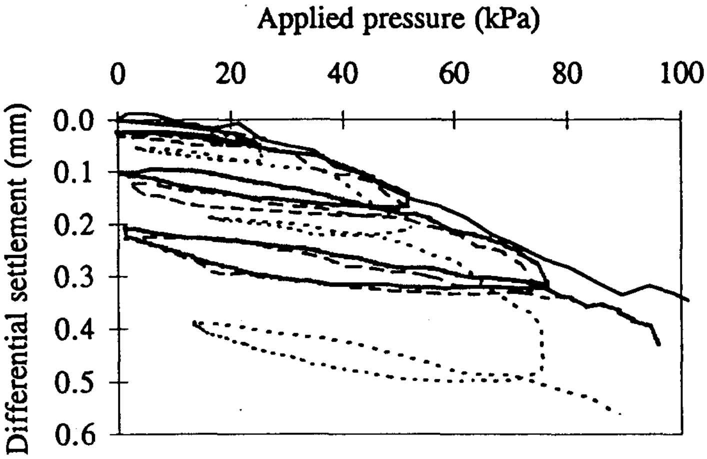
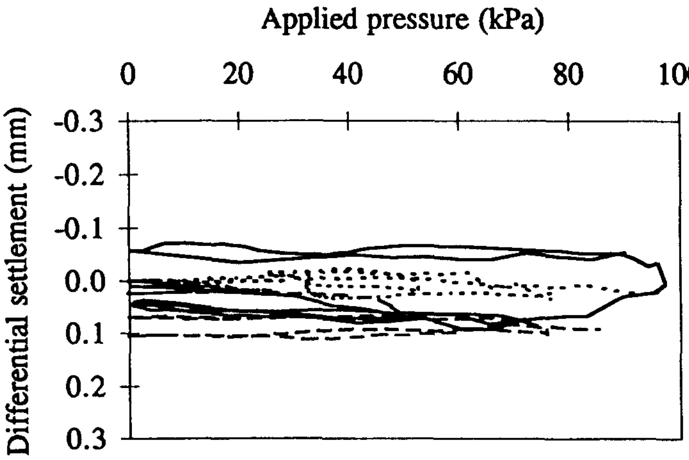



Figure 14 Average settlement of raft during loading

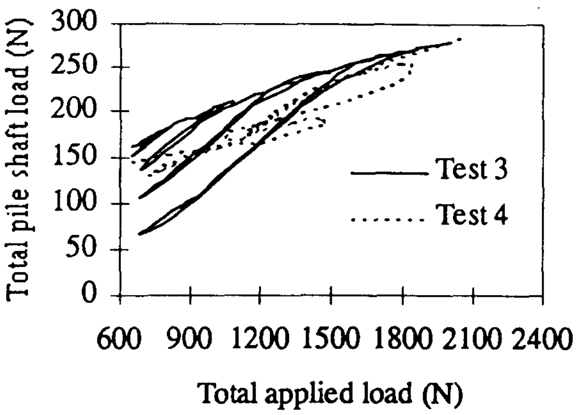



(a) Unpiled raft (Tests 1, 3, 4 and 5)

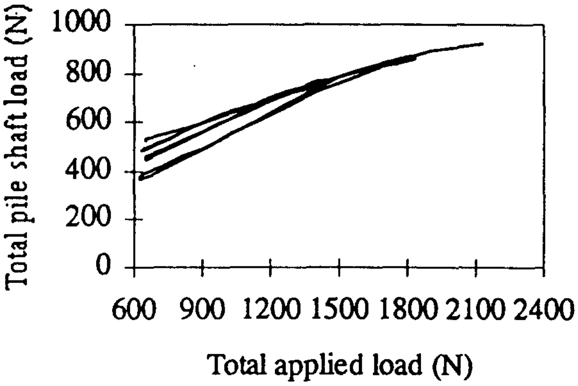



(b) Piled raft, n = 9 (Tests 2, 3 and 4)

Figure 15 Differential settlement of rafts during loading




(a) Piled raft, n =9




(b) Piled raft, n =21

Figure 16 Proportion of load transferred to piles

## 7. OPTIMUM PILED RAFT DESIGN

The model tests presented above provide a clear indication of the benefits of central pile support in eliminating differential settlements, even where the piles carry as little as 15 % of the total load. The key design decision is then the number, size and plan distribution of the pile support, since this is no longer dictated by capacity considerations. A parametric study, carried out by Horikoshi (1995), provides some guidance as to the optimum pile layout.

