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Robust Tuning of Quadratic Criterion-based Iterative Learning Control for Linear Batch System

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Abstract We propose a robust tuning method of the quadratic criterion based iterative learning control(Q-ILC) algorithm for discrete-time linear batch system. First, we establish the frequency domain representation for batch systems. Next, a robust convergence condition is derived in the frequency domain. Based on this condition, we propose to optimize the weighting matrices such that the upper bound of the robustness measure is minimized. Through numerical simulation, it is shown that the designed learning filter restores robustness under significant model uncertainty.

Keywords Iterative Learning Control, Batch Process Control, Robust Control, Discrete Fourier Transform

1. Introduction

Since first formalized by Arimoto *et al.*(1984), iterative learning control(ILC) has been settled as a new control paradigm for repetitive systems such as robot manipulators or rotating machines. Recently, application of ILC starts to extend to chemical batch processes, too. Although ILC has many attractive mathematical features such as asymptotically zero-error tracking and perfect rejection of repetitive disturbances under imprecise knowledge of process model, the existing ILC algorithms still have some shortcomings which prevent them from being widely applied in practice. They are highly sensitive to high frequency output disturbances and not suitable to nonsquare MIMO systems, and can not handle constraints on input and output variables. As a way to overcome the above shortcomings, Lee *et al.*(1996) have proposed the so-called quadratic criterion based iterative learning control(Q-ILC) algorithm. It is a model-based algorithm and finds input update by minimizing a one-step ahead quadratic cost which includes input change as well as tracking error penalty terms. The weightintg matrices for the penalty terms are used to tune Q-ILC.

The objective of this paper is placed on developing formal procedures to tune the weighting matrices in the frequency domain. For this purpose, we first establish the frequency domain representation of a linear time-varying batch system. Based on this relationship, we propose a robust convergence condition and a design procedure for the weighting matrices which minimizes the upper bound for the robustness measure.

In section 2 and 3, we briefly review the Q-ILC algorithm and discuss the properties relevant to our study. In section 4, the robust condition is derived and the tuning method based on this condition is presented. Numerical illustration is given in section 5 and finally, conclusion is drawn.

2. Q-ILC Algorithm

2.1 Process Model and ILC Problem

Consider an discrete-time time-varying linear batch process defined over $T = \{1, 2, \ldots, N\}$. Since the input and output sequences over T form finite dimensional vectors, we can describe the input-output relationship by the following linear static model composed of finite pulse response coefficients.

$$\mathbf{y}_k = \mathbf{G}\mathbf{u}_k + \mathbf{d}_k \tag{1}$$

where

$$\mathbf{y} = \left[y^T(1)\ y^T(2) \ldots y^T(N)\right] \tag{2}$$

d and **u** are defined similarly, and

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_{1,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{g}_{2,1} & \mathbf{g}_{2,2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{g}_{N,1} & \mathbf{g}_{N,2} & \cdots & \mathbf{g}_{N,N} \end{bmatrix} \tag{3}$$

$$\mathbf{g} = \begin{bmatrix} g^{11} & \cdots & g^{1n_u} \\ \vdots & \vdots & \vdots \\ g^{n_y 1} & \cdots & g^{n_y n_u} \end{bmatrix} \tag{4}$$

In the above, the subscript $_k$ denotes the k^{th} batch: u, y, and d represent the input, output, and output disturbance, respectively; $g^{ij}_{t,\tau}$ denotes the pulse response coefficient of the i^{th} output at time t for the j^{th} input at time τ. For time-variant systems, $\mathbf{g}_{t,\tau}$ is equal to $\mathbf{g}_{t-\tau}$. We call the above representation a static gain model.

