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Optimal Design of Smart Actuator by using of GA for the Control of a Flexible Structure Experiencing White Noise Disturbance

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Abstract

This paper deals with the problem of placement/sizing of distributed piezo actuators to achieve the control objective of vibration suppression. Using the mean square response as a performance index in optimization, we obtain optimal placement and sizing of the actuator. The use of genetic algorithms as a technique for solving optimization problems of placement and sizing is explored. Genetic algorithms are also used for the control strategy. The analysis of the system and response moment equations are carried out by using the Fokker-Planck equation. This paper presents the design and analysis of an active controller and optimal placement/sizing of distributed piezo actuators based on genetic algorithms for a flexible structure under random disturbance, shows numerical example and the result.

I. Introduction

In this paper we consider the problem of placement and sizing of distributed piezo actuators to achieve the control objective of vibration suppression. The recent application of piezo materials by Crawley and others for the use of actuators of flexible structures has added new lights to control problems. Piezoelectric materials are light and therefore can be used for vibration control without significantly modifying the dynamic properties of the structure. Crawley et. al. have presented mechanical models for the interaction of piezoelectrics with one and two dimensional structures. Piezoelectric materials, which generate an electric charge in response to a mechanical deformation, conversely undergo mechanical strain when an electric field is applied across them, seem to be promising candidates for the role of distributed actuators in the active control of flexible structures[1]. Kondoh et al. and many researchers used the linear quadratic optimal control framework to perform actuator placement, but formulated the problem such that the solution is initial conditions dependent[2]. However, how do they know real initial conditions of the flexible system under random disturbance such as earthquake and so on. This paper proposes using the mean square response as a performance index in optimization.

In many cases of engineering interest, it has become quite common to use stochastic processes to model loading such as earthquakes [6]. The structural response needs to be adequately evaluated in the probabilistic sense by means of the cumulants or the moments of any order of the response. For linear or nonlinear systems driven by random prosesses, the problem of finging the probabilistic characterization of the response can be faced by means of Ito's stochastic calculus which, in connection with Ito's rule allows the complete characterization of the response process. It can be obtained by means of the moment equation approach. The method consists in the converting the equation of motion into an Ito-type stochastic differential equation and the applying Ito's rule in evaluating the differential equations govering the response moments of any order[5].

Genetic algorithms are search procedures based on the mechanics of natural genetics. All natural species survive by adapting themselves to the environment. This natural adaptation is the underlying theme of GA. Genetic algorithm search combines a Darwinian survival-of-the-fittest strategy to eliminate unfit characteristic and uses random information exchange. The advantage of genetic algorithms is that they decrease to have local minimum in probabilistic using global search and don't need restrictions such as continuity of searching space and differentiability.[3]

Genetic algorithms are diffrent from normal search methds encountered in engineering optimization in the following ways: 1) GAs work with a coding og the parameter set, not the parameter themselves. 2) GAs search from a population of points, not a single point. 3) Gas use probabilistic transition rules, not deterministic transition rules. Another important difference between GAs and the classical approaches is in the selection of the transition rules. In classical methods of optimization, the transition rule is deterministic. In contrast, GAs use probabilistic operator to guide their search.

A simple genetic algorithm is composed of three operators: 1) reproduction, 2) crosover and 3) mutation. Reproduction is a process whrer an old string is carried through into a new population depending on the performance index,or fitness value. Due to this move, strings with better fitness values get larger numbers of copies in the next generation. Seleting good strings for the reproduction operation can be implement in many different ways. Crossover recombines pair of chromosomes by swapping parts of them from a randomly selected point, to create two new string(offspring), and a change of the value in a string position is evolved by mutation. These algorithms use probabilistic rule to produce a new generation.

The other emphasis of this paper is the design and analysis of a GA based controller for a cantilever beam as a flexible structure. The considered feedback controller using genetic algorithms are applied to the vibration suppression of the flexible structure under random disturbance.