Furthermore, the following dimensionless values were found to be important:

| | |
|---|---|
| Relative pile length | $\ell/a$ (or $\ell/a_{eq}$ for rectangular rafts) |
| Pile group-raft area ratio | $a_{gr} = A_g/A_r$ |
| Equivalent pier-soil stiffness ratio | $E_{eq}/E_s$ |
| Pile group-raft stiffness ratio | $K_{pr} = k_p/k_r$ |

where a is the radius of the circular raft, $a_{eq}$ is the radius of a circular raft of equivalent area to a rectangular raft, defined as $a_{eq} = \sqrt{(BL/\pi)}$ for rectangular rafts, $A_r$ is the raft area, and $k_p$ and $k_r$ are the overall stiffness of the pile group (or the equivalent pier) and the unpiled raft respectively. The pile group-raft area ratio defines the proportion of the central area over which piles are installed. This may also be expressed in terms of the ratio, S/B, where S is the width of the pile group, and B is the width of the raft, since $a_{gr} = (S/B)^2$.

Among the non-dimensional parameters, it was found that the pile group-raft stiffness ratio, $K_{pr}$, is critical in designing optimum piled rafts. The magnitude of the differential settlement is largely governed by this parameter, and optimum results correspond to a ratio close to unity. The overall stiffness of a piled raft can be estimated through the interaction factor between the pier and the raft, $\alpha_{rp}$, as discussed earlier. However, the accuracy of the estimated overall stiffness was checked with results using the computer code HyPR developed by Clancy (1993).

In the design process, the critical decisions in regard to the pile support are (a) pile geometry (areal extent of the pile group, length of the piles in relation to the raft geometry), and (b) pile capacity in relation to the local and total loading applied to the raft. There are four separate, but interlinked, load ratios that need to be considered:

| | |
|---|---|
| Ratio of piled area load to pile capacity | $P_g^* = pA_g/(nq_p) = a_{gr}P_t^* = ma_{gr}/p_{gt}$ |
| Ratio of total raft load to pile capacity | $P_t^* = P_t/(nq_p) = P_g^*/a_{gr} = m/p_{gt}$ |
| Proportion of load carried by piles | $p_{gt} = P_g/P_t = m/P_t^* = ma_{gr}/P_g^*)$ |
| Degree of mobilisation of pile capacity | $m = P_g/(nq_p) = p_{gt}P_g^*/a_{gr} = p_{gt}P_t^*$ |

where p is the uniform pressure applied over the raft, $q_p$ is the single pile capacity, $P_t$ is the total applied load which is equal to $pA_r$, $P_g$ is the total load transferred to the pile group. The first two ratios may be considered as design decisions, since they are quantifiable before analysis, while the second two ratios may only be calculated after analysis.

The differential settlement of rectangular rafts is defined as the difference between the central and the mid-side settlements, which was denoted (1-0.5) settlement by Clancy (1993). This differential settlement is normalised by the average settlement of the raft (alone) with the same raft-soil stiffness ratio. The normalised differential settlement is defined as $w^*_{1\text{-}0.5}$ in the figures presented later.

The parametric study was carried out using the code HyPR, developed by Clancy (1993), which allows for full interaction between piles, soil and raft, and also models gradually developing slip between piles and soil. Figure 17 shows the configurations that were analysed in the parametric study, with essentially a 9-pile group at different spacing, s, at the centre of a square raft of semi-width, b, (with $s/b = S/B = \sqrt{a_{gr}}$). In practice, there may be considerably more than 9 piles in the central group, but the effect of the group may still be represented by 9 equivalent piers. All results presented here are for a raft with stiffness ratio $K_{rs} = 0.1$. A ratio of shear modulus, G, to shaft friction, $\tau_s$, was taken as $G/\tau_s = 500$, although the results to be presented are actually independent of $G/\tau_s$, as discussed by Horikoshi (1995).

Figure 18 shows the effect of pile spacing and length on the centre-midside differential settlement. It should be noted that the horizontal axes, denoting $1/P_t^*$, represent the loading level, with low loads (and elastic conditions) at the right hand end ($P_t^*$ of unity represents a situation where the total load applied to the raft equals the capacity of the

