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Adaptive Control for Linear Systems with Parameter Uncertainty Using Switching

Midori Maki and Kojiro Hagino

Dept. of Communications and Systems Engineering, The University of Electro-Communications,
1-5-1 Chofugaoka, Chofu-shi,Tokyo 182, Japan
Tel: 0424(83)2161, Fax: 0424(98)0541, E-mail: midori@cocktail.cas.uec.ac.jp

Abstract: This paper deals with the problem of designing an adaptive regulator in order to improve transient performance in time-response when the linear state-space model of the plant contains unknown parameters which vary within prescribed bounds. The whole possible parameter space is divided into some subspaces and multiple models and controllers are established from the view point that each controller gives satisfactory transient behavior for systems corresponding to each parameter subspace. Based on time-response and an associated cost function, an appropriate controller is selected on-line out of multiple controllers.
Keywords: adaptive control, unknown parameters, switching, time-response, multiple controllers

1 Introduction

In the last decade, a considerable amount of work has been done in the field of controlling uncertain dynamical systems. Roughly speaking, these methods are classified into the robust control and the adaptive control according to different assumptions and approaches. The fundamental distinction between them is that the adaptive control approach explicitly involves structure for reducing uncertainty, while the robust control approach does not.

The almost conventional robust controller design methods have focused on stability robustness, in which a single and fixed controller is used in order to stabilize a system of which parameters vary within prescribed bounds (e.g. Petersen and Hollot, 1986, Khargonekar et al., 1990). In these control schemes, however, attainable performance such as transient behavior in time-response is not obvious. Recently, although attention is being given to robust control methods with additional performance robustification (e.g. Bernstein and Haddad, 1989, Khargonekar and Rotea, 1991, Luo et al., 1994), since robust control schemes do not reflect a posteriori information in the control law, they result by nature in the worst case design. In attaining the higher performance, it is necessary to obtain on-line information relevant to each control objective and utilize it in an active fashion.

On the other hand, adaptive control systems consist of a parameterized controller and an identifying mechanism (Ortega and Tang 1989). The typical adaptive control scheme is the parameter adaptive control, in which unknown parameters are estimated explicitly, and control parameters are determined based on these estimates. Depending on the estimation and control schemes, various possibilities exist for designing such adaptive controllers. However, since these methods must have accurate dynamics or stochastic model of unknown parameters, they are not necessarily practical. Moreover, it is also pointed out that even in the so called "ideal case", stable adaptive controller does not necessarily gurantee good transient response. Adaptive control methods to improve transient response are now in progress (e.g. Narendra and Balakishnan 1994).

In this paper, we present an adaptive control design method in order to improve the transient performance in time-response. No knowledge of the accurate model or stochastic behavior concerning parameter uncertainties is assumed. The only a priori information available is that unknown parameters vary within prescribed bounded ranges. First, in addition to the nominal model, multiple linear models are selected a priori from all possible plants, and a family of optimal controllers is determined, in which each controller is designed for the individual linear model so as to achieve the desired time-response. The whole possible parameter space is divided into some subspaces and multiple models and controllers are established from the view point that each controller gives allowable transient behavior for systems corresponding to each parameter subspace. Since each controller designed for a typified system in each parameter subspace ensures the quadratic stability and an upper bound of certain cost function for the possible systems corresponding to each parameter subspace, an appropriate controller might be selected on-line out of multiple controllers based on time-response and an associated cost function.

The remaining sections of this paper proceed as follows. In section 2, the problem is formulated. In Section 3, multiple models and controllers are considered ,and the possible parameter space is devided based on attainable time-response by each controller. In section 4, a switching rule based on the result obtained in the section 3 is discussed. Finally, section 5 offers some conclusions.

2 Problem Formulation

Consider the following linear multi-input multi-output system described by the state equation;

$$\begin{aligned}\dot{x}(t) &= [A_0 + \Delta A(t)]x(t) + [B_0 + \Delta B(t)]u(t) \\ &= \left[A_0 + \sum_{i=1}^{p} \theta_i(t) D_i\right] x(t) + \left[B_0 + \sum_{i=p+1}^{p+q} \theta_i(t) E_i\right] u(t)\end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the input vector, A_0, B_0 are the nominal system matrix and input matrix of appropriate dimensions respectively, and the pair (A_0, B_0) is assumed to be controllable. The time-varying parameters $\theta_i(t)$ represent unknown parameters which vary within the bounded ranges as

$$\underline{\theta}_i \leq \theta_i(t) \leq \bar{\theta}_i, \quad i = 1, \ldots, p+q \tag{2}$$

The constant matrices D_i, E_i represent the structure of uncertainties. We use the notation $\bar{S}$ to denote the possible parameter space;

$$\bar{S} = \left\{ \left(A_0 + \sum_{i=1}^{p} \theta_i(t) D_i, B_0 + \sum_{i=p+1}^{p+q} \theta_i(t) E_i \right) \right| \\ \left. \underline{\theta}_i \leq \theta_i(t) \leq \bar{\theta}_i, i = 1, \ldots, p+q \right\} \tag{3}$$

The desired response $x^*(t)$ is supposed to be described by

$$\dot{x}^*(t) = \Lambda x^*(t), \; x^*(0) = x(0) \tag{4}$$

where Λ is a stable matrix. Our control objective is to achieve the transient behavior close to the desired time-response (4) by switching multiple controllers on-line.

