# An Identification of Transfer Function of A Single Point Cutting Process In A Recursive way

김 학인 제 1 연구 개발 본부 2부 2실 국 방 과 학 연 구 소

## 개 요

기계 가공을 위해선 보통 많은 절삭 에너지가 필요하다. 공작 기계는 대부분 매우 적은 감쇄 특성을 지니는데, 앞서 언급한 절삭 에너지는 공작 기계 구조물에 전달되게 된다. 이때 기계 가공에서의 특유 메카니즘인 채터 (chatter)라는 자생적인 진동(self-excited vibrations) 현상이 종종 발생한다. 이러한 채터 현상에서의 절삭 가공은 절삭 공구의 물리적 한계에 직면한다. 그러므로, 실시간 제어기의 적용을 필요로 한다. 이 연구 보고서는 chatter 진동을 적절히 제어하기 위해, 가공 시간에 따라 변하는 절삭공구와 피삭제간의 transfer function을 recursive 방식을 통해 update하도록 시도하였다. 이 방식은 디지털 신호처리 기법과 제어 설계 기법을 적용하였다. 아울러, 일련의 시뮬레이션이 재시되며, 현재, 실험적으로 접근하기 위한 연구를 진 3 중에 있다.

#### **ABSTRACT**

Most of machining process will take a large amount of cutting energy to accomplish the process. Then, self-excited vibration (called chatter) is often developed when the energy is fed through a machine structure that has a small damping characteristics. When the process is involved in chatter, the cutting process will face a basic performance limitation of the cutting tool. Therefore, it is necessary to implement a real-time controller. This paper presents a recursive way of finding the transfer function of the machine tool-workpiece interaction, based on digital signal processing technique, and a design of active chatter controller in real time. Currently, verification of the analytical work is being pursued by means of experimental approach.

## 1. INTRODUCTION

Both parametric method and non-parametric way have been used for a system identification of cutting dynamics. Parametric approach is represented by AR, ARMA and etc., and non-parametric method is characterized by FFT. With a broad application, the methods have been used for controller design as well as modal analysis and digital signal processing applications. Development and implementation of a filter based on recursive algorithm using non-parametric method was applied to a closed loop cutting process such as lathe, milling and grinding, etc. This paper shows how the recursive filter can continuously update the estimates of transfer function of the process and illustrates how the estimates can be used for controlling chatter vibrations of a machine tool.

## 2. BACKGROUND

Most of machining process will take a large amount of cutting energy to accomplish the process. Then, self-excited vibration (called chatter) is often developed when the energy is fed through a machine structure that has a small damping characteristics. When the process is involved in chatter, the process will face the poor surface quality in general as well as a basic performance limitation of the cutting tool.

It has been reported that the chatter vibration, caused by three types of mechanisms such as surface regeneration, mode coupling, and velocity-dependence (Wu, 1985a,b and Kim, 1995) typically lead to instabilities of the process and low production rates.

There are extensive researches to the problems of chatter in machining in the past several decades.

Gurney and Tobias (1962) reported a graphical method for regenerative chatter based on the harmonic response locus of the machine tool structure and allows the determination of the stable and unstable cutting speed ranges.

Subsequently, Bartalucci and Lisini (1969) worked on a theoretical investigation on chatter vibrations of cylindrical plunge grinding using a closed loop system with two feedback paths, one due to the machine tool structure, the other to the regenerative effect of the grinding wheel.

Besides these works in the 1960s', there are great number of investigations on the machine chatter vibrations and the stability by Lemon and Ackermann (1965), Long and Lemon (1965), Merritt (1965), R. Sridhar et al (1968a,b), followed by Nachtigal (1972), Mitchell and Harrison (1974), and Srinivasan and Nachtigal in the field of control for the cutting process in 1970s'.

Merritt (1965) proposed a model of a closed-loop feedback control system with regenerative chatter, investigating the subject on the asymptotic and lobed stability problems for an interaction between machining machine and cutting tool with *multi-*degrees of freedom. In this paper, for the purposes of exploring different controller design, a lumped model of machining system with second-order is simulated.

It is often assumed that for simple turning operations the cutting force  $F_c$ , are linear relationship with the chip thickness b and the depth of cut a as,

$$F_{c} = Kba \tag{1}$$

where K represents a cutting force parameter and is a function of cutting conditions and workpiece materials (Kim. 1995). The lumped second order system of cutting process can be represented as,

$$M_{...}\ddot{v} + C_{...}\dot{v} + K_{...}v = F_{..} \tag{2}$$

where y is the deflection of the system and  $K_m$  is the machine stiffness.