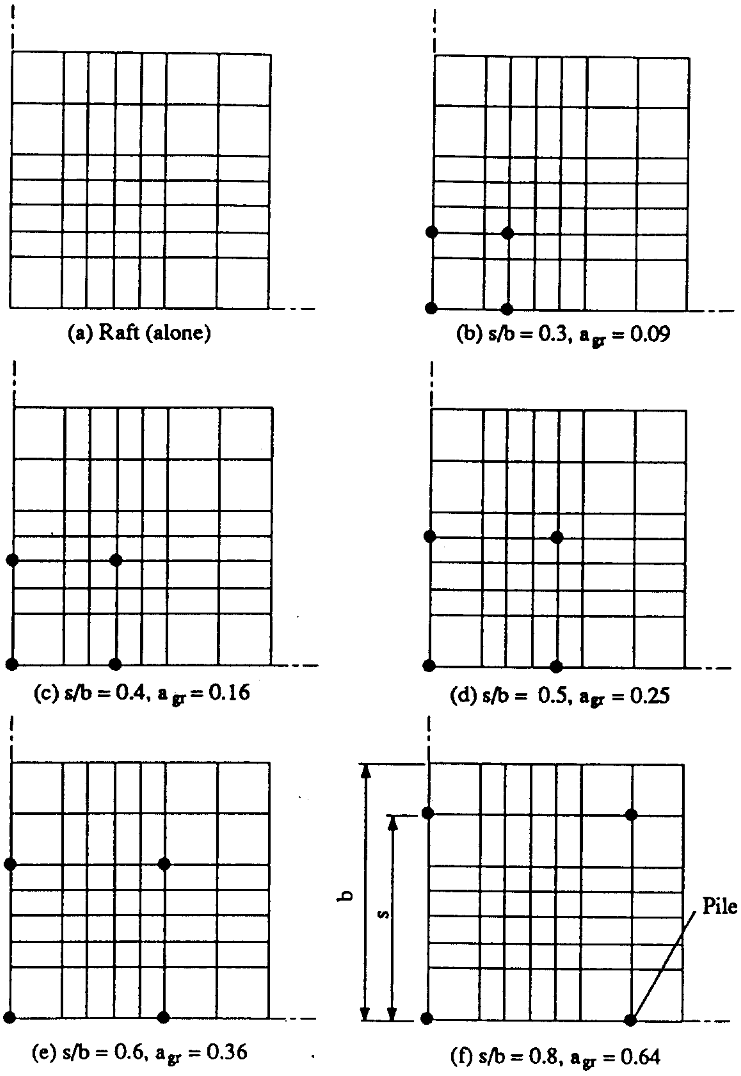



Figure 17 Raft meshes used for parametric study

pile group). The non-linear effects become obvious at the left hand side of each graph, where $P_t^*$ increases to 5 or more.

The differential settlement of the unpiled raft has also been included on each figure. Since the raft-soil behaviour was assumed fully elastic, this profile is independent of the applied load. The figures show clear differences in the differential settlements for cases with different pile spacing (i.e. different pile group area ratio). Although the results are not presented here, it was found that the average settlement (or overall stiffness) was essentially independent of the pile spacing for given values of $K_{rs}$ and $\ell/a_{eq}$. As the total applied load increases (i.e. $1/P_t^*$ decreases), the differential settlement approaches that of the unpiled raft.

The proportion of load transferred to the piles is shown in Figure 19 for the case of $\ell/a_{eq}$ = 3 and $K_{rs}$ = 0.1. As the total applied load increases, the proportion of load carried by the piles decreases gradually, since the non-linear pile behaviour causes the stiffness of the pile group relative to the soil to become smaller. As the total pile load becomes closer to the total pile capacity, the calculated curves converge to the 'm = 1' line.