Under the condition that the disturbance vector $\mathbf{d}_k$ is same at every batch, the output transition from the k^{th} to the $(k+1)^{th}$ batches can be written as follows:

$$\mathbf{y}_{k+1} = \mathbf{y}_k + \mathbf{G}\left(\mathbf{u}_{k+1} - \mathbf{u}_k\right) \tag{5}$$

If we denote $\mathbf{y}_d$ as the desired reference output trajectory vector and define

$$\begin{aligned} \mathbf{e}_k &= \mathbf{y}_d - \mathbf{y}_k \\ \Delta \mathbf{u}_{k+1} &= \mathbf{u}_{k+1} - \mathbf{u}_k \end{aligned} \tag{6}$$

(5) can be rearranged in terms of the tracking error and input change as follows:

$$\mathbf{e}_{k+1} = \mathbf{e}_k - \mathbf{G}\Delta \mathbf{u}_{k+1} \tag{7}$$

Under the above model description, in order to accomodate nonsquare batch processes as well, the following minimizing objective is considered as the ILC objective in this study.

$$\|\mathbf{e}_k\| = \min_{\mathbf{u}} \|\mathbf{e}\| \text{ as } k \to \infty \tag{8}$$

Here, $\|\cdot\|$ represents an appropriately defined norm.

2.2 Q-ILC Algorithm

As a way to reduce noise sensitivity in parallel with achieving the objective (8), we consider the following quadratic subproblem which contains a penelty term on input change.

$$\min_{\Delta \mathbf{u}_{k+1}} \left\{ J_k = \frac{1}{2} \left[\mathbf{e}_{k+1}^T \mathbf{Q} \mathbf{e}_{k+1} + \Delta \mathbf{u}_{k+1}^T \mathbf{R} \Delta \mathbf{u}_{k+1} | \mathcal{I}_k \right] \right\} \tag{9}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are symmetric positive definite matrices and $\mathcal{I}_k$ denotes all the information available after the k^{th} batch run. The cost function has a penalty term on the input change between two adjacent batches in order to suppress excessive input movements caused by high frequency output disturbances and large errors during the initial few runs. This penalty term will, however, vanish as the input trajectory converges, and therefore not cause output offset in the limit as in the algorithms by Tao *et al.*(1994) or Sogo and Adachi(1994).

We can readily obtain the least squares solution by substituting (7) into the cost function (9). The result is

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \mathbf{H}^Q \mathbf{e}_k \tag{10}$$

where

$$\mathbf{H}^Q = \left(\mathbf{G}^T \mathbf{Q} \mathbf{G} + \mathbf{R}\right)^{-1} \mathbf{G}^T \mathbf{Q} \tag{11}$$

In (10), $\mathbf{u}_{k+1} = \mathbf{u}_k$ when $\mathbf{e}_k = \mathbf{0}$. This indicates there will be no output offset if the proposed ILC algorithm converges.

3. Properties of Q-ILC

3.1 Noise Sensitivity

In fact, *learning* is a procedure to find the inverse process map in an iterative way using input/output data. Because of this trait, the learning filters have differentiating property (in the sense of continuous time) and is prone to amplify high frequency components in the output. If we measure the magnitude of signals using the Euclidean norm, noise sensitivity can be quantified by the sup norm of the learning filter. The sup norm of Q-ILC can be shown to satisfy the following inequality

$$\begin{aligned} \|\mathbf{H}^Q\|_\infty &= \|\left(\mathbf{G}^T \mathbf{Q} \mathbf{G} + \mathbf{R}\right)^{-1} \mathbf{G}^T \mathbf{Q}\|_\infty \\ &\leq \frac{\sigma_{max}(\mathbf{G})\sigma_{max}(\mathbf{Q})}{\sigma_{min}(\mathbf{R})} \end{aligned} \tag{12}$$

where σ_{max} and σ_{min} denote maximum and minimum singular values of a matrix, respectively.

The above relationship implies the noise sensitivity is bounded from above irrespective of the sampling period and can be adjusted by the selection of the weighting matrices. This is a definite advantage over the other existing algorithms based on pure differentiators or model inversion, which gives infinite gain at infinite frequency.

3.2 Convergence

If we premultiply $\mathbf{G}$ on (10) and substitute (7) into the resulting equation, we obtain the evolution equation of the output error

$$\mathbf{e}_{k+1} = \left(\mathbf{I} - \mathbf{G}\mathbf{H}^Q\right) \mathbf{e}_k \tag{13}$$

and the output error converges to zero if the following condition is satisfied.

$$\left|\lambda_i \left(\mathbf{I} - \mathbf{G}\mathbf{H}^Q\right)\right| < 1 \ \ \forall i \tag{14}$$

where λ_i denotes the eigenvalue.