Using Laplace transform, Equation (2) can be shown as the ratio between the deflection and the cutting force as,

$$\frac{Y(s)}{F_c(s)} = \frac{G_m(s)}{K_m} = \frac{1}{K_m} \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}$$
(3)

where  $\zeta_m$  and  $\omega_m$  are the damping ratio and natural frequency of the machine tool system, respectively. The instantaneous depth of cut a(t) during oscillations of tool-workpiece system can be described in Figure 1 and can then be expressed as,

$$a(t) = a(0) - y(t) + \mu y(t - T)$$
(4)

where the term  $-y(t) + \mu y(t-T)$  represents the deviation of the relative motions resulting from the preceding revolution and the current revolution, and  $\mu$  describes the amount of overlap with the previous cut. Normally, it is taken for  $\mu = 0$  for threading operation and  $\mu = 1$  for orthogonal cutting operation.

Equation 1 through 3 are combined in order to model the uncontrolled process with chatter. Most of works in this paper is try to compare with Merritt's result. Based on his conclusion, stability was confirmed when the minimum real value of the forward loop transfer function is greater than -0.5 and in the frequency domain this description is written as,

$$\operatorname{Re}\left[\frac{K}{K_{m}}\frac{\omega_{m}^{2}}{s^{2}+2\zeta\omega_{m}s+\omega_{m}^{2}}\right]_{\min} > -0.5$$
(5)

For a characteristics of Equation (5) in the frequency domain, it is substituted for s by  $j\omega$ . For the uncontrolled process, the critical frequency  $\omega_c$  is obtained by taking the first derivative of Equation (5) as,

$$\omega_c = \omega_m \sqrt{1 + 2\zeta} \tag{6}$$

Then, the ratio of  $K/K_m$  is obtained at the stability limit as.

$$\frac{K}{K_{m}} = 2(\zeta + \zeta^{2}) \tag{7}$$

Two typical types of controllers are illustrated for active controllers: 1) control scheme for the error signal of the displacement (Mitchell and Harrison, 1974) and 2) control scheme for the cutting force signal (Nachtigal, 1972). When the error signal of the displacement is the control variable, the signal would contain the information on the overall deviation of displacement from the reference. Therefore, the controller is applied to minimize the error signal to maintain a constant cutting depth. In this case, the location of the measurement sensor is very critical and may produce noise a lot.

Controlling cutting force signal is an another type of active chatter controller. The measurement system uses strain gauges to measure the cutting forces so that the force signal is to use control the machine tool vibration. An active dynamic compliance is superposed in parallel with that of the machine structure. Using this type of controller, a natural difficulty is arising for estimating an accurate machine tool transfer function. It is normally understood that the machine tool transfer function varies with the cutting point.

In this analysis, a controller as illustrated in Figure 3 will be used. For this system, it is assumed that a second-order transfer function can accurately model the machine tool workpiece combination and the controller/actuator combination, denoted  $G_{as}(s)$ , will have the same second-order form:

$$G_{as}(s) = K_c G_c(s) = \frac{K_c \omega_c^2}{s^2 + 2\zeta \omega s + \omega^2}$$
 (8)

Using this type of controller, the following closed-loop transfer function can be derived:

$$\frac{Y(s)}{A_o(s)} = \frac{K[G_m(s) / K_m - K_c G_c(s)]}{1 + K[1 - e^{-sT}][G_m(s) / K_m - K_c G_c(s)]}$$
(9)

Thus, it is easily seen that as  $K_cG_c(s)$  approaches  $G_m(s)/K_m$ , the controller will eliminate chatter; however, if  $K_cG_c(s)>G_m(s)/K_m$ , then the resulting system will become unstable. For this case, stability will be ensured if the following

condition is met:

$$\frac{K}{K_{m}} \text{Re} [G_{m}(s) - K_{m} K_{c} G_{c}(s)]_{\min} > -0.5$$
(10)

For a stable operations, it would be very difficult to determine the machine tool parameters precisely and this issue is even further complicated by the fact that the machine tool parameters are constantly changing during the machine tool process.