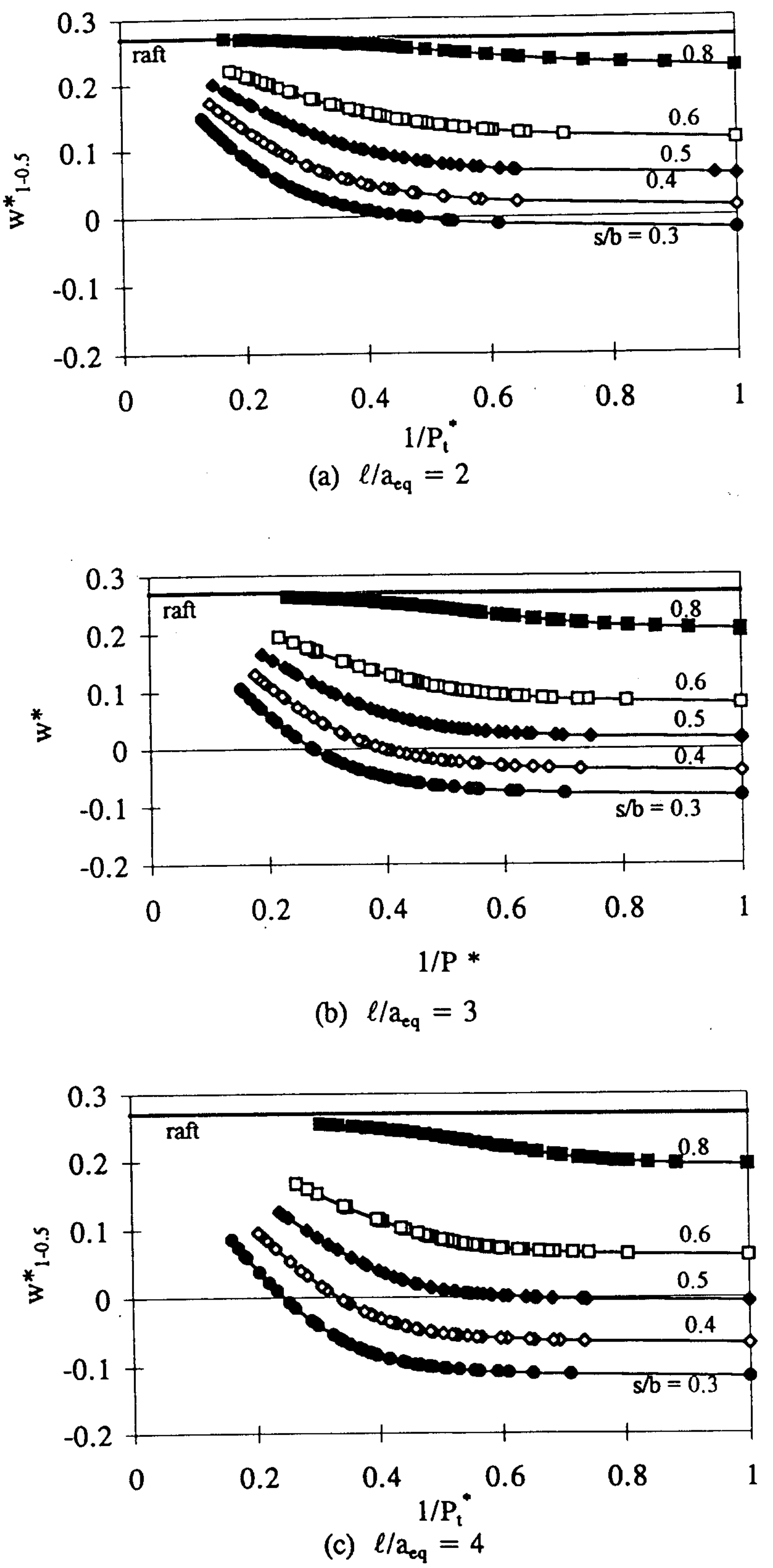



Figure 18 Effects of pile spacing and length on differential settlements

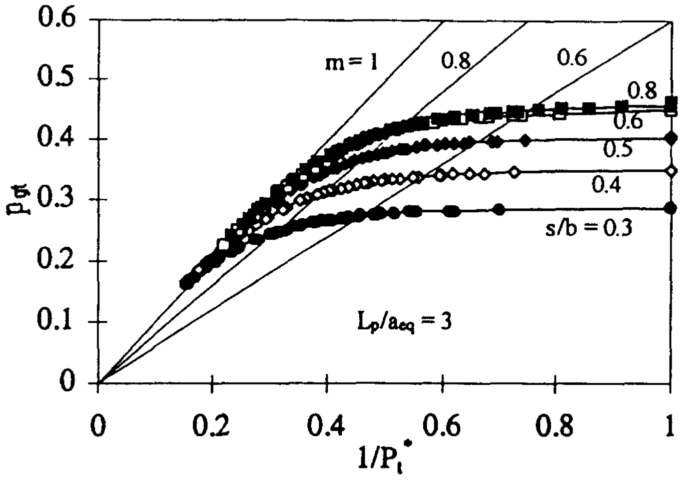



Figure 19 Effect of pile spacing on proportion of load taken by piles

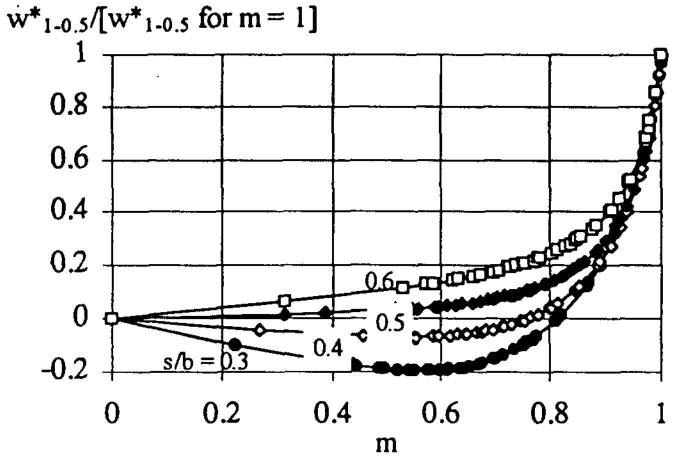



Figure 20 Variation of differential settlement with mobilisation ratio, m

The increase in differential settlement is summarised in Figure 20 for the case of $\ell/a_{eq}$ = 3. The differential settlements are normalised with respect to the differential settlement at m = 1. The figure shows that differential settlements increase significantly for values of m greater than 0.8, and that piled rafts should therefore be designed for this parameter to be less than 0.8.

These results indicate that a set of optimum conditions for piled rafts appears to be $\ell/a_{eq}$ = 3, s/b = 0.4 (i.e. $a_{gr}$ = 0.16) and $K_{rs}$ = 0.1. These result in very small differential settlement after moderate mobilisation of pile capacity (see Figure 18). The pile group may be loaded up to load ratios of $P_g^* \sim 0.4$ and thus $P_t^* \sim 2.5$ to give m ~ 0.8. When $P_g^* = 0.4$ and $P_t^* = 2.5$, the corresponding $p_{gt}$ (proportion of total load to be carried by piles) is calculated as 0.32.