II. Analytical Investigation

Mathematical modeling

Consider a flexible cantilever beam with a laminated piezo actuator. The beam is subject to random disturbance as the base excitation at the clamped end. The system under investigation is shown schematically in Fig 1.

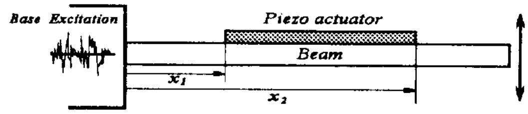

Fig. 1. A sketch of the piezo actuated beam

Hamiton's principle is used to derive the partial differential equation of motion describing the distributed parameter system above (See Appendix). Applying Galerkin's method the following ordinary modal equations can be written as

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = B_i v(t) + P_i \ddot{z}(t) \qquad (1)$$

where ψ_i determined the modal stiffness and other coefficients are defined by

$$\psi_i = \int_0^L \phi_i \frac{d^4\phi_i}{dx^4} dx \,,\quad \omega_i = \sqrt{\frac{E_b I_b \psi_i}{\rho_b A_b}}\,,$$

$$B_i = \frac{1}{\rho_b A_b}[\phi'_i(x_2) - \phi'_i(x_1)]K_p \qquad (2)$$

$$P_i = -\frac{1}{\rho_b A_b}\int_0^L \phi_i(x)dx$$

Let the state feedback be defined by

$$v(t) = -g_i q_i(t) - h_i \dot{q}_i(t) \qquad (3)$$

Substituting Eq(3) into Eq(1) yields the closed-loop equation:

$$\ddot{q}_i(t) + B_i h_i \dot{q}_i(t) + B_i(\omega_i^2 + g_i) q_i(t) = P_i \ddot{z}(t) \quad (4)$$

The goal of the control system design in this study is to determine actuator location x_1, x_2 involved in B_i and controller feedback gain g_i, h_i

Mean Square Response

The thrust of this paper is to investigate the optimal mean square response of the flexible system to a stationary, white noise input disturbance. For a linear system excited with a white noise input, random vibration theory shows that the mean square response of any output y is related to the spectral density amplitude of the input S_0 and the frequency response function H_y.

$$E[y^2] = S_0 \int_{-\infty}^{+\infty} |H_y(\omega)|^2 d\omega \qquad (5)$$

From Eq(4), we get the frequency response function H_{qi} as following:

$$H_{q_i}(\omega) = \frac{P_i}{-\omega^2 + j\omega B_i h_i + B_i(\omega_i^2 + g_i)} \qquad (6)$$

In order to find the mean square response, we use Eqs(5) and (6), then

$$E[q_i^{\,2}]=\frac{P_i^{\,2}}{B_i h_i(\omega_i^2+g_i)}\pi S_z \tag{7}$$

S_z is the spectral density amplitude of the input disturbance. The integral may be evaluated with the help of Newland[4].

$$\int_{-\infty}^{+\infty}\left|\frac{i\omega B_1+B_0}{-\omega^2A_2+i\omega A_1+A_0}\right|^2 d\omega = \frac{(B_0^2/A_0)A_2+B_1^2}{A_1A_2}\pi \tag{8}$$

where the ratio inside the absolute value is just the second order transfer function evaluated at $s=j\omega$.

Differential Equations of Response Statistic

The random input disturbance is characterized by a power spectral density for which empirical expressions can be used or an assumed Markov field may be introduced. The use of empirical expressions requires numerical example of the equations of motion, and the Markov field approximation yields a set of differential equations for the response moments.