3 Multiple Models and Controllers

In addition to the nominal system (A_0, B_0), N linear models $(A_j, B_j), j = 1, \ldots, N$ are supposed to be extracted from the possible system behavior patterns in $\bar{S}$, where the pair (A_j, B_j) is assumed to be controllable. We consider multiple controllers, each of which are designed for the individual linear model by applying the regulator theory so as to make the plant output follow the desired trajectory in time-response. That is, each controller is determined for the augmented system

$$\begin{pmatrix} \dot{x}(t) \\ \dot{x}^*(t) \end{pmatrix} = \begin{pmatrix} A_j & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} x(t) \\ x^*(t) \end{pmatrix} + \begin{pmatrix} B_j \\ 0 \end{pmatrix} u_j(t) \tag{5}$$

$$\Longleftrightarrow \quad \dot{X}(t) = \tilde{A}_j X(t) + \tilde{B}_j u_j(t) \tag{6}$$

so as to minimize the cost function

$$\begin{aligned} J &= \int_0^\infty \left\{ \left(x(t) - x^*(t)\right)^T Q \left(x(t) - x^*(t)\right) \right. \\ &\quad \left. + u_j{}^T(t) R u_j(t) \right\} dt \\ &= \int_0^\infty \left(X^T(t) \tilde{Q} X(t) + u_j{}^T(t) R u_j(t)\right) dt \end{aligned} \tag{7}$$

where

$$Q \geq 0, R > 0, \tilde{Q} = \begin{pmatrix} Q & -Q \\ -Q & Q \end{pmatrix} \tag{8}$$

are weighting matrices of appropriate dimensions. Applying the standard LQ regulator theory gives the optimal solution for the minimization problem of (7) subject to (6) as

$$u_j(t) = \tilde{K}_j X(t) \equiv \tilde{K}_{j1} x(t) + \tilde{K}_{j2} x^*(t) \tag{9}$$

$$\tilde{K}_j = -R^{-1} \tilde{B}_j^T \tilde{P}_j \tag{10}$$

$$\tilde{A}_j^T \tilde{P}_j + \tilde{P}_j \tilde{A}_j - \tilde{P}_j \tilde{B}_j R^{-1} \tilde{B}_j^T \tilde{P}_j + \tilde{Q} = 0 \tag{11}$$

Since there may exist some parameter spaces around (A_j, B_j) such that $u_j(t)$ gives allowable transient behavior, we can find vertices of this space by iterative numerical simulations. That is, $\underline{\theta}_i^j \leq 0 \leq \bar{\theta}_i^j$, the upper and lower bounds of θ_i^j respectively, could be searched by numerical iterations such that time-responses of the systems corresponding to these vertices

$$\dot{X}(t) = (\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j^v) X(t) \\ v = 1, \ldots, 2^{p+q} \tag{12}$$

are asymptotically stable and satisfy

$$\max_{t \in [0, \infty)} \|x(t) - x^*(t)\| < \varepsilon \tag{13}$$

where ε is an allowable error bounds specified in advance, and $\Delta F_j^v, v = 1, \ldots, 2^{p+q}$ are vertex matrices of

$$\Delta F_j(\theta_i^j) = \begin{pmatrix} \sum_{i=1}^{p} \theta_i^j D_i & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \sum_{i=p+1}^{p+q} \theta_i^j E_i \\ 0 \end{pmatrix} \tilde{K}_j \tag{14}$$

Remark 3.1: From continuity of the eigenvalues of $\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j(\theta_i^j)$ with respect to θ_i^j, there exist neighborhood around (A_j, B_j) such that (13) is satisfied.

