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Fault Diagnosis using Multiple PI observers

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Abstract Fault diagnosis problem is currently the subject of extensive research and numerous survey paper can be found. Although several works are studied on the fault detection and isolation observers and the residual generators, those are concerned with only the detection of actuator failures or sensor failures. So, the perfect detection and isolation is strongly required for practical applications. In this paper, a strategy of fault diagnosis using multiple proportional integral (PI) observers including the magnitude of actuator failures is provided. It is shown that actuator failures are detected and isolated perfectly by monitoring the integrated error between actual output and estimated output by a PI observer. Also in presence of complex actuator and sensor failures, these failures are detected and isolated by multiple PI observers.

Keywords Fault Detection, Isolation, FDI observer, PI observers, Actuator failures, Sensor failures

1. Introduction

The detection and the isolation of actuator and sensor failures have received considerable attention during the last two decade in both theoretical reasearch and practical applications[1]-[4]. Although several works are studied on fault detection and isolation (FDI) observers and residual generators, those are concerned with only the partly detection and isolation of actuator failures or sensor failures.

In the case of FDI observer, multiple observers has been used to determine the localization of faulty elements of a process, where unknown input observers are constructed as many as number of inputs[3]. But, this method guarantees only the detection, but not the isolation in the presence of complex actuator and sensor failures. So, perfect detection and isolation method is strongly required for practical applications, if possible including estimation of the fault magnitude.

In this paper, a strategy of faults diagnosis including the detection, isolation, and fault magnitude is proposed. The new method is based on intensive use of knowledge on the characteristic of proportional integral (PI) observer, which estimates and cancels the step actuator failures.

First, the actuator failures are detected and isolated perfectly by monitoring the integrated error between actual output and estimated output by using one PI observer in the presence of only actuator failures. Second, in the presence of complex actuator and sensor failures, the fault detection is easily achieved by applying the multiple PI observers which are constructed as many as number of outputs. And the failures is isolated by comparing the magnitude of estimated output error with a pre-specified magnitude of thresholds, which are related with the system's characteristics. Finally, simulation result shows that the proposed strategy is effectiveness to detect and isolate the complex actuator and sensor failures.

2. Problem formulation

We consider the following linear time-invariant system without failures

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1-a}$$
$$y(t) = Cx(t) \tag{1-b}$$

where, $x(t) \in \Re^n$ is state vector, $u(t) \in \Re^m$ is input vector, and $y(t) \in \Re^p$ is output vector. The matrices A, B, and C are of appropriate dimensions. It is assumed that (C, A) is observable.

In this paper, we consider the two types of failures ; actuator failure and sensor failure which are defined as follows:

(i) Actuator failures : Let $u(t)$ represent the desired output of the actuator when no failures are presented. Let $\bar{u}(t)$ be the actual output of the actuator. Then we have

$$\bar{u}(t) = u(t) + a(t) \tag{2}$$

where $a(t) \in \Re^m$ is actuator failure vector.

(ii) Sensor failures : We model sensor failures in the same way. Let $y(t)$ and $\bar{y}(t)$ represent the desired and the actual output respectively. Then

$$\bar{y}(t) = y(t) + s(t) \tag{3}$$

where $s(t) \in \Re^p$ is sensor failure vector.

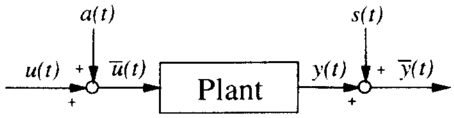

Fig. 1. System with sensor and actuator failures

On the definition of two types of failures, the system (1) is considered the following model with failures of actuators and sensors in which additive signals appear at input and output as shown in Fig. 1.

$$\dot{x}(t) = Ax(t) + Bu(t) + Ba(t) \tag{4-a}$$

$$y(t) = Cx(t) \quad (4\text{-b})$$
$$\bar{y}(t) = y(t) + s(t) \quad (4\text{-c})$$

The aim of this paper to propose a scheme for the detection and isolation of the actuator failures and the sensor failures perfectly using PI observers.

3. Structure of PI observer

In this section, we consider a proportional integral (PI) observer which have additional degree of freedom in the given system (1). A PI observer is described as

$$\dot{\hat{x}}(t) = (A-KC)\hat{x}(t)+Ky(t)+Bu(t)+B\omega(t) \quad (5\text{-a})$$
$$\dot{\omega}(t) = H(y(t)-C\hat{x}(t)) \quad (5\text{-b})$$

where $\hat{x}(t) \in \Re^n$ is estimated state vector and $\omega(t) \in \Re^p$, and K and H are the proportional gain and the integral gain, respectively.