For a two-mode iteration, the differential equations of the first two modes are

$$\begin{aligned}&\ddot{q}_1(t)+B_1h_1\dot{q}_1(t)+B_1(\omega_1^2+g_1)q_1(t)=P_1\dot{z}(t)\\&\ddot{q}_2(t)+B_2h_2\dot{q}_2(t)+B_2(\omega_2^2+g_2)q_2(t)=P_2\dot{z}(t)\end{aligned} \tag{9}$$

The external random disturbance is assumed uncorrelated, Gaussian, wide-band random process with zero mean. The white noise process, say, the external random disturbance can be expressed as the formal derivative of the Brownian motion:

$$\dot{z}(t) = \frac{dB_z(t)}{dt} \tag{10}$$

where $B_z(t)$ is Brownian motion process. The correlation function of this process in the study is

$$E[dB_z^2(t)] = 2S_z dt \tag{11}$$

Through the coordinate transformation

$$\{q_1, q_2, \dot{q}_1, \dot{q}_2\} = \{X_1, X_2, X_3, X_4\} \tag{12}$$

Eq(9) may be written in terms of Ito stochastic differential equation in the state vector form

$$\begin{aligned}dX_1 &= X_3\,dt\\ dX_2 &= X_4\,dt\\ dX_3 &= \{-B_1(\omega_1^2+g_1)X_1-B_1h_1X_3\}dt+P_1dB_z\\ dX_4 &= \{-B_2(\omega_2^2+g_2)X_2-B_2h_2X_4\}dt+P_2dB_z\end{aligned} \tag{13}$$

The evolution of the joint probability density of the response coordiates $p(\boldsymbol{X},t)$ can be described by the Fokker-Planck equation

$$\frac{\partial p(\boldsymbol{X},t)}{\partial t} = -\sum_{i=1}^{4}\frac{\partial}{\partial t}\{a_i(\boldsymbol{X},t)p(\boldsymbol{X},t)\} + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\frac{\partial^2}{\partial X_i\partial X_j}\{b_{ij}(\boldsymbol{X},t)p(\boldsymbol{X},t)\} \tag{14}$$

where a_i and b_{ij} are incremental moments known as drift and diffusion coefficients, respectively. It is not possible to solve the system Fokker-Planck equation for the response probability density even for the stationary case. Instead, one may derive a general first order differential equation for the response dynamic moments by using the Fokker-Planck equation approach or the Ito stochastic calculus[5]. This equation is derived by multiplying both sides of the system Fokker-Planck equation by the scalar function

$$\Phi(\boldsymbol{X}) = X_1^iX_2^jX_3^kX_4^s \tag{15}$$

and integrating over the entire space $-\infty<\boldsymbol{X}<\infty$. This results in the general moment equation

$$\begin{aligned}\dot{m}_{1000} &= m_{0010}\\ \dot{m}_{0100} &= m_{0001}\\ \dot{m}_{0010} &= -B_1h_1m_{0010}-B_1(\omega_1^2+g_1)m_{1000}\\ \dot{m}_{0001} &= -B_2h_2m_{0001}-B_2(\omega_2^2+g_2)m_{0100}\\ \dot{m}_{1001} &= m_{0011}-B_2h_2m_{1001}-B_2(\omega_2^2+g_2)m_{1100}\\ \dot{m}_{1010} &= m_{0020}-B_1h_1m_{1010}-B_1(\omega_1^2+g_1)m_{2000}\\ \dot{m}_{1100} &= m_{0110}+m_{1001}\\ \dot{m}_{0101} &= m_{0002}-B_2h_2m_{0101}-B_2(\omega_2^2+g_2)m_{0200}\\ \dot{m}_{0110} &= m_{0011}-B_1h_1m_{0110}-B_1(\omega_1^2+g_1)m_{1100}\\ \dot{m}_{0011} &= -B_1h_1m_{0011}-B_1(\omega_1^2+g_1)m_{1001}\\ &\quad -B_2h_2m_{0011}-B_2(\omega_2^2+g_2)m_{0110}+4P_1P_2S_z\\ \dot{m}_{2000} &= 2m_{1010}\\ \dot{m}_{0200} &= 2m_{0101}\\ \dot{m}_{0020} &= -2\{B_1h_1m_{0020}+B_1(\omega_1^2+g_1)m_{1010}-P_1^2S_z\}\\ \dot{m}_{0002} &= -2\{B_2h_2m_{0002}+B_2(\omega_2^2+g_2)m_{0101}-P_2^2S_z\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}m_{ijks} &= \int\int\int\int_{-\infty}^{+\infty}X_1^iX_2^jX_3^kX_4^s dX_1dX_2dX_3dX_4\\ &= E[X_1^iX_2^jX_3^kX_4^s]\end{aligned} \tag{17}$$