As in the case of (3), the parameter set S_j characterized by (A_j, B_j) and $\underline{\theta}_i^j, \bar{\theta}_i^j$ can be written as

$$S_j = \left\{ \left(A_j + \sum_{i=1}^{p} \theta_i^j(t) D_i, B_j + \sum_{i=p+1}^{p+q} \theta_i^j(t) E_i \right) \right| \\ \left. \underline{\theta}_i^j \leq \theta_i^j(t) \leq \bar{\theta}_i^j, i = 1, \ldots, p+q \right\} \tag{15}$$

Since $\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j^v$, $v = 1, \ldots, 2^{p+q}$ are stable matrices, $\tilde{P}_j > 0$, the positive definite solution of (11) satisfies

$$(\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j^v)^T \tilde{P}_j + \tilde{P}_j (\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j^v) < 0 \\ for \quad \forall v = 1, \ldots, 2^{p+q} \tag{16}$$

Equivalently, $\tilde{P}_j > 0$ satisfies

$$\left(\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j(t)\right)^T \tilde{P}_j \\ + \tilde{P}_j \left(\tilde{A}_j + \tilde{B}_j \tilde{K}_j + \Delta F_j(t)\right) < 0 \\ for \quad \forall \theta_i^j(t) \in [\underline{\theta}_i^j, \; \bar{\theta}_i^j] \tag{17}$$

since the left hand side of (17) depends affinely on $\theta_i^j(t)$, where $\Delta F_j(t)$ is the time-varying version of $\Delta F_j(\theta_i^j)$ in (14). That is, $u_j(t)$ ensures the quadratic stability of all possible systems corresponding to the $j-$th parameter set S_j.

Remark 3.2: Althogh the quadratic stability of all possible plants corresponding to S_j is ensured by utilizing $u_j(t)$, (13) may not always be satisfied.

In order to cover the all possible plants corresponding to $\bar{S}$, the following condition should be satisfied;

$$\bigcup_{j=0}^{N} S_j \supset \bar{S} \tag{18}$$

Remark 3.3: The number of models to satisfy (18) depends on both how to select family of models and the size of allowable error bound ε in (13). As ε become smaller, the N increases.

4 Switching Method

In this section, based on the multiple models and controllers obtained in the previous section, when to switch and what controller should be selected are discussed. Since $u_j(t)$ ensures the quadratic stability of all possible systems corresponding to the $j-$th parameter set S_j, there exist L_j and $\bar{P}_j > 0$ which satisfy

$$(\tilde{A}_j + \tilde{B}_j\tilde{K}_j)^T\bar{P}_j + \bar{P}_j(\tilde{A}_j + \tilde{B}_j\tilde{K}_j) + \tilde{K}_j^T R\tilde{K}_j + \tilde{Q} + L_j^T L_j = 0 \tag{19}$$

$$L_j^T L_j > \Delta F_j^T(t)\bar{P}_j + \bar{P}_j\Delta F_j(t) \quad for \quad \forall\theta_i^j(t) \in [\underline{\theta}_i^j,\ \bar{\theta}_i^j] \tag{20}$$

(See Luo et al.,1994). Using the identity

$$\int_0^\infty \frac{d}{dt}(X^T\bar{P}_jX)dt - X^T(\infty)\bar{P}_jX(\infty) + X^T(0)\bar{P}_jX(0) = 0 \tag{21}$$

the following holds;

$$\begin{aligned} J(u_j \to S_j, x(0), \infty) &= \int_0^\infty (X^T(t)\tilde{Q}X(t) + u_j{}^T(t)Ru_j(t))dt \\ &= X^T(0)\bar{P}_jX(0) - X^T(\infty)\bar{P}_jX(\infty) \\ &+ \int_0^\infty \{\dot{X}^T\bar{P}_jX + X^T\bar{P}_j\dot{X} + X^T\tilde{Q}X + u_j^T Ru_j\}dt \end{aligned} \tag{22}$$

From (19),(20) and $X(\infty) = 0$

$$\begin{aligned} J(u_j \to S_j, x(0), \infty) &< X^T(0)\bar{P}_jX(0) \\ &\equiv \bar{J}(u_j \to S_j, x(0), \infty) \end{aligned} \tag{23}$$

Since $\bar{J}(u_j \to S_j, x(0), \infty)$ is an upper bound of (7) when $u_j(t)$ is applied to systems corresponding to the $j-$th parameter set S_j, this upper bound can be used as a criterion for judging the validity of the selected controller.

Now, let t_s represent a switching instant. Then, we determine the next switching instant t_{s+1} as

$$t_{s+1} = t_s + min\ \tau,\ t_0 = 0 \tag{24}$$

such that

$$J(u_j \to S_a, x(t_s), t_s + \tau) > \bar{J}(u_j \to S_j, x(t_s), t_s + \tau) \tag{25}$$

where

$$\begin{aligned} J(u_j &\to S_a, x(t_s), t_s + \tau) \\ &= \int_{t_s}^{t_s+\tau} \Big\{ (x(t) - x^*(t,t_s))^T Q(x(t) - x^*(t,t_s)) \\ &+ u_j^T(t)Ru_j(t) \Big\} dt \end{aligned} \tag{26}$$

$$\dot{x}^*(t,t_s) = Ax^*(t,t_s), \quad x^*(t_s,t_s) = x(t_s) \tag{27}$$

is the actual performance, and

$$\begin{aligned} \bar{J}(u_j &\to S_j, x(t_s), t_s + \tau) \\ &= X^T(t_s)\bar{P}_jX(t_s) - X^T(t_s+\tau)\bar{P}_jX(t_s+\tau) \end{aligned} \tag{28}$$

is the worst case performance when $u_j(t)$ is applied to the possible systems corresponding to the $j-$th parameter subspace.