The system (5) is defined as a PI observer for the system (1) if

$$\lim_{t\to\infty} e(t) = 0, \quad \forall \hat{x}(0_-), x(0_-)$$
$$\lim_{t\to\infty} \omega(t) = 0, \quad \forall \omega(0_-)$$

where $e(t) = x(t) - \hat{x}(t)$ represents the observer error.

Lemma 1[5] **:** The system (5) is a full-order PI observer for the system (1) if and only if all the eigenvalues of the matrix

$$R = \begin{bmatrix} A-KC & B \\ -HC & 0 \end{bmatrix}$$

have negative real parts.

Lemma 2[5] **:** The existence condition of the gains K and H satisfying Lemma 1 is given as

$$\text{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + m$$

Remark : Lemma 2 involves the condition of $p \geq m$, that is, the number of sensors must not be less than the number of actuators.

4. Fault Detection and Isolation by PI Observer

4.1. Detection and isolation of only actuator failures

We have the dynamics of system with only actuator failures as

$$\dot{x}(t) = Ax(t) + Bu(t) + Ba(t) \quad (6\text{-a})$$
$$y(t) = Cx(t) \quad (6\text{-b})$$

Defining the error

$$e(t) = x(t) - \hat{x}(t)$$

then, it can be easily shown that

$$\dot{e}(t) = (A - KC)e(t) + B(a(t) - \omega(t)) \quad (7)$$

and

$$\dot{\omega}(t) = HCe(t)$$

Let $\zeta(t)$ be defined as

$$\zeta(t) = a(t) - \omega(t) \quad (8)$$

then, under the assumption of step actuator failures ($\dot{a}(t) = 0$),

$$\dot{\zeta}(t) = -\dot{\omega}(t) = -HCe(t) \quad (9)$$

From (7) and (9), an augmented system is constructed as

$$\begin{bmatrix} \dot{e}(t) \\ \dot{\zeta}(t) \end{bmatrix} = \begin{bmatrix} A-KC & B \\ -HC & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ \zeta(t) \end{bmatrix} \quad (10)$$

And, if Lemma 1 is satisfied, then $e(t)$ and $\zeta(t) \to 0$ $(t \to \infty)$. Also, from (8) the actuator failures can be estimated as

$$\hat{a}(t) = \omega(t) \quad (11)$$

Thus the magnitude of actuator failure is estimated and the actuator failure is perfectly isolated. But, note that it is impossible to estimate the actuator failures in the presence of complex actuator and sensor failures.

4.2. Detection and isolation of complex actuator and sensor failures

As the mentioned in the above section, if the system has only actuator failures, the states of system and the actuator failures can be estimated by only one PI observer. But, if the system has the sensor failures, it is impossible because of existing the observation's errors.

In this case, we consider a PI observer which can not be effected by specified sensor failure. This means that we construct the PI observer just using partly output information which eliminating the specified component of outputs. So, it is possible that the PI observer estimate the states of system and actuator failures inspire of sensor failure. First of all, we define the following variables :

${}^k\bar{y}(t)$: kth component of $\bar{y}(t)$

$\bar{y}^k(t)$: $(p-1)$ vector obtained from $\bar{y}(t)$ by deleting ${}^k\bar{y}(t)$

$$\hat{a}_k(t) = \omega_k(t) = \begin{bmatrix} \omega_{k1}(t) \\ \vdots \\ \omega_{km}(t) \end{bmatrix}$$

: estimated actuator failures by kth PI observer

$$e_k(t) = \bar{y}^k(t) - \hat{\bar{y}}^k(t) = \bar{y}^k(t) - C^k\hat{x}_k(t) = \begin{bmatrix} e_{k1} \\ \vdots \\ e_{kp} \end{bmatrix} \text{ (where } e_{kk} \text{ is not included)}$$

: output estimated error of kth PI observer

where C^k is $(p-1) \times n$ matrix obtained from C by deleting kth row of C and $\hat{x}_k(t)$ is estimated states of system by kth PI observer.

For estimating the states of system with complex actuator and sensor failures, we first consider the ith sensor failure. Then, the real output of system $\bar{y}^i(t)$ which eliminating the ith sensor failures is considered as follows :

$$\bar{y}^i(t) = C^i x(t) + s^i(t) = \begin{bmatrix} \bar{y}_1(t) \\ \vdots \\ \bar{y}_{i-1}(t) \\ \bar{y}_{i+1}(t) \\ \vdots \\ \bar{y}_p(t) \end{bmatrix}$$

Using the output and all of inputs, we construct a PI observer as

$$\dot{\hat{x}}_i(t) = (A - K_i C^i)\hat{x}_i(t) + K_i \bar{y}^i(t) + Bu(t) + B\omega_i(t) \quad \text{(12-a)}$$

$$\dot{\omega}_i(t) = H_i(\bar{y}^i(t) - C^i \hat{x}_i(t)) \quad \text{(12-b)}$$

where K_i and H_i are the ith PI observer's gains.