Performance Index

In this section we will determine feedback gains and optimal actuator placement/sizing using GA. We select the performance index or the fitness function in genetic algorithms. In Eq(18), α_i are the weighting values that is smaller than 1.

$$J = \frac{\pi S_0}{\sum_{i=1}^{\infty} \alpha_i E[q_i^2]} \tag{18}$$

Genetic algorithms are to determine optimal gains, g_i and h_i and locations, x_1 and x_2 using the above fitness function represented by the mean square responses.

IV. Numerical Example and Results

The properties of beam and piezo material under consideration is shown in tables. See Appendix II. In this paper, genetic algorithm optimization techniques involves vibration control system optimization problem. Genetic algorithms can find both optimal placement/sizing of the piezo actuator and controller feedback gains. In genetic algorithm the number of the maximum generation is 200. The population size is fixed at 50. The probabilities of crossover and mutation are 0.2 and 0.05, respectively. For simplicity the first two modes are considered. This assumption is reasonable because the first and second modes are dominant in a flexible system. The optimal location x_1 and x_2 using genetic algorithms are 0.103009 and 0.173872, respectively. Even though 28% piezo actuator of total beam area is used, the flexible structure under random disturbance is controlled succefully. Fig. 2 reveals the successful performance of the considered feedback controller using genetic algorithms on the vibration suppression control of the flexible structure under random disturbance applied to the base of the flexible structure.

A technique is discussed in which genetic algorithms are used to design the controller for a flexible structure under random disturbance. It can be concluded that genetic algorithms are efficient and a powerful tool both in a controller gain tuning and in the optimization of placement/sizing simultaneously. Thus genetic algorithm is successfully used to design the controller for stochastic system. More refined development is going on.