It has been shown[2] the above condition is always satisfied for $\mathbf{G}$ with full row rank. When $\mathbf{G}$ does not have full row rank, the system has uncontrollable modes, which implies it is impossible to enforce $\mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k$ to zero in general. Only the outputs in the reachable subspace can be enforced to zero while the outputs in the unreachable subspace remain intact. Mathematically, the convergcne property of Q-ILC can be it can be written as

$$\mathbf{e}_k^T \mathbf{U}_c^T \mathbf{Q} \mathbf{U}_c \mathbf{e}_k \ \to \ 0 \ \text{as} \ k \ \to \ \infty \tag{15}$$

where $\mathbf{U}_c$ is a projection matrix onto the image space of $\mathbf{G}$. In this way, the objective (8) can be proved to be fulfilled regardless of singularity of $\mathbf{G}$.

When there is model error, (13) is written as

$$\mathbf{e}_{k+1} = \left(\mathbf{I} - \mathbf{G}_{true}\mathbf{H}^Q\right) \mathbf{e}_k \tag{16}$$

where $\mathbf{G}_{true}$ denotes the true process, and the convergence is determined by $\mathbf{I} - \mathbf{G}_{true}\mathbf{H}^Q$. Obviously, the robust convergence depends on the choices of the weighting matrices.

4. Robust Tuning of Q-ILC

In this section, we present the robust tuning method for the weighting matrices. Since model uncertainty is very often understood in the frequency domain, we first establish the frequency domain transformation of the batch process model

4.1 Discrete Fourier Transformation

For any finite sequence $\{u(t), t = 1, 1, \cdots, N\}$, the discrete Fourier transform is defined as follows:

$$\tilde{u}(\omega_k) = \sum_{n=0}^{N-1} u(n) e^{-j\omega_k n T_s} \quad (17)$$

where

$$\omega_k = \left(\frac{2\pi}{NT_s}\right) k, \qquad k = 0, 1, \cdots, N-1 \quad (18)$$

Here, T_s is the sampling period. The above relationship can be expressed by a vector form as follows:

$$\tilde{\mathbf{u}} = \mathbf{F}_{n_u} \mathbf{u} \quad (19)$$

where

$$\tilde{\mathbf{u}}^T = [\tilde{u}^T(\omega_0)\ \tilde{u}^T(\omega_1) \cdots \tilde{u}^T(\omega_{N-1})]$$

$$\mathbf{F}_{n_u} = \begin{bmatrix} \mathbf{I}_{n_u} & \mathbf{I}_{n_u} & \cdots & \mathbf{I}_{n_u} \\ \mathbf{I}_{n_u} & e^{-j\omega_1}\mathbf{I}_{n_u} & \cdots & e^{-j\omega_{N-1}}\mathbf{I}_{n_u} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_{n_u} & e^{-j\omega_{N-1}}\mathbf{I}_{n_u} & \cdots & e^{-j\omega_{N-1}(N-1)}\mathbf{I}_{n_u} \end{bmatrix} \quad (20)$$

where $\mathbf{I}_n$ denotes an $n \times n$ identity matrix. Accordingly, the inverse transform is

$$\mathbf{u} = \mathbf{F}_{n_u}^{-1} \tilde{\mathbf{u}}, \quad \mathbf{F}_{n_u}^{-1} = \frac{1}{N^2} \mathbf{F}_{n_u}^H \quad (21)$$

where the superscript H denotes the conjugate transpose.

From (19), it is obvious that any linear relationship between N sequence vectors

$$\mathbf{y} = \mathbf{A}\mathbf{u} \quad (22)$$

can be transformed to

$$\tilde{\mathbf{y}} = \tilde{\mathbf{A}}\tilde{\mathbf{u}}, \quad \tilde{\mathbf{A}} = \mathbf{F}_{n_y} \mathbf{A} \mathbf{F}_{n_u}^{-1} \quad (23)$$

By the same reasoning, a serial connection of two time-domain static gain models can be transformed to

$$\mathbf{y} = \mathbf{B}\mathbf{A}\mathbf{u} \rightarrow \tilde{\mathbf{y}} = \tilde{\mathbf{B}}\tilde{\mathbf{A}}\tilde{\mathbf{u}} \quad (24)$$

Using the above rules, we can directly transform the time domain equations of Q-ILC into the frequency domain by preserving the functional relationships.