We note that the PI observer is not effected by the ith sensor failure, *i.e.*, the states of system can be estimated by PI observer (12) perfectly inspire of ith sensor failure. Hence, for constructing the faults detection and isolation system, p number of PI observers (multiple PI observers) are necessary. And the following theorem must hold.

Theorem 1 : The existence condition of multiple PI observer for constructing the fault detection and isolation system is given as

$$\text{rank} \begin{bmatrix} A & B \\ C^i & 0 \end{bmatrix} = n + m, \quad i = 1, \cdots, p$$

The fault diagnosis system based on multiple PI observer in Fig. 2.

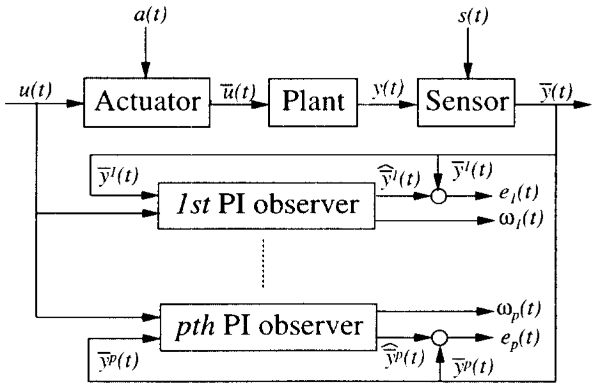

Fig. 2. Configuration of fault diagnosis system based on multiple PI observer

Now, we consider the algorithms for faults detection and isolation. We assume that the condition of Theorem 1 is satisfied and the p number of PI observers are designed respectively. If the ith sensor is failed, then the real states of system can be estimated by only ith PI observer. The other PI observers can not estimate the states of system and there exists the observation error. From this results, if all of ith output estimated error is great than pre-specified threshold except for ith PI observer, then we can conclude that the ith sensor is failed. Therefore, the detection and isolation algorithm of sensor failures is summarized as follows :

$$r_{si} = \prod_{k=1(k \neq i)}^{k=p} s_{ki} \quad (13)$$

where

$$s_{ki} = \begin{cases} 1, & \text{if } |e_{ki}| \geq Th_{si} \\ 0, & \text{else} \end{cases}$$

and Th_{si} denotes the threshold for detecting the ith sensor failures. From (13), if $r_{si} = 1$ then the ith sensor is failed, else ith sensor is not failed.

In the similar way, the actuator failures is detected and isolated as follows :

$$r_{ai} = \prod_{k=1}^{k=p} a_{ki} \quad (14)$$

where

$$a_{ki} = \begin{cases} 1, & \text{if } |\omega_{ki}| \geq Th_{ai} \\ 0, & \text{else} \end{cases}$$

and Th_{ai} denotes the threshold for detecting the ith actuator failures. From (14), if $r_{ai} = 1$ then ith actuator is failed, else ith actuator is not failed.

5. Example

Consider the following linear time-invariant system.

$$A = \begin{bmatrix} 0 & 1.0 & 0.3096 & 0 \\ 0 & -1.0541 & 1.1395 & -14.2955 \\ 0 & -0.0238 & -0.2981 & -0.7717 \\ 0.1338 & 0.2957 & -0.9550 & -0.0981 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 \\ -7.8013 & -1.1160 \\ 1.8013 & -0.6584 \\ 0.1454 & 0.0169 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It can be easily checked that the system $(C^i, A, B), (i = 1, 2, 3)$ satisfy the conditions of Theorem 1. Therefore, the fault detection and isolation system by multiple PI observer can be constructed. Here, each of the PI observers are designed with same eigenvalues

$$\lambda_i = \{-1.1,\ -1.2,\ -1.3,\ -1.4,\ -5,\ -10\}$$

by pole assignment method in MATLAB software, where the input is

$$u(t) = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix}$$

and the sampling time is 0.1 [Sec].

The magnitude of actuator failures is assumed to be 0.05 steps at the 2 [Sec] and the sensor failures is 0.5 at 4 [Sec] respectively. For detecting the failures, we set the thresholds as

$$Th_a = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad Th_s = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Simulation results are shown in Figures 3 to 11. Figures 3 and 4 represent respectively the detection of actuator failures, and Figures 5 to 7 show the detection of sensor failures. When the actuator and the sensor are failed, Figures 8 to 11 show the detection and the isolation of the actuator and sensor failures.