For elastic conditions, the ratio of raft to pile group stiffnesses was found to be $K_{pr}$ = 1.02 for the above set of parameters. The principle of maintaining approximately equal stiffness of the raft and the pile group provides a useful guideline for the design of centrally supported piled rafts. However, as the plan area of the pile group increases, the optimum raft-pier stiffness to give minimum differential settlement also increases, reaching $K_{pr}$ = 1.5 for $a_{gr}$ = 0.33.

## 8. CASE HISTORY 1 - QV1 BUILDING, PERTH

The potential for reducing the amount of pile support will be illustrated with reference to the QV1 building, in Perth. The foundation design and analysis has been discussed by Smith and Randolph (1990) and by Randolph and Clancy (1994).

The Figure 21 shows the settlements measured at slab level in July, 1991, at completion of the building . The average settlement of the core is 37 mm, while that of the outer foundations is about 23 mm. There is thus about 14 mm differential settlement between the central and outer foundation units (compared with a design estimate of 15 - 20 mm) and also significant dishing of the long hammerhead foundations.

The large number of piles precluded the use of a complete piled-raft analysis for the prediction of differential settlements and overall stiffness. However, sufficiently accurate results may be obtained by replacing the 280 piles by 42 'equivalent piers' (Randolph and Clancy, 1994). The piers overestimate the pile groups which they replace by typically 5 - 10 %.

The displacement contours resulting from this analysis are shown in Figure 22, and compare reasonably well with those measured. The computed differential settlements are a little higher than measured. However, this is considered due to the fact that no allowance has been made in the analysis for stiffening of the raft by the superstructure.

### 8.1 Alternative Pile Layout

In keeping with the concept of central pile support, a more efficient pile layout for the reduction of differential settlements may be achieved by only placing piles under the central region of the raft (150 piles). To analyse this, the equivalent pier approach was again used, but with piers under the core alone. The displacement contours have been plotted in Figure 23, showing that overall differential settlements have been reduced to 20 mm from the previous value of 27 mm. This reduction comes at the price of increasing the gradient of differential settlements at the extremities of the raft. Again, it can be expected that the structural stiffening effect of the building would go some way towards reducing this problem. It should be emphasised that the average settlements are essentially identical to those with the full 280 pile group.

## 9. CASE HISTORY 2 - STONEBRIDGE PARK

A second example is drawn from the instrumented foundation study reported by Cooke et al (1981), for a 16-storey block of flats at Stonebridge Park in London. The foundation of the building was constructed by installing 351 bored piles with a diameter of 0.45 m and length of 13 m, over the full raft area. A concrete raft with a thickness of 0.9 m, and size of 20.1 m by 43.3 m (therefore $a_{eq}$ = 16.6 m) was constructed over the piles, and was in contact with the ground. Schematic views of the building are shown in Figure 24. The raft and the piles were instrumented extensively to measure settlement and load transfer from the superstructure.

Detailed numerical analyses of the building were performed by Padfield and Sharrock (1983), which included re-design of the building by using only 40 piles of the same size over the central area of the raft. Such a re-design is consistent with the ideas presented here, and will be explored below.