## 3. RECURSIVE ESTIMATION

As described in Figure 3, we want to make the output, relative displacement between the tool and the workpiece, to be zero as in the frequency domain

$$Y(\omega) = Y_{w}(\omega) - Y_{t}(\omega) = 0 \tag{11}$$

where

$$Y_{\mathcal{W}}(\omega) = FFT[y_{\mathcal{W}}(t)]; Y_{\mathbf{f}}(\omega) = FFT[y_{\mathbf{f}}(t)]$$
(12)

The tool displacement would be related to the cutting force by the following relationship:

$$Y_{c}(\omega) = K_{c}G_{c}(\omega)F_{c}(\omega) \tag{13}$$

Since  $F_c(\omega)$  is modulated by the change in the machine tool dynamics in closed-loop process model, the only  $G_{as}(s)$  can be actively controlled variable. Recognizing this fact, Equation 12 can be expanded in a Taylor series about an operating point:

$$Y_{w}(\omega) - Y_{t}(\omega) = \hat{Y}_{w}(\omega) - \hat{Y}_{t}(\omega) - \frac{\partial \hat{Y}_{t}(\omega)}{\partial G_{c}(\omega)} \left[ G_{c}(\omega) - \hat{G}_{c}(\omega) \right]$$
(14)

Since the left-hand side is identically equal to zero, we must have

$$\hat{Y}_{u}(\omega) - \hat{Y}_{r}(\omega) - \frac{\partial \hat{Y}_{r}(\omega)}{\partial G_{r}(\omega)} \left[ G_{c}(\omega) - \hat{G}_{c}(\omega) \right] = 0 \tag{15}$$

In incremental form, this equation can be rearranged to give the desired frequency based control equation:

$$G_{c,j}(\omega) = \hat{G}_{c(j-1)}(\omega) + \frac{Y_{w,j-1}(\omega) - Y_{t,j-2}(\omega)}{Y_{t,j-1}(\omega) - Y_{t,j-2}(\omega)} \left[ G_{c,j-1}(\omega) - G_{c,j-2}(\omega) \right]$$
(16)

Equation 16 represents a frequency based adaptation of Newton's method for solving a nonlinear equation for a root. Note that in the frequency domain, multiplication and division are point by point operations between components with the same frequencies; consequently, each and every component of the one-dimensional frequency based arrays are processed by a single application of Equation 16. It should be also noted that two estimates of  $G_{as}(\omega)$  must be obtained before Equation 16 can be implemented.

One of the most important advantage of using this scheme is that this type of controller works equally well for both linear and nonlinear system and since most machining processes are indeed highly nonlinear, this controller is ideally suited to the task.

## 4. SIMULATION

The author has experienced some of analytical and experimental work in the area of transfer function estimation and control and the results has shown that it is possible to control chatter based on tool position measurements, cutting forces, and head stock spindle deflections.

In this paper, a general single turning process with a feed forward controller illustrated in Figure 3. The system was assumed to be  $K/K_m = 0.1$ , damping ration  $\zeta_m = 0.02$ , natural frequency  $\omega_m$  is 350 rad/sec. First, setting the controller gain  $K_c = 0$  as

described in Figure 3, a time response of displacement of the machining system for the uncontrolled process is obtained as shown in Figure 4. Simulation has been performed in such a way that the transfer function is estimated at every 2.048 second and is affected the process for the next 2.048 seconds as described by Equation 16. The sampling interval is 0.001 seconds and the spectral estimation used the hamming window with 512 FFT points in a block.

At the end of the first time segment (2.048 seconds), the controller natural frequency and damping ratio were estimated to be 327 rad/sec and 0.05, respectively. Also, the ratio of  $K/K_m$  was estimated to be 0.15. This value of  $K/K_m$  was used throughout the rest of simulation. Using these estimated values in the feed forward controller, the simulation is allowed to proceed to the next time step. This estimation returned a natural frequency of 332 rad/sec and a damping ratio of 0.11. Using these two estimates of the machine tool transfer function, Equation 16 was used to determine a new transfer function with a damping ratio of 0.06 and a natural frequency of 335 rad/sec as shown in Figure 6. At the next time step (4.096 seconds) this initial estimates was refined even further by another application of Equation 16 yielding a natural frequency of 346 rad/sec and a damping ratio of 0.07 in Figure 7. Meantime, the time domain response is shown in Fiure 5. For the rest of the simulation, these parameters did not change significantly.

## 5. CONCLUSION

As seen in the simulation, the proposed system identification algorithm works quickly and efficiently - correctly identifying the machine tool parameters and adjusting to match the system dynamics in as little as four seconds. By the end of the 6.144 second simulation period, the controller has virtually estimated all of the chatter. Although the proposed controller was simulated on a linear dynamical system and the resulting transfer function was determined based on linear system theory, this type of controller can be adapted to work on nonlinear systems by computing the desired cutting tool position as a function of time by inverse transforming the product of the cutting force time sequence and the estimated transfer function.

