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# THE MODEL FOLLOWING CONTROL SYSTEMS FOR DESCRIPTOR SYSTEM

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Abstract In this paper, a designing method of model following control system for linear descriptor system with disturbances is proposed. The features of this method are: 1) both the physical structure of the system and the physical system variables properties can be preserved because there is no necessary to make transformation of this system. 2) boundedness of internal states are proved by means of coprime factorization of descriptor system.

Key word descriptor system, model following control system, disturbances, inner states, boundedness

## 1. INTRODUCTION

Descriptor system (DS) are often found in engineering system(such as electrical circuit network, power system, aerospace engineering, and chemical processing), social system, economic systems, biological systems network analysis, time-series analysis, singular singularly perturbed systems with which the singular system has a great deal to do, etc. their form also makes them useful in system modeling [1].

The descriptor system has been studied as a more natural description of dynamical system than the state-space system [2) 3)]. Using the descriptor system ,we can explicitly describe impulsive modes and algebraic constraints of a system. Owing to its versatility for describing various system, the descriptor-based system theory will play an important role bridging a gap between practical problem and the control theory based on the state-space form.

The theory of DS has been studied intensively in recent years [1)~3)], but little attention was paid to the studying of the model following control system (MFCS) for DS. In this paper, a design method of linear multi input multi output MFCS with disturbances for descriptor system was proposed. The internal states stability is proved by means of the coprime factorization of DS. Because there is no necessary to make transformation of this system, the physical structure of the system and the physical properties of the system variables can be preserved. The effectiveness of this method has been verified by numerical simulations.

## 2. SYSTEM DESCRIPTION

The controlled object is described in (1), which is a linear descriptor system with disturbances, the reference model is given in (2). The reference model is assumed controllable and observable.

$$E\dot{x}(t) = Ax(t) + Bu(t) + d(t) \tag{1.a}$$

$$y(t) = Cx(t) + d_0(t) \tag{1.b}$$

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_m(t) \tag{2.a}$$

$$y_m(t) = C_m x_m(t) \tag{2.b}$$

where $x(t) \in R^n$ is the descriptor variable, $u(t) \in R^l$ is the control input, $d(t) \in R^n$ and $d_0(t) \in R^l$ are the disturbances, $E(t) \in R^{n\times n}$, $x_m(t) \in R^{n_m}$, $r_m(t) \in R^{l_m}$, $y_m(t) \in R^l$, $A, B, C, A_m, B_m, C_m$ are appropriate dimensional matrices.

The control object is assumed

(1) Controllable and observable, i.e. the following conditions hold

$$rank[Ep-A,B]=n, \quad \forall p \in C_z, p \text{ finite} \quad (3.a)$$

$$rank[E,B]=n \quad (3.b)$$

$$rank\begin{bmatrix} Ep-A \\ C \end{bmatrix}=n, \quad \forall p \in C_z, p \text{ finite} \quad (4.a)$$

$$rank\begin{bmatrix} E \\ C \end{bmatrix}=n, \quad (4.b)$$

where $C_z$ is complex set.

(2) In order to guarantee the existence and uniqueness of the solution and have exponential function mode but a impulse one for (1), the following conditions are assumed

$$\det(Ep-A) \neq 0 \quad (5)$$

$$\text{rank } E = \text{degree det } (Ep-A)=r \quad (6)$$

(3) Zero points of $C(Ep-A)^{-1}B$ are stable.

In this paper, we propose a design of model following control for linear descriptor system with disturbances, of which all the internal states are bounded and output error

$$e(t)=y(t)-y_m(t) \quad (7)$$

is asymptotically converge to zero.

## 3. A DESIGN OF MFCS FOR DESCRIPTOR SYSTEM

Let $p= d/dt$, then the transfer function of controlled system and model are as followings

$$C(Ep-A)^{-1}B = N(p)/D(p) \quad (8)$$

$$C_m(Ep-A_m)^{-1}B_m = N_m(p)/D_m(p) \quad (9)$$

where

$$D(p)=|Ep-A|, \quad D_m(p)=|Ip-A_m|$$

transfer functions are proper. From this, (10), (11) and (12) are obtained, and (12) represents disturbances part. $u(t)$ has no relation with any initial condition $x(0)$ with this method, so the initial condition $x(0)$ is omitted, to simplify procedure of calculating $u(t)$.

