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FUZZY POLYNOMIAL NEURAL NETWORK MODEL AND ITS APPLICATION TO WASTEWATER TREATMENT SYSTEM

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Abstracts In this paper, a fuzzy PNN algorithm is proposed to estimate the structure and parameters of fuzzy model, using the PNN based on GMDH algorithm. New algorithm uses PNN algorithm and fuzzy reasoning in order to identify the premise structure and parameter of fuzzy implications rules, and the leastsquare method in order to identify the optimal consequence parameters. Both time series data for gas furnace and data for wastewater treatment process are used for the purpose of evaluating the performance of the fuzzy PNN. The results show that the proposed technique can produce the fuzzy model with higher accuracy than other works achieved previously.

Keywords Fuzzy, Neural Network, PNN, GMDH, Activated Sludge

1. INTRODUCTION

Recently, many researchers have had much interest in various methods for system modeling. Among them, mathematical modeling method such as regression technique was widely used to identify and predict the linear systems based on input-output data. But mathematical models to express dynamic analysis of nonlinear, complex and real system have lots of problems in the selection of the variables to construct the model among many input-output variables and of the model structure.

In general, higher-order equations require too many data for estimation of system parameters in mathematical models. To solve the problems, the PNN based on GMDH was first introduced by A.G. Ivakhnenko. The GMDH has been used to synthesize the PNN - the building blocks of modeling methodology. This approximation technique based on the perceptron principle with a neural network-type architecture is used to model, identify and predict the input-output relationship of a complex process system.

Fuzzy modeling is another method in order to describe the static and dynamics of nonlinear system. Fuzzy modeling divides entire spaces into several spaces and model the spaces. As known, when this method multiplies the fitness, according to the inputs, it has excellent performances for plants with intense non-linearlity.

In this paper, the fuzzy PNN is proposed that combines fuzzy inference with PNN algorithm. Both time series data for gas furnace and data for activated sludge process are used to evaluate the performance of the fuzzy PNN algorithm. Simulations show that this method can produce models with higher accuracy and feasibility than other works presented before.

2. PNN PROCESS MODELING

The highly complex nonlinear systems have limited the success of process modeling from measured data. Recently, polynomial neural networks [PNNs] [1], as well as fuzzy systems[2-6] have emerged as a more attractive alternative to physical models and empirical statistical methods. The PNNs, however, are still in the first stage of their research on the complex process modeling. The inherent property of the PNN is to model complex systems using simple building blocks.

The PNN strategy are employed to address the complex process modeling via GMDH [1]. Here, the PNN methodology is implemented on the random input-output sets of measured training and testing data obtained from the system. An input matrix, consisted of measured (first layer) or calculated (second layer and more) input data, is presented to the each layer of

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a PNN, and the output at each node of each layer is calculated by combining the inputs of each layer as pairs (or singles or triplets) and filtering this, using the least square method. The calculated output vector is then compared to the measured output data vector, and the mean squared difference between these two vectors determines the error of the complex process system. Then, an optimal model of the system is selected in the viewpoint of this error.

2.1. The Structure of PNN

The PNN based on the perceptron principle with a neural network-type architecture is used to model the input - output relationship of a complex process system. At each layer, new generations of complex equations are constructed from simple forms. Survival of the fittest principle (appropriate thresholds) determines the equations that are passed on to the next layer and those that are discarded, that is, only the best combination of input properties (new variables) are allowed to pass through to the next layer. The model obtained after each layer is progressively more complex than the model at the preceding layers. To avoid an overfit, the data sample is divided into a) the training set, which is used for the generation of several computing alternative models and b) the testing set, which is used to test the accuracy of the models generated and for the selection of the best models at each layer.

The number of layers is increased until the newer models begin to have poorer powers of predictability than their predecessors. This indicates overspecialization of the system. The final model is an estimate of each performance function as a function of two or three variables, which are themselves functions of two more variables, and so on. The network result is a very sophisticated model from a very limited data set.

In a PNN technique, a simple form function is usually combined at each node of a polynomial neural network to obtain a more complex form. This function as an appro-ximation represents the current model for the given training and testing sets of input-output data. This approximation is written as a second degree regression equation like eqn.(1) in a case of combining two inputs at each node.

$$y = A + BX_i + CX_j + DX_i^2 + EX_iX_j \qquad (1)$$

where y is the output and Xi and Xj are the two inputs.