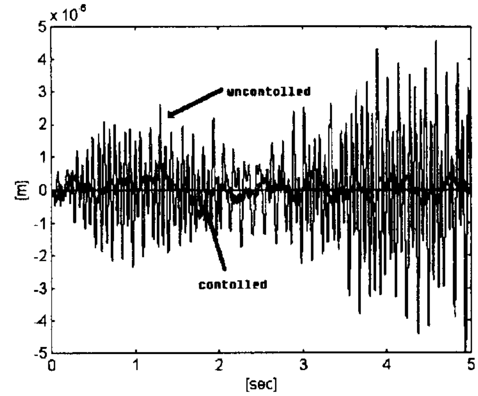

Fig. 2 Tip Displacememt of a Piezo Actuated Beam

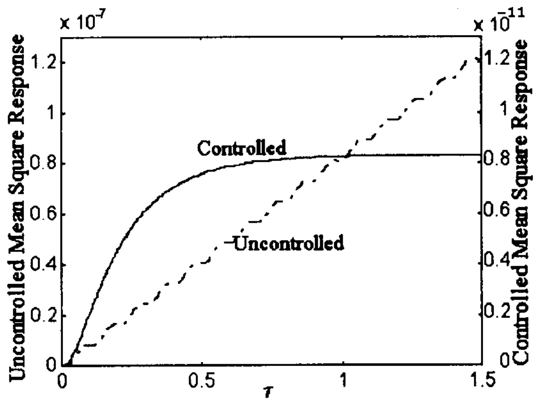

Fig. 3 Mean Square Responses of the 1st mode

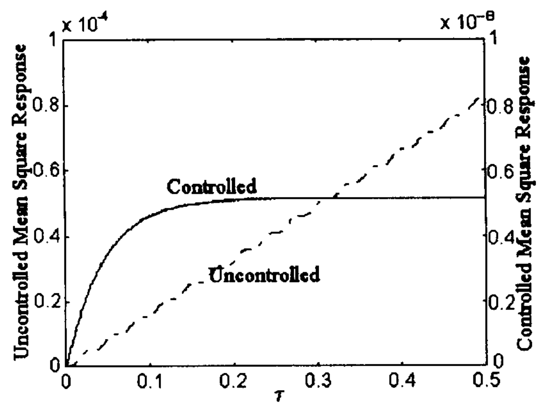

Fig. 4 Mean Square Responses of the 2nd mode

Appendix I : A Piezo Actuated Beam

The mathematical model for such a mechanically flexible beam is modeled by an

Euler-Bernoulli type partial differential equation having terms due to the control associated with bending moment in the study, and is given by the following hyperbolic partial differential equation:

$$\frac{\partial^2}{\partial x^2}\left(E_b I_b \frac{\partial^2 y(x,t)}{\partial x^2}\right) + \rho_b A_b \frac{\partial^2 y(x,t)}{\partial t^2} = -\rho_b A_b \ddot{z}(t) + F_M(x,t) \quad \text{for } 0<x<L \tag{A1}$$

where $y(x,t)$ is the transverse displacement from its equilibrium state at the position x and time t, $F_M(x,t)$ is a distributed control input associated with bending moments $M(x,t)$, $z(t)$ the support displacement due to the random base excitation. The initial and the boundary conditions are given as

$$I.C.'s: \quad y(x,0) = y_0(x), \quad \frac{\partial y(x,0)}{\partial t} = \dot{y}_0(x)$$

$$B.C.'s: \quad y(0,t) = \frac{\partial y(0,t)}{\partial x} = 0 \tag{A2}$$

$$\frac{\partial^2 y(L,t)}{\partial x^2} = \frac{\partial^3 y(L,t)}{\partial x^3} = 0$$

Consider a segment of the piezo beam as shown in Fig.1. The bending moment M around the neutral axis, D of the composite beam generated by the piezo actuated beam as follows [1]

$$M = b d_{31} E_p D v(t) \triangleq K_p v(t) \tag{A3}$$

For torquers located at the piezo endpoint x_1 and x_2, one can express the external excitation as [4]

$$F_M(x,t) = -\frac{\partial \tau_s(x,t)}{\partial x} = M(t)[\delta'(x-x_2) - \delta'(x-x_1)] \tag{A4}$$

in which τ_s is the spatial torque distribution. $M(t)$ denoted the amplitudes of the torque inputs of actuators located at the piezo endpoint x_1 and x_2, and δ' is the spatial derivative of the Dirac delta function.

Our assumption of a finite beam model guarantees a modal decomposition of the form

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t) \tag{A5}$$

where the $q_i(t)$ are the modal amplitudes and the $\phi_i(x)$ are the corresponding mode shape functions which are selected from the linear eigenvalue solution.

$$\phi_i(x) = \cosh(\beta_i x) - \cos(\beta_i x) - \sigma_i[\sinh(\beta_i x) - \sin(\beta_i x)] \tag{A6}$$

where

$$\sigma_i = \frac{\cosh(\beta_i L) + \cos(\beta_i L)}{\sinh(\beta_i L) + \sin(\beta_i L)} \tag{A7}$$

Appendix II: Specifications of a Beam Used

Parameter	Value
Length, L	0.293 [m]
Width, b	0.04 [m]
Thickness, t_b	0.0013 [m]
Density, ρ_b	1.2661×10^3 [kg/m^3]
Young's Modulus, E_b	4.7538 [GPa]

Table 1. Properties of the Flexible Beam

Parameter	Value
Thickness, t_p	110 [μm]
Density, ρ_p	1.78×10^3 [kg/m^3]
Young's Modulus, E_p	2 [GPa]
Piezo Strain constant, d_{31}	23×10^{-12}[C/N]
Piezo Stress constant, g_{31}	216×10^{-3}[Vm/N]

Table 2. Properties of the Piezomaterial

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