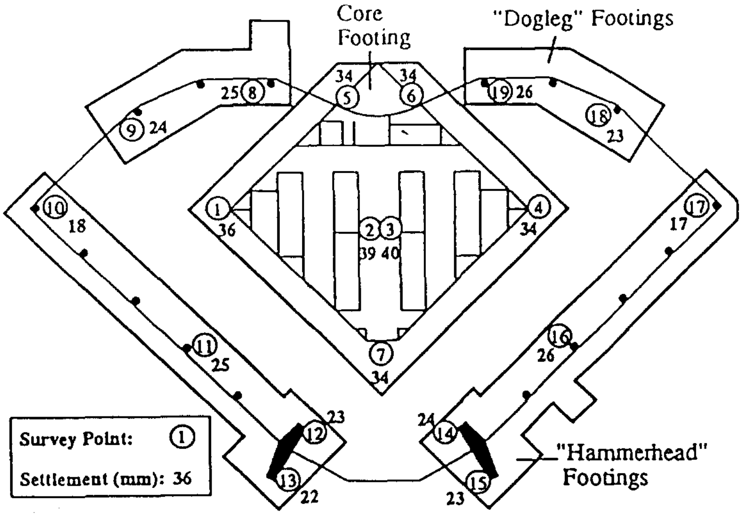



Figure 21 QV1 Building: Measured settlements at building completion

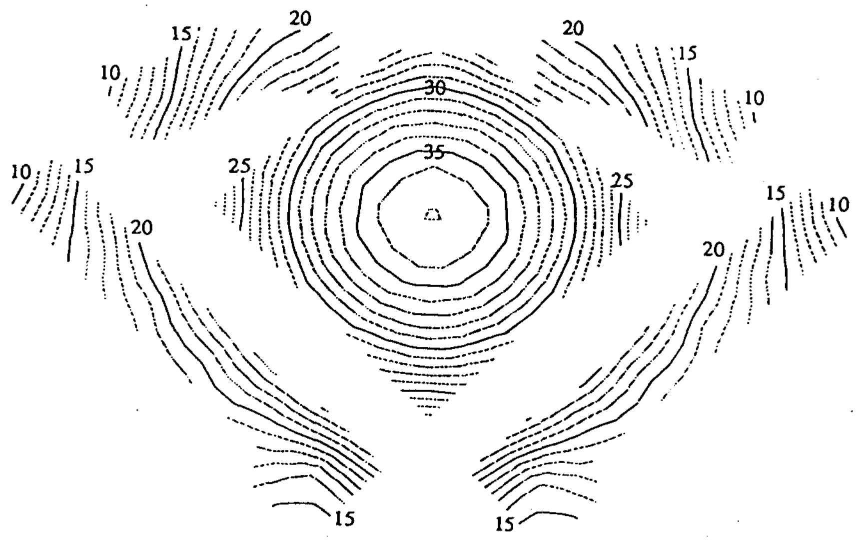


Figure 22 QV1 Building: Calculated settlements using equivalent piers

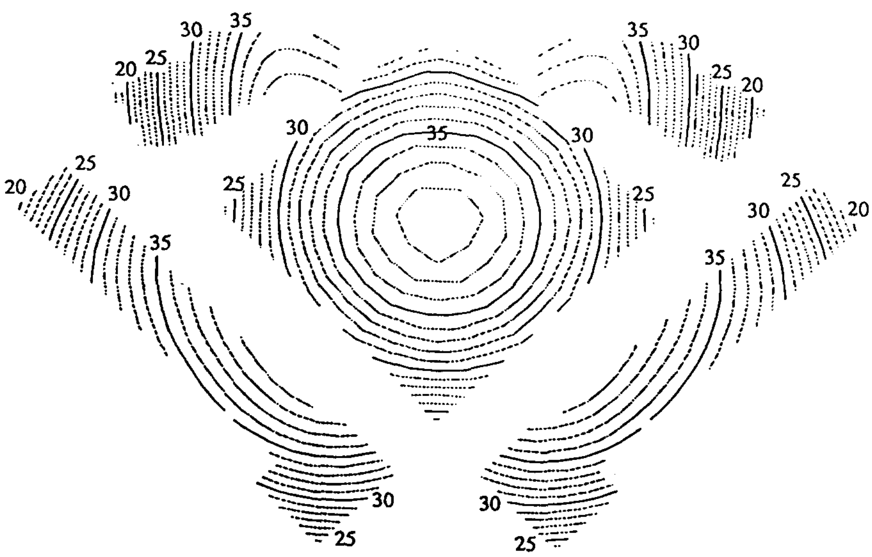