$$D(p)y(t)=N(p)u(t)+w(t) \quad (10)$$

$$D_m(p)y_m(t)=N_m(p)r_m(t) \quad (11)$$

$$w(t)=Cadj(Ep-A)d(t)+D(p)d_0(t) \quad (12)$$

$$N(p)=diag(p^{\sigma_i})N_r+\tilde{N}(p) \quad (i=1,2,..l) \quad (13)$$

$$N_m(p)=diag(p^{\sigma_{mi}})N_{m_r}+\tilde{N}_m(p) \quad (i=1,2,...l) \quad (14)$$

where $\partial_{r_i}(\tilde{N}(p))<\sigma_i$, $\partial_{r_i}(\tilde{N}_m(p))<\sigma_{m_i}$, $N_r$ assumed to be regular. The disturbances $d(t)$, $d_0(t)$ are satisfy the following conditions

$$D_d(p)d(t)=0, D_d(p)d_0(t)=0$$

where $D_d(p)$ is a scalar characteristic polynomial of disturbances, $\partial D_d(p)=n_d$ (i.e. degree of $D_d(p)$) and $D_d(p)$ is monic polynomial. Thus

$$D_d(p)w(p)=0 \quad (15)$$

Choose a stable polynomial $T(p)$ which satisfy the following condition

1) Degree of $T(p)$ is $\rho, \rho \geq n_d+2r-n_m-1-\sigma_i$

2) The coefficient of maximum degree term of $T(p)$ is the same as $D(p)$. From (16), $R(p)$, $S(p)$ can be obtained

$$T(p)D_m(p)=D_d(p)D(p)R(p)+S(p) \quad (16)$$

where degree of each term as the following

$$\partial T(p)=\rho, \ \partial D_m(p)=n_m, \ \partial D_d(p)=n_d,$$

$$\partial D(p)=\deg|Ep-A|=r, \ \partial S(p)\leq n_d+r-1$$

$$\partial R(p)=\rho+n_m-n_d-r$$

from (7), (10), (11), (16), the following form is obtained

$$T(p)D_m(p)e(t)=D_d(p)R(p)N(p) - Q(p)N_r u(t) + S(p)y(t) - T(p)N_m(p)r_m(t) \quad (17)$$

therefore from $e(t)=0$, the next control law $u(t)$ is obtained as form (18)

$$u(t)=-N_r^{-1}Q(p)^{-1}\{D_d(p)R(p)N(p) - Q(p)N_r(p)\}u(t) - N_r^{-1}Q(p)S(p)y(t) + N_r^{-1}Q(p)^{-1}T(p)N_m(p)r_m(t) \quad (18)$$

where

$$Q(p)=diag(p^{\rho+n_m-r+\sigma_i})+\tilde{Q}(p) \quad (i=1,2,...l) \quad (19)$$

is a selected polynomial matrix, of which $|Q(p)|$ is stable, $\partial_{r_i}(Q(p))=\rho+n_m-r+\sigma_i$, $\Gamma_r(Q(p))=I$.

For using no derivatives of signals in control input $u(t)$, next constraints of polynomial degree must be satisfied.

$$n_m-\sigma_{m_i}\geq r-\sigma_i, \quad (i=1,2,....l),$$

$$\rho \geq n_d+2r-n_m-1-\sigma_i, \quad (i=1,2,....l)$$

Therefor $e(t)$ satisfies (20) and converges to zero.

$$T(p)D_m(p)e(t)=0 \quad (20)$$

$$e(t)\to 0, (t\to\infty) \quad (21)$$

Control input u(t) can be described by using internal states as following

$$u(t)=-H_1\xi_1(t)-E_2y(t)-H_2\xi_2(t)+E_3r_m(t)+H_3\xi_3(t) \quad (22)$$

where

$$H_1(pI-F_1)^{-1}G_1=N_r^{-1}Q(p)^{-1}\{D_d(p)R(p)N(p) - Q(p)N_r\} \quad (23)$$

$$E_2+H_2(pI-F_2)^{-1}G_2=N_r^{-1}Q(p)^{-1}S(p) \quad (24)$$

$$E_3 + H_3(pI - F_3)^{-1}G_3 = N_r^{-1}Q(p)^{-1}T(p)N_m(p) \tag{25}$$

since the right side of form (23) is strictly proper and that of (24), (25) are proper, the realization of right side of above form is given as the following forms