(1) The cases of actuator failures

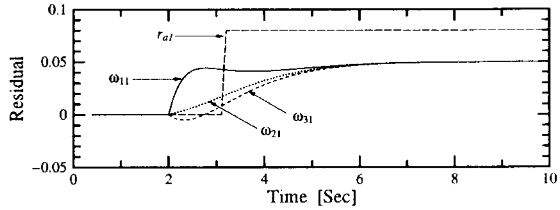

Fig. 3. Failure of actuator 1 at $t = 2$[Sec]

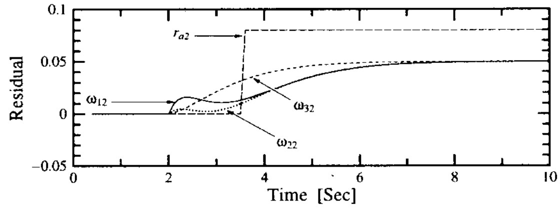

Fig. 4. Failure of actuator 2 at $t = 2$ [Sec]

(2) The cases of sensor failures

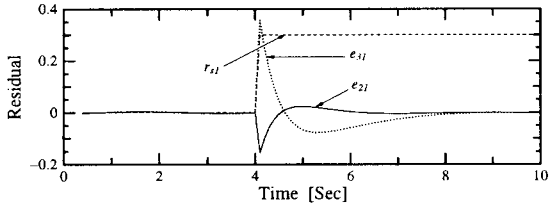

Fig. 5. Failure of sensor 1 at $t = 4$ [Sec]

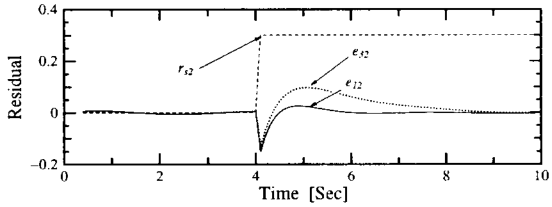

Fig. 6. Failure of sensor 2 at $t = 4$ [Sec]

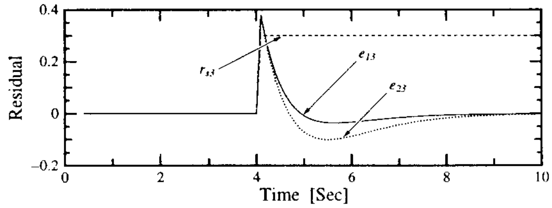

Fig. 7. Failure of sensor 3 at $t = 4$ [Sec]

(3) The cases of actuator and sensor failures

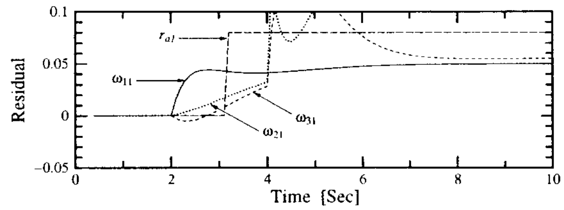

Fig. 8. Failures of actuator 1 and sensor 1 at $t = 2$ and 4 [Sec]

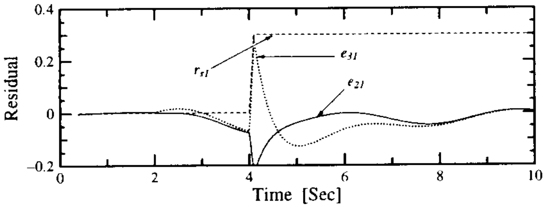

Fig. 9. Failures of actuator 1 and sensor 1 at $t = 2$ and 4 [Sec]

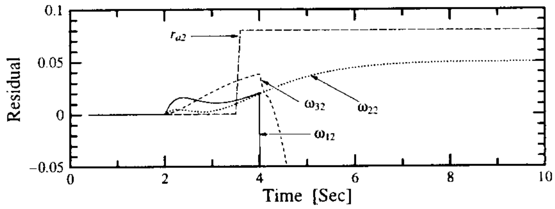

Fig. 10. Failures of actuator 2 and sensor 2 at $t = 2$ and 4 [Sec]

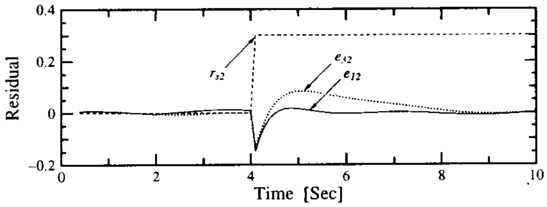

Fig. 11. Failures of actuator 2 and sensor 2 at $t = 2$ and 4 [Sec]

In these Figures, r_a and r_s which denote the detection of actuator failures and sensor failures are scaled down to 0.08 and 0.3 respectively. By detecting r_a and r_s signals, all of failures including the complex actuator and sensor are detected and isolated perfectly.

6. Conclusions

In this paper, we have proposed a fault detection and isolation method by multiple PI observer techniques. Based on this method, we can detect and isolate the actuator and the sensor failures perfectly in presence of complex failures.

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