**Figure 23 QV1 Building: Calculated settlements with piles under central core only**

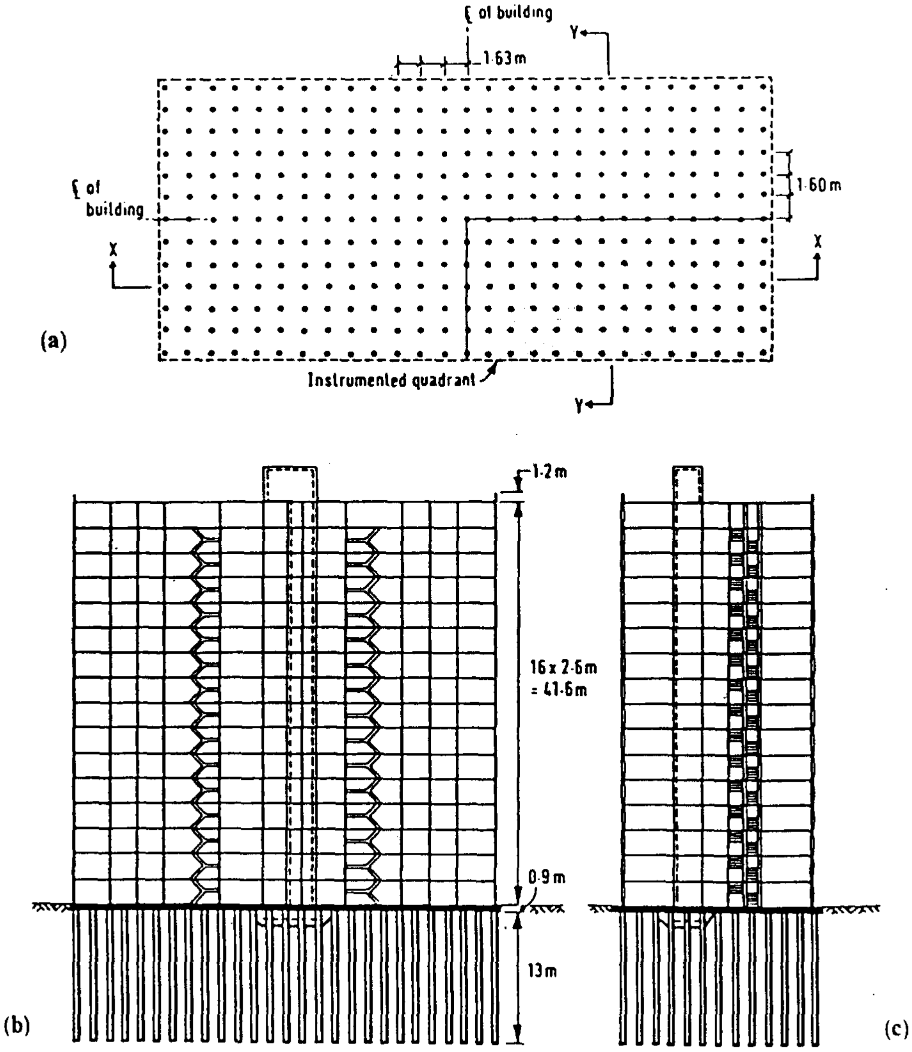



Figure 24 Plan (a) and elevation (b) of Stonebridge Park Flats (Cooke et al, 1981)

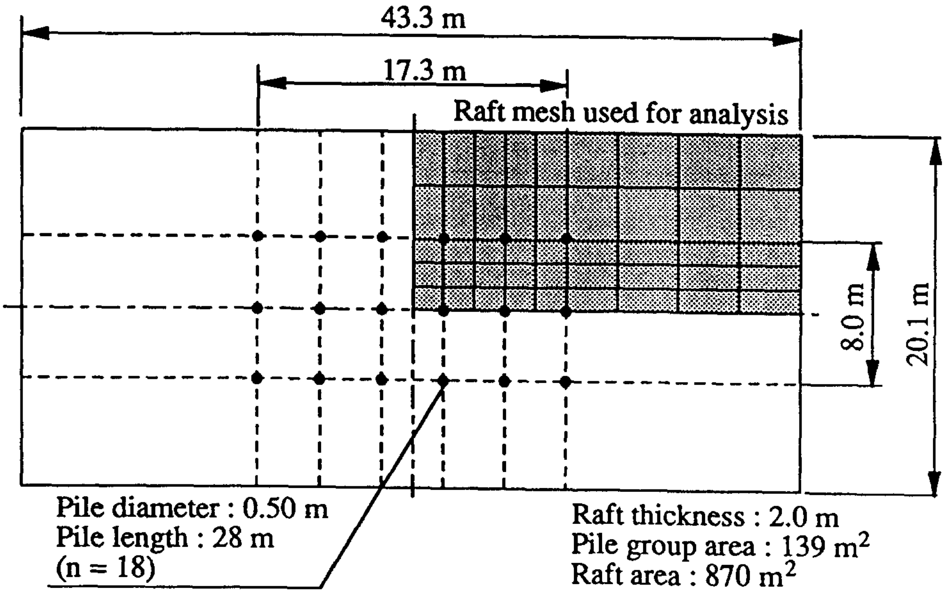



Figure 25 Foundation model for optimum design of Stonebridge Park Flats foundations