The outputs obtained from each of these nodes are then combined to obtain a higher-degree polynomial so that the best model may be achieved which represents the input-output data. The degree of the polynomial increases by the number of selected inputs at each layer. Here, a complex ploynomial is obtained from the above process, called a Ivakhnenko polynomial. This function uaually has a form such as;

$$y = A + \sum_{i=1}^{n} B_i Xi + \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_iX_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}X_iX_jX_k + \cdots \qquad (2)$$

where X_i, X_j and X_k is nodal input variables , and y is the output of an individual neuron(node). A, B_i, C_{ij} and D_{ijk} are the coefficients of the Ivakhnenko polynomial.

2.2. The Algorithm of PNN

The design of a variable control strategy requires the availability of a reasonable accurate model of the process system. Such models had not been available for the complex process model due to the process dimensionality and the complexity of the interacting physical phenomena. In this paper, the input-output data for the gas furnace and wastewater treatment processes are becoming available and open-up the possibility for robust modeling tools that adopt a modeling paradigm that is based on a **PNN**.

The **PNN** synthesis activities have focused over the past years on the development of self-organizing, minimal polynomial networks with good generation capabilities. We considered nine types of model equations as shown in TABLE 1.

TABLE 1. The node equations cosidered for polynomial neuralnetwork synthesis

		Number of Input		
		1	2	3
degree	1	linear	bilinear	trilinear
	2	quadratic	biquadratic	triquadratic
	3	cubic	bicubic	tricubic

- trilinear = $x_0 + w_1x_1 + w_2x_2 + w_3x_3$
- triquadratic = trilinear $+ w_4x_1x_2 + w_5x_1x_3 + w_6x_2x_3 + w_7x_1^2 + w_8x_2^2 + w_9x_3^2$
- tricubic = triquadratic $+ w_{10}x_1x_2x_3 + w_{11}x_1^3 + x_{12}x_2^3 + w_{13}x_3^3$

Searching for the optimal configuration in the space of all possible polynomial neural networks is untractable and requires a set of heuristics.

The PNN leads to self-organizing heuristic hierarchical models of high degree with automatic elimination of undesirable variable interactions. In contrast with the conventional regression technique, this scheme has several distinct advantages. A smaller data set is required, the computational time and resources are reduced and the final structure of the PNN does not need to be specified. In addtion, high-order regression often leads to a severely ill-conditioned system of equations. However, the PNN avoids this by constantly eliminating variables and variable interactions at each layer, and helps to reduce linear dependencies. Therefore, complex systems can be modeled without specific knowledge of the system or massive amounts of data.

3. STRUCTURE AND ALGORITHM OF FPNN

In this section, the differences between proposed fuzzy PNN and conventional PNN are considered. While conventional PNN obtains the output using the second order equation of two variables, fuzzy PNN extract the output from each node of conventional PNN, using fuzzy models with constants or first-order linear equations in the consequence. Each node is operated as a small fuzzy system.

The consequence of each node is expressed by constants in fuzzy PNN using the simplified inference and by first-order linear equation in Fuzzy PNN using linear inference. If premise input variables and parameters are given, the optimal consequence parameters which minimizes PI can be determined. PI is a criterion which means differences between output data of original system and output data of fuzzy PNN model. When input-output data set are given, the consequence parameters can be determined by least-square method in a similar way of fuzzy systems[7].

3.1. Fuzzy GMDH by simplified inference

The consequence part of the simplified inferenceis expressed by constants is given as eqn.(3).

$$R^i: \text{If } x_1 \text{ is } A_{i1}, \cdots, x_k \text{ is } A_{ik}, \text{Then } y = a_i$$

$$y^* = \frac{\sum_{i=1}^{n} \mu_i a_i}{\sum_{i=1}^{n} \mu_i} = \sum_{i=1}^{n} \hat{\mu}_i a_i \tag{3}$$

where R^i is the i-th fuzzy rule, x_j is input variables, A_{ij} is a membership function of fuzzy sets, a_i is a constant, n is the number of the fuzzy rules, y^* is the infered value, μ_i is the premise fitness of R^i and $\hat{\mu}_i$ is the normalized premise fitness of R^i. If premise input variables and parameters are given in consequence parameter identification, the optimal consequence parameters which minimizes PI can be determined. PI is a perfomance index which means differences between output data of original system and model. It can be defined by eqn.(4).