As mentioned in sectuion 2, the system matrix in (22) may represent time-varying as well as time-invariant dynamics. Considering that transfer functions are defined only for time-inaˑariant cases in continuous processes, the above frequency domain transformation is somewhat surprising.

4.2 Frequency Dependent Tuning of Q-ILC

One of the applications of the above transformation is frequency dependent tuning of of the weighting matrices. Using (19), the Q-ILC objective (9) can be rewritten as

$$J_k = \frac{1}{2N^2} \left\{ \tilde{\mathbf{e}}_{k+1}^T \tilde{\mathbf{Q}} \tilde{\mathbf{e}}_{k+1} + \Delta\tilde{\mathbf{u}}_{k+1}^T \tilde{\mathbf{R}} \Delta\tilde{\mathbf{u}}_{k+1} \right\} \quad (25)$$

Once transformed as above, $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ can be assumed to be block diagonal matrices without loss of generality. The k^{th} diagonal element (matrix) is associated with the frequency component at ω_{k-1}. Therefore, we can penalize signals at each frequency by assigning values to the diagonal elements. After $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ are designed, they are transformed to the time domain according to

$$\mathbf{Q} = \mathbf{F}_{n_y}^H \tilde{\mathbf{Q}} \mathbf{F}_{n_y} / N^2, \quad \mathbf{R} = \mathbf{F}_{n_u}^H \tilde{\mathbf{R}} \mathbf{F}_{n_u} / N^2 \quad (26)$$

4.3 Robust Tuning

We assume the model uncertainty is given in the unstructured output multiplicaitve form in the frequency domain. Other types of uncertainties are, however, dealt with similarly. If we let $\Delta w(\omega_k)$ with $||\tilde{\Delta}||_\infty < 1$ be the model uncertainty at ω_k, the transformed form of the real process can be represented by

$$\tilde{\mathbf{G}}_{real} = (\mathbf{I} + \tilde{\Delta}\tilde{\mathbf{W}})\tilde{\mathbf{G}} \quad (27)$$

where

$$\tilde{\mathbf{W}} = \begin{bmatrix} w(\omega_0) & 0 & \cdots & 0 \\ 0 & w(\omega_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(\omega_{N-1}) \end{bmatrix} \quad (28)$$

with $||\tilde{\Delta}||_\infty < 1$. Each element shown in (28) is an $n_y \times n_u$ matrix.

From (16) and the realtionship in (24), the robust convergence condition can be written as

$$||\mathbf{I} - \tilde{\mathbf{G}}_{\mathbf{real}} \tilde{\mathbf{H}}^Q||_\infty < 1 \quad (29)$$

where $||\mathbf{A}||_\infty = \sigma_{max}(\mathbf{A})$. Using the triangular inequality, we obtain the following relationship.

$$\begin{aligned} ||\mathbf{I} - \tilde{\mathbf{G}}_{\mathbf{real}} \tilde{\mathbf{H}}^Q||_\infty &\leq ||\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty + ||\Delta\mathbf{W}\tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty \\ &\leq ||\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty + ||\mathbf{W}\tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty \end{aligned} \quad (30)$$

where

$$\tilde{\mathbf{H}}^Q = (\tilde{\mathbf{G}}^T \tilde{\mathbf{Q}} \tilde{\mathbf{G}} + \tilde{\mathbf{R}})^{-1} \tilde{\mathbf{G}}^T \tilde{\mathbf{Q}} \quad (31)$$

From the above inequality, the robust tuning problem becomes the following minization problem.

$$\begin{aligned} &\min \left[||\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty + ||\mathbf{W}\tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q||_\infty \right] = \\ &\min \left[\sigma_{max}(\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q) + \sigma_{max}(\mathbf{W}\tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q) \right] \end{aligned} \quad (32)$$

The minimization can be done both for $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$. In practice, however, only the relative magnitude between $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ is important. Therefore, we may set $\tilde{\mathbf{Q}}$ at a constant matrix (for scaling purpose, for example) and solve the minimization with respect to only $\tilde{\mathbf{R}}$, or solve the minimization with respect to both $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ with constraints imposed on the matrices. Once optimum $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ are found, they are converted to the time domain forms according to (26).