$$\dot{\xi}_1(t) = F_1\xi_1(t) + G_1 u(t) \tag{26}$$

$$\dot{\xi}_2(t) = F_2\xi_2(t) + G_2 y(t) \tag{27}$$

$$\dot{\xi}_3(t) = F_3\xi_3(t) + G_3 r_m(t) \tag{28}$$

Multiplying $I = Q(p)N_r N_r^{-1}Q(p)^{-1}$ on the left and right side of form (17),the following form is obtained

$$e(t) = \frac{Q(p)N_r}{T(p)D_m(p)}\{u(t) + H_1\xi_1(t) + E_2 y(t) + H_2\xi_2(t) - E_3 r_m(t) - H_3\xi_3(t)\}$$

If the internal states of control system are bounded, the design of MFCS for descriptor system can be realized.

## 4. THE PROOF OF BOUNDEDNESS OF INTERNAL STATES AND THE CONVERGENCE OF OUTPUT ERROR

In the control system , external signals are $r_m(t)$ , $d(t)$ and $d_0(t)$. It is clear that $r_m(t)$ is bounded. The $d(t)$ $d_0(t)$ are all bounded because form (15) are hold in the limit times , even though characteristic polynomial of disturbances have the roots in the right half plan. Remove the $u(t)$ from (1),(18),(26)~(28) forms, then the representation of an overall system can be obtained as following

$$\frac{d}{dt}\begin{bmatrix} E & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} x(t) \\ \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 \\ -G_1E_2H_3 & F_1 - G_1H_1 & -G_1H_1 \\ G_2C & 0 & F_2 \end{bmatrix}\begin{bmatrix} x(t) \\ \xi_1(t) \\ \xi_2(t) \end{bmatrix} + \begin{bmatrix} BH_3 \\ G_1H_3 \\ 0 \end{bmatrix}\xi_3(t) + \begin{bmatrix} BE_3 \\ G_1E_3 \\ 0 \end{bmatrix} r_m(t) + \begin{bmatrix} d(t) - BE_2d_0(t) \\ -G_1E_2d_0(t) \\ G_2d_0(t) \end{bmatrix} \tag{29}$$

$$\dot{\xi}_3(t) = F_3\xi_3(t) + G_3 r_m(t) \tag{30}$$

$$u_m(t) = E_3 r_m(t) + H_3\xi_3(t) \tag{31}$$

$$y = \begin{bmatrix} C & 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ \xi_1(t) \\ \xi_2(t) \end{bmatrix} + d_0(t) \tag{32}$$

in the form (30), the $\xi_3(t)$ are bounded because $|Ip - F_3| = |Q(p)|$ is stable polynomial and $r_m(t)$ are reference input. Let $z(t)$, $\tilde{E}$ are as following respectively

$$z^T(t) = [x(t), \ \xi_1(t), \ \xi_2(t)]^T \tag{33}$$

$$\tilde{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{34}$$

with the consideration that $r_m(t)$ and $\xi_3(t)$ are bounded, the necessary parts to the easy proof of boundedness are arranged as the followings

$$\tilde{E}\ \dot{z}(t) = A_s z(t) + d_s(t) \tag{35}$$

$$y = C_s z(t) + d_0(t) \tag{36}$$

where the contents of $A_s, C_s, d_s(t)$ are clear from (29),(32). The boundedness of internal states is turned out to be that $z(t)$ is bounded. This is then changed into the proof that $|\tilde{E}p - A_s|$ is stable polynomial. For the calculation of $|\tilde{E}p - A_s|$, the following theorem 1 is introduced.