$$PI = \frac{1}{m}\sum_{k=1}^{m}\{ y(k) - y^o(k) \}^2 \tag{4}$$

where y^o is output of fuzzy model, k is number of input variables, and m is total number of data. In fuzzy model of Type 1, the consequence parameters can be estimated by least-square method as eqn.(5).

$$\hat{a} = (X^T X)^{-1} X^T Y \tag{5}$$

3.2. Fuzzy GMDH by linear inference

The consequence is expressed by first-order linear equation, and uses the linear (or complex) inference given by eqn.(6).

$$R^i: \text{If } x_1 \text{ is } A_{i1}, \cdots, x_k \text{ is } A_{ik}, \text{ Then } y = f_i(x_1, \cdots x_k)$$

$$f_i(x_1, \cdots, x_k) = a_{i0} + a_{i1}x_1 + \cdots + a_{ik}x_k$$

$$y^* = \frac{\sum_{i=1}^{n} \mu_i f_i(x_1, \cdots, x_k)}{\sum_{i=1}^{n} \mu_i} = \sum_{i=1}^{n} \hat{\mu}_i f_i(x_1, \cdots, x_k) \tag{6}$$

4. SIMULATIONS AND RESULTS

4.1 Gas Furnace

In this section, the density of burned carbon dioxide is modeled using the time series data of gas furnace. While the delayed terms of gas flow rate u(t) and CO_2 density y(t) are input variables such as u(t-1), u(t-2), u(t), y(t-3), y(t-2) and y(t-1), y(t) is the output variable. This system uses PI as performance index. Varying the nodal polynomial, the results of fuzzy PNN are analyzed. Triquadratic type shows the best performance among the nine nodal polynomials. In general, Fuzzy PNN gives a very sophisticated model from a very limited data set.

Fuzzy PNN model has more performance in linear reasoning method than simplified reasoning method. This model also provided good results in performance, when the number of membership function is properly chosen in modeling system. In this process, though we use all the inputs, optimal model chooses only 4 inputs such as u(t), y(t-3), y(t-2) and y(t-1). TABLE 2 compares fuzzy PNN with other fuzzy modeling methods. TABLE 2 compares the output of identified model using fuzzy PNN with real measured data

TABLE 2. Comparison of identification error with conventional fuzzy modeling methds

Model	PI
Tong's model[3]	0.469
Pedrycz's model[4]	0.56
Xu's model[9]	0.35
Sugeno's model[8]	0.355
Oh's model[7]	0.098
Fuzzy GMDH[11]	0.06
FPNN(Our model)	0.039

4.2 . Activated sludge process

Sewage treatment systems utilize activate sludge process in general. Recently, almost sewage treatment systems use mathematical model in order to obtain regulation data from control process. However, a mathematical model does not design the relationships between variables of sewage treatment process and parameters of its model, accurately and effectively. The accurate modeling of sewage treatment process should be required vigorously. The accurate model can provide control information for the control operators so that they should treat sewage efficiently.

In this paper, a sewage treatment system plant in Seoul KOREA, is chosen as a model. The modeling by fuzzy PNN is done in the activated sludge process , using the sewage data for one year.

We really measured input variables such as Mixed Liquid Suspended (MLSS), Waste Sludge Ratio WSR), Recycled Return Sludge (RRSP) and Dissolved Oxygen Set Point (DOSP) , and output variable such as Effluent Suspended Solids (ESS). Criterion is the same as gas furnace.

TABLE 3. Comparison of identification error with conventional intelligent modeling methds

Model	PI
Conventional model[7]	1.34
Fuzzy-Neural model[10]	0.56
Fuzzy GMDH[11]	0.35
FPNN(Our model)	0.016

5. CONCLUSIONS

In this paper, the fuzzy PNN was proposed to combine conventional PNN to fuzzy inference, in order to model the nonlinear system with small data sets. Both data for gas furnace of time series and wastewater treatment process are used for the purpose of evaluating the performance of the proposed modeling method.

Some results are drawn from computer simulation as follows:

a) Fuzzy PNN gives a very sophisticated model from a very limited data set

b) When the number of membership function is properly chosen, fuzzy PNN has more excellent results than conventional PNN.

c) Fuzzy PNN with linear inference can obtain more results in performance than fuzzy PNN with simplified inference.

d) Fuzzy PNN can easily give author information that determines the number of layers on the basis of some prescribed small quantity.

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