When $\mathbf{G}$ does not have full row rank, $\tilde{\mathbf{G}}$ also loses row rank and $\sigma_{max}(\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q)$ can be shown to remain at 1. In this case, the solution becomes trivial, and more importantly, the above minimization loses its meaning. To avoid this problematic case, we need to introduce a projection matrix $\mathbf{U}_c$ as discussed in subsection 3.2 and formulate the above probelm by replacing $\tilde{\mathbf{G}}$ with $\tilde{\mathbf{U}}_c\tilde{\mathbf{G}}$.

5. Numerical Example

For numerical demonstration of the proposed robust tuning method, we consider a discrete-time batch process which is obtained by discretizing the following SISO continuous time model with the sampling period of 2 over [0,40].

$$G(s) = \frac{1}{15s^2 + 8s + 1} \tag{33}$$

We assume that the frequency dependent uncertainty weighting at ω_k is

$$w(\omega_k) = \left| 0.2 \frac{5s+1}{0.5s+1} \right|_{\omega_k} \tag{34}$$

In terms of maginitude, model error ranges from 20% at zero frequency to about 400% at high frquencies. In this example, we fix $\mathbf{Q} = \mathbf{I}$ and optimize only $\tilde{\mathbf{R}}$. $\mathbf{R}^*$ denote the solution of the optimization problem in eq. (32). For comparion of the design results, we consider the cases with $\mathbf{R} = 2\mathbf{I}$, $0.02\mathbf{I}$, and $0.005\mathbf{I}$. For each case including the optimum design,

$$L_i = \sigma_i(\mathbf{I} - \tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q) + \sigma_i(\tilde{\mathbf{W}}\tilde{\mathbf{G}}\tilde{\mathbf{H}}^Q), \quad i = 1, 2, \cdots, N \tag{35}$$

are plotted from the maximum to minimum in Fig. 1. We can observe the cases with $\mathbf{R} = 0.005\mathbf{I}, 0.02\mathbf{I}$, and $2\mathbf{I}$ violate convergence condition. But the proposed robust tuning restores the convergnce. Figure 2 shows the simulation results of the robustly tuned Q-ILC for a given $\mathbf{y}_d$. We can see the output converges to the desired trajectory as number of batches incereases.

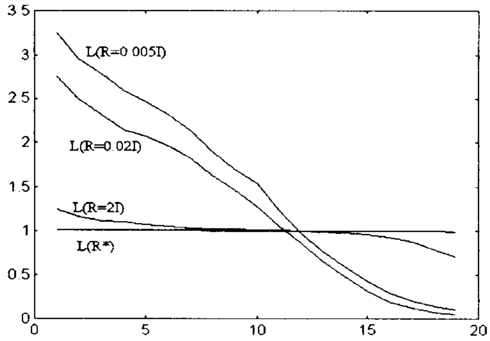

Fig. 1. The plots of $L(\mathbf{R})$ and $L(\mathbf{R}^*)$.

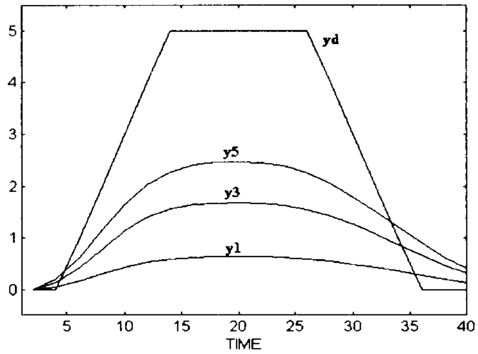

Fig. 2. Output trajectory of the Q-ILC tuned by the proposed method

5. Conclusions

We have studied tuning methods for quadratic criterion-based iterative learning control(Q-ILC) in the frequency domain. For this purpose, we first establish the frequency domain transformation using discrete Fourier transform on finite pulse sequences. Based on this transfomation, we propose a frequency dependent tuning method and then a robust tuning method of Q-ILC. The proposed robust tuning method has been evaluated through numerical simulation for a SISO linear system.

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