Theorem 1 [5)]

Considering the following descriptor system

$$E\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

this system is assumed both controllable and observable, then the transmission poles and transmission zeros of singular minimal multivariable system are the same as the system poles and invariant zeros of it respectively, that is,

$$\alpha|\ D_R(p)\ | = |Ep - A|$$

$$\det\prod(p) = \beta\det[\ \tilde{N}_R(p) + (D + \tilde{G}_2(p))\tilde{D}_R(p)\ ] = \beta\det N_R(p)$$

where, $G(p) = C(Ep - A)^{-1}B = \tilde{G}_1(p) + D + \tilde{G}_2(p)$
$= \tilde{N}_R(p)\tilde{D}_R^{-1}(p) + D + \tilde{G}_2(p)$
$= [\ \tilde{N}_R(p) + (D + \tilde{G}_2(p))\tilde{D}_R(p)\ ]\tilde{D}_R^{-1}(p)$
$= N_R(p)D_R^{-1}(p)$

$\tilde{G}_1(p)$ is strictly proper matrix and $\tilde{N}_R(p)$, $\tilde{D}_R(p)$ are polynomial matrices which are right coprime factorization of $\tilde{G}_1(p)$, $D$ is constant matrices, $\tilde{G}_2(p)$ is polynomial matrix, $N_R(p)$, $D_R(p)$ are polynomial matrix of right coprime factorization of $G(p)$, $\prod(p)$ is the system matrix, $\alpha, \beta$ are constant.

With the theorem 1, the characteristic polynomial of form (35) is calculated in the next equation

$$\left|\widetilde{E}p - A_s\right| = T(p)^l D^l{}_m(p) |Q(p)| \frac{|N(p)||N_r^{-1}|}{D(p)^{l-1}}$$
$$= \alpha \ |N_r^{-1}| \ T(p)^l \ D^l{}_m(p) \ |Q(p)| \ |N_R(p)|$$

it is clear that $A_s$ is a stable matrix because $T(p)$, $D_m(p)$, $|Q(p)|$, $|N_R(p)|$ are stable polynomials.

In general, the above results are summarized in a next theorem.

Theorem 2

With controlled object (1) and reference model (2), all the internal state are bounded and output error $e(t) = y(t) - y_m(t)$ is asymptotically converge to zero in the design of model following control for a linear descriptor system, if the following conditions hold

(1) Both the object and the model are controllable and observable.

(2) $\det(Ep - A) \neq 0 \quad \forall p \in C$, p finite

$\operatorname{rank} E = \operatorname{degree} \det(Ep - A)$

where $C$ is complex set

(3) Zero points of $C(Ep - A)^{-1}B$ are stable.

## 5. NUMERICAL SIMULATIONS

A example is given as (37), (38). We show a result of simulation in **Fig. 1**.

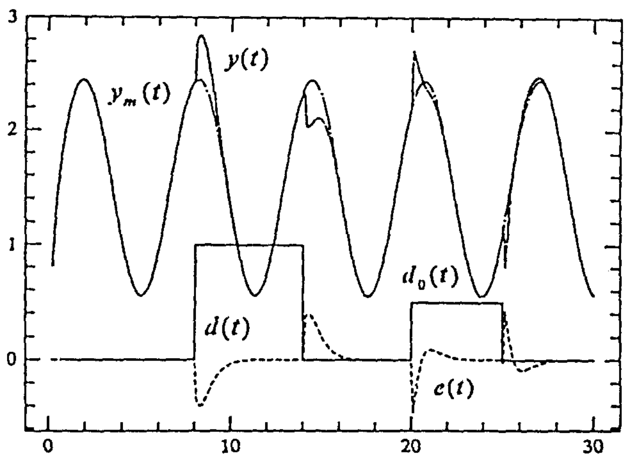


**Fig.1** System responses of descriptor system

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ d(t) \end{bmatrix} \quad (37.a)$$

$$y(t) = \begin{bmatrix} 5 & 1 \end{bmatrix} x(t) + d_0(t) \quad (37.b)$$

reference model is given in (38)

$$\dot{x}_m(t) = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x_m(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(t) \quad (38.a)$$

$$y_m(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} x_m(t) \quad (38.b)$$

$$r_m(t) = 3\sin t + 4.5 \quad (38.c)$$

In this example, $d(t)$, $d_0(t)$ are step disturbances. $T(p) = 4p + 16$, $Q(p) = (p+3)^3$. System responses are shown in **Fig.1**. It can be concluded that output signal follows the reference even disturbance existed in system.

## 6. CONCLUSION

A method of MFCS for descriptor system is proposed and the effectiveness of this method has been verified by the numerical simulation. Because the scope of description with descriptor system is wider than the traditional state-space system does, the method proposed in this paper extends the application scope of MFCS based on state-space [4].

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