Also, as shown in Figure 3, one of the parameters that are used to estimate the machine tool transfer function is the workpiece deflection, and as most people would agree, this measurement is very difficult (if not impossible) to measure accurately in an actual shop environment. However, this problem can be overcome by incorporating accelerometers into the headstock to measure the vibrations of the spindle during the cutting process. The resulting vibrational information can then be used in the same manner as the workpiece deflection to estimate the transfer function parameters. With all analytical developments, the real test comes with experimentation and a closed loop active chatter control system is being designed and built.

#### REFERENCES

- 1. Bartalucci, B., and Lisini, G. G., 1969, "Grinding Process Instability", Journal of Engineering for Industry, ASME, pp. 597 606.
- 2. Gurney, J. P. and Tobias, S. A., 1962, "A Graphical Analysis of Regenerative Machine Tool Instability", Journal of Engineering for Industry, ASME, pp. 201 206.
- 3. Kim, H., 1995, "Dynamics of ceramic grinding: regeneration and stability", Ph.D. Dissertation, The University of Arizona
- 4. Long, G. W., and Lemon, J. R., 1965, "Structural Dynamics in Machine-Tool Chatter: Contribution to Machine Tool Chatter", Journal of Engineering for Industry, ASME ,pp. 455 463.
- 5. Merritt, H. E., 1965, "Theory of Self-Excited Machine Tool Chatter: Contribution to Machine-Tool Chatter, Research 1", Journal of Engineering for Industry, Series B, Vol. 87, No. 4, pp. 447-454.

- 6. Mitchell, E. E. and Harrison., E., 1974, "Active Machine Tool Controller Requirements for Noise Attenuation", Journal of Engineering for Industry, pp.  $261 \sim 267$ .
- 7. Nachtigal, C., 1972, "Design of Force Feedback Chatter Control System", Journal of Dynamic Systems, Measurement, and Control, Trans. of ASME, pp. 5~10
- 8. Sridhar, R., Hohn, R. E., and Long, G. W., 1968, "A General Formulation of the Milling Process Equation", Journal of Engineering for Industry, ASME, pp. 317 324.
- 9. Srinivasan, K. and Nachtigal, C. L., 1978, "Investigation of the Cutting Process Dynamics in Turning Operations", Journal of Engineering for Industry, ASME, Vol. 100, pp. 323 331.
- 10. Wu, D. W. and Liu, C. R., 1985, "An analytical Model of Cutting Dynamics. Part 1: Model Building", Journal of Engineering for Industry, ASME, Vol. 107, pp. 107 111.
- 11. Wu, D. W. and Liu, C. R., 1985, "An analytical Model of Cutting Dynamics. Part 2: Verification", Journal of Engineering for Industry, ASME, Vol. 107, pp.  $112 \sim 118$

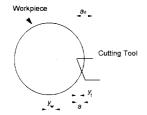


Figure 1. Configuration of tool - workpiece system: i) dashed line for no motion and solid line for displaced configuration

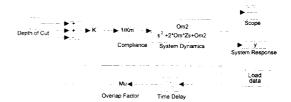


Figure 2 Uncontrolled single point cutting process

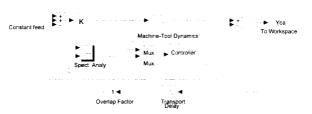


Figure 3 Controlled single point cutting process

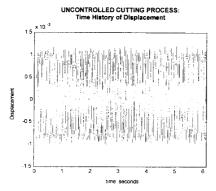


Figure 4. Time history of uncontrolled chatter displacement (0~6.144seconds)

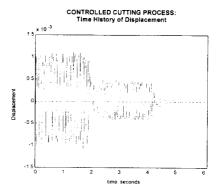


Figure 5. Time history of controlled chatter displacement

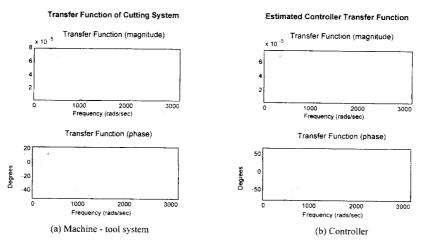


Figure 6. Transfer Function During Controlled Chatter For 2<sup>nd</sup> Period (2.049 ~ 4.096 sec.)

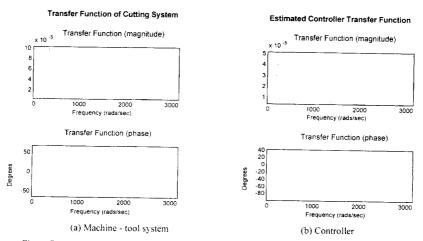


Figure 7. Transfer function during controlled chatter vibrations for  $3^{rd}$  Period (4.097  $\sim$  6.144 sec.)