Next, we select an appropriate controller by comparing the time-response of the plant with those of multiple models over the time interval between $\hat{t}$ and t_{s+1}. $\hat{t}$ is defined as

$$\begin{aligned} \hat{t} = max\{t \in (t_s, t_{s+1}) \mid \|x(t_1) - x^*(t_1)\| < \varepsilon,\ t_1 < t \\ and\ \|x(t_2) - x^*(t_2)\| > \varepsilon,\ t < t_2\} \end{aligned} \tag{29}$$

, which indicates the latest time instant when the trajectory of the plant deviate from the allowable range defined in (13). Then the $j-$th controller is switched at t_{s+1} to $i-$th ($i \neq j$) controller, where the trajectory $x_i(t), t \in [\hat{t}, t_{s+1}]$ for $i-$th model

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_j(t), \quad x_i(\hat{t}) = x(\hat{t}) \tag{30}$$

gives the minimum value of the following cost function

$$\int_{\hat{t}}^{t_{s+1}} \|x(t) - x_i(t)\|^2 dt \tag{31}$$

The configuration of proposed adaptive control system is shown in Fig. 1.

Remark 4.1: In switching schemes, there may be the case that extremely frequent switching results in poor performance. In this proposed method, since the upper bounds of the associated cost functions are used as a switching criterion, this phenomenon might be avoided. On the other hand, if the upper bounds of the cost functions are set too large, then enough number of times of switching may not occured. However, in view of the fact that at least the worst case performance is guaranteed, infrequent switching may not lead instability.

Remark 4.2: The proposed method can be used to complement the defect of the conventional robust control methods considering only stability robustness. In this case, the minimum number of models enough to prevent remarkable performance deterioration are used.

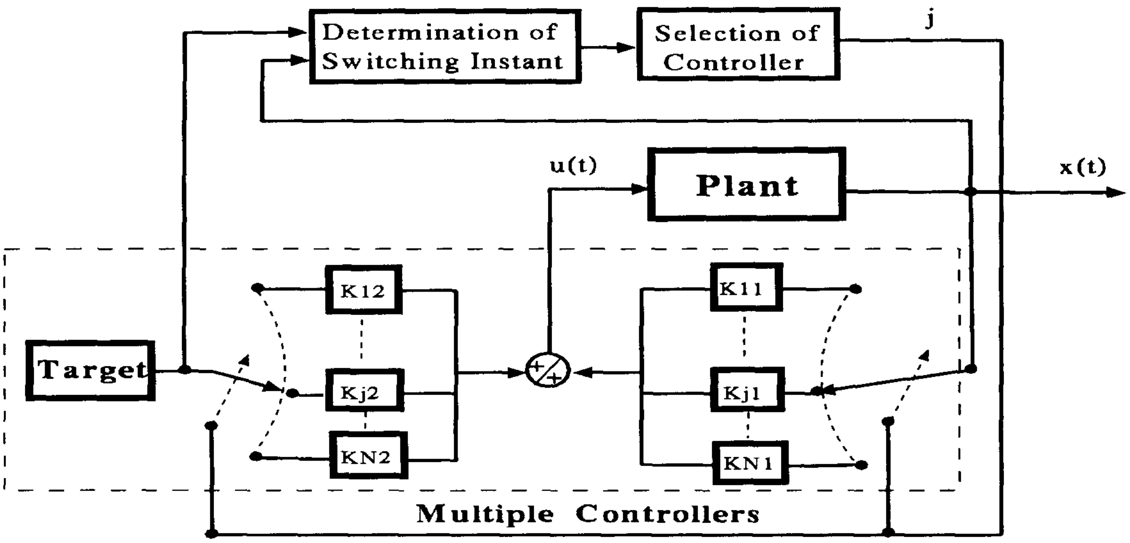

Fig. 1 The configuration of the adaptive control system

5 Concluding remarks

In this paper, an adaptive control method in order to improve the transient behavior in time-response is presented. The whole possible parameter space is divided into some subspaces and multiple models and controllers are established from the view point that each controller gives satisfactory transient behavior for systems corresponding to each parameter subspace. Further, based on time-response and an associated cost function, an appropriate controller is selected out of multiple controllers. The following subjects are left for future research area;

1. Rigorous stability analysis considering the rate of variation of unknown parameters.
2. Development of the effective algorithm for the selection of multiple models and for the division of the possible parameter space.
3. Stability robustness analysis in the presence of disturbances

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