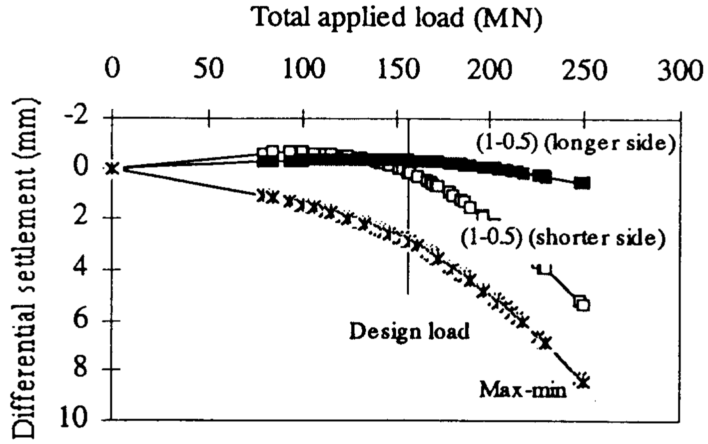



Figure 26 Differential settlement of centrally piled raft

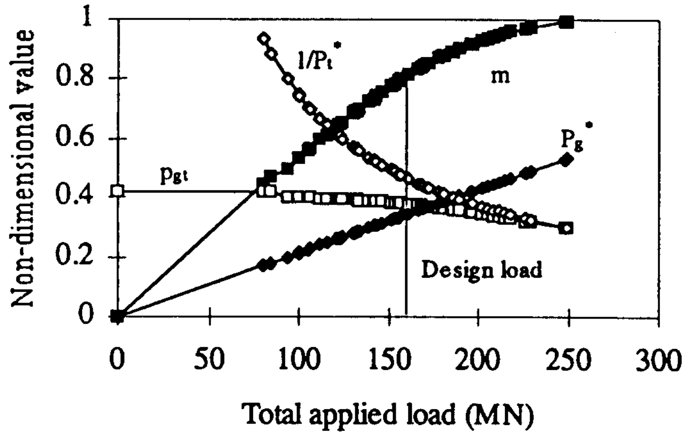



Figure 27 Variation of non-dimensional parameters

The ground conditions were assumed with reference to Fleming et al (1992). The profile of shear strength with depth was approximated by $s_u = 100 + 7.2z$ kPa, where z is the depth in m below foundation level (2.5 m below ground level). The shear modulus of the soil was assumed as $G_s = 200s_u$. For analysis using the program, HyPR, which is restricted to uniform soil conditions, a representative shear modulus of $G_s = 44$ MPa was adopted, equivalent to the actual values at a depth of $z = a_{eq}$. Poisson's ratio of the soil was taken as $\nu_s = 0.1$, and the adhesion factor for pile shaft friction was taken as 0.41 to be consistent with the report by Padfield and Sharrock. The total load ($P_t = 157$ MN) was assumed to be distributed uniformly over the raft, giving a pressure of 180 kPa.

The assumed foundation model and raft discretisation are shown in Figure 25. The design conditions are also summarised in Table 3. Although the raft assumed is relatively thick, the corresponding raft-soil stiffness ratio, $K_{rs}$ (see equation (6)) is calculated as 0.157. A raft thickness of $t_r = 1.7$ m would correspond to a value of $K_{rs} = 0.1$. Since the pile group area ratio, $a_{gr}$, is 0.16, the required pile group-raft stiffness ratio, $K_{pr}$, is about 1.0 considering non-linear pile behaviour.

The stiffness ratio $K_{pr}$ is first estimated using a simple approach. The stiffness of the unpiled raft, $k_r$, is calculated as $k_r = 3{,}150$ MN/m, from equation (14). The equivalent pier analysis described by Randolph (1994) gives the overall stiffness of the independent pile group as $k_p$ = 3,140 MN/m. The value of $K_{pr}$ is therefore calculated as 1.00.

**Table 3 Parameters assumed for Stonebridge Park example**

| PROPERTIES | VALUES | NOTE |
|---|---|---|
| Raft | | |
| Thickness, t (m) | 2.0 | $K_{rs}$ = 0.16 |
| Young's modulus, $E_r$ (GPa) | 40 | |
| Poisson's ratio, $\nu_r$ | 0.16 | |
| Pile | | |
| Young's modulus, $E_p$ (GPa) | 40 | |
| Length, $\ell$ (m) | 28 | $\ell/a_{eq}$ = 1.7 |
| Diameter, d (m) | 0.5 | $\ell/d$ = 56 |
| Number of piles, n | 18 | |
| Pile group area ratio, $a_{gr} = A_g/A_r$ | 0.16 | |

The overall stiffness of the piled raft is calculated as $k_{pr}$ = 3,800 MN/m, for the estimated interaction factor, $\alpha_{rp}$, of 0.66. This leads to an average settlement of the piled raft of 41 mm under the design load of 157 MN. The total capacity of the individual piles is calculated as 3.62 MN (shaft) plus 0.53 MN (base), giving 4.15 MN in total. The various non-dimensional parameters are thus calculated as $P_g^* = pA_g/(nq_p) = 0.33$ and $1/P_t^* = a_{gr}/P_g^* = 0.48$.

An analysis using HyPR gave an average settlement of 42 mm (2 % higher than the estimate above) and calculated differential settlements as shown in Figure 26. At the design load of 157 MN, the values at the mid-point of each side are negligible. In fact, the maximum settlement occurs between the pile group and the raft edge, but even there, the differential settlement is only 3 mm at the working load.

The variation of non-dimensional parameters with load level is shown in Figure 27. Under the design load, the two non-dimensional values not previously calculated are found to be m = 0.80 and $p_{gt}$ = 0.38, which also fall within the optimum range discussed earlier.

## 10. CONCLUSIONS

This paper has reviewed a variety of methods which may be used to estimate the settlement of pile groups and piled rafts. The use of equivalent piers to model either the entirety, or some part, of a pile group provides a simple, but reasonably accurate, alternative to full computer analysis. The same approach may be used for piled rafts, where the pile cap transfers load directly to the ground.

A key aspect of any foundation design is the likely differential settlement. Excessive differential settlement is often the basis for opting for a piled foundation rather than a raft. It has been shown, both through centrifuge model tests, and also by analysis, that the use of piles in the centre of a raft can provide an optimum solution, largely eliminating differential settlements, with the minimum amount of piling.

## 11. REFERENCES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