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THE IDENTIFICATION OF CONTINUOUS-TIME SYSTEMS WITHIN A CLOSED-LOOP

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Abstract Physical systems are generally continuous-time in nature. However as the data measured from these systems is generally in the form of discrete samples, and most modern signal processing is performed in the discrete-time domain, discrete-time models are employed. This paper describes methods for estimating the coefficients of continuous-time system within a closed loop control system. The method employs a recursive estimation algorithm to identify the coefficients of a discrete-time bilinear-operator model. The coefficients of the discrete-time bilinear-operator model closely approximate those of the corresponding continuous-time Laplace transform transfer funtion.

Keywords continuous-time system, bilinear-operator, FIR and IIR filter

1 Introduction

In recent years, a great deal of attention has been given to the parameter estimation of closed-loop system.

In this paper, a method for identifying continuous time systems operating within a closed control loop is discussed. The difficulty of the estimation problem depends on whether the controller is digital or analogue. Generally, most identification methods for closed-loop systems assume a digital controller because the identification problem is simplified when a sampler is contained. In the paper we consider the identification of continuous time systems operating in a closed-loop with an analogue controller. Since identification techniques of usual discrete-time systems have been discussed and applied widely, it is a good idea to obtain an approximated discrete-time estimation model with the continuous system parameter. Then we can estimate the continuous system parameters applying the existing recursive identification techniques such as the least squares(LS), instrumental variable(IV) methods for usual discrete-time systems. The discrete-time model approximation is based on the bilinear transformation. Some advantages of the bilinear-operator recursive estimation method in this application are that it allows the coefficients of the transfer function to be estimated quickly and accurately directly from the sampled input-output signals.

2 Problem statement

We consider a closed loop system, as shown in Fig.

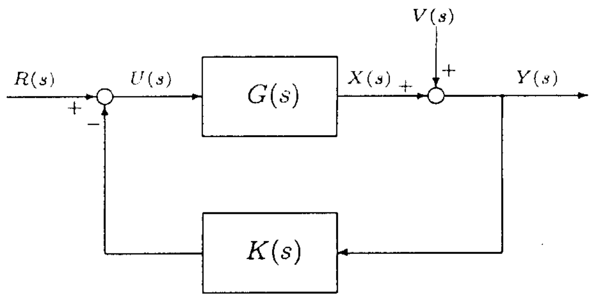

Figure 1: Continuous-time system within a closed-loop

where $G(s)$ and $K(s)$ are the controlled object and controller respectively. Our objective here is to identify the unknown parameter of $G(s)$ from the estimate of $U(s)$ and $Y(s)$. The transfer function $G(s)$ is given by

$$G(s) = \frac{b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n} \tag{1}$$

The relationship between the input and output signals are represented in the time domain as

$$A(p)x(t) = B(p)u(t) \tag{2}$$

where $A(p)$ and $B(p)$ are polynomials in the differential operator p.

$$\begin{aligned} A(p) &= p^n + a_1 p^{n-1} + \cdots + a_n \\ B(p) &= b_1 p^{n-1} + b_2 p^{n-2} + \cdots + b_n \end{aligned}$$

To avoid direct differentiation of the system input-output signals, a digital filter $H(p)$ is introduced:

$$A(p)H(p)x(t) = B(p)H(p)u(t) \qquad (3)$$

Eqn.(3) can be transformed by the bilinear transformation as

$$A(w)H(w)x_k = B(w)H(w)u_k \qquad (4)$$

where u_k, x_k are respectively the sampled input and output signals at time $t = kT$ (T is the sampling period), $A(w)$ and $B(w)$ are bilinear-operator polynomials given by

$$A(w) = w^n + a_{n-1}w^{n-1} + \cdots + a_1 w + a_0 \qquad (5)$$

$$B(w) = b_m w^m + b_{m-1}w^{m-1} + \cdots + b_1 w + b_0 \qquad (6)$$

and w is bilinear operator [6]

$$w \triangleq \frac{2}{T}\frac{q-1}{q+1} \qquad (7)$$

where q is forward-shift operator.

3 Estimation model

In this section, we describe the design of digital filters and the approximated discrete-time estimation model of the underlying continuous-time system model.

3.1 $F(w)$ filter approach

Consider the discrete-time bilinear-operator model (4)

$$A(w)H(w)x_k = B(w)H(w)u_k$$

In order to remove the differentiation effect of the bilinear-operator on the sampled input-output signals, a monic filter polynomial $F(w)$, is defined.

$$F(w) = w^n + f_{n-1}w^{n-1} + \cdots + f_1 w + f_0 \qquad (8)$$

If the $F(w)$ filter polynomial is selected such that $H(w) = 1/F(w)$, then, expanding the term $A(w)/F(w)$ gives

$$\frac{A(w)}{F(w)} = 1 - \frac{F(w) - A(w)}{F(w)}$$

which from (4) yields

$$x_k = \frac{F(w) - A(w)}{F(w)}x_k + \frac{B(w)}{F(w)}u_k \qquad (9)$$

or

$$x_k = \left[\frac{(f_{n-1} - a_{n-1})}{F(w)}w^{n-1} + \cdots + \frac{(f_0 - a_0)}{F(w)}\right]x_k + \left[\frac{b_{n-1}}{F(w)}w^{n-1} + \cdots + \frac{b_0}{F(w)}\right]u_k \qquad (10)$$

3.2 FIR filter approach

Introduce a low-pass digital filter $H(w)$ as

$$H(w) = Q_F(q)\left(\frac{T}{2}(1+q^{-1})\right)^n \qquad (11)$$

where $Q_F(q)$ is the FIR filter:

$$Q_F(q) = \sum_{m=0}^{N-1} q_m z^{-m} \qquad (12)$$

Define $\xi_{Fix}(k)$ and $\xi_{Fiu}(k)$ as follows

$$\begin{aligned} \xi_{Fix}(k) &\triangleq H(w)w^{n-i}x_k \\ \xi_{Fiu}(k) &\triangleq H(w)w^{n-i}u_k \end{aligned}$$

and we can rewrite (4) as

$$\xi_{F0x}(k) + \sum_{i=1}^{n} a_i \xi_{Fix}(k) = \sum_{i=0}^{n} b_i \xi_{Fiu}(k)$$

$$= Q_F(q)\left(\frac{T}{2}\right)^i (1+q^{-1})^i(1-q^{-1})^{n-i}x(k) \quad (i = 0, 1, \cdots, n)$$

$$= Q_F(q)\left(\frac{T}{2}\right)^i (1+q^{-1})^i(1-q^{-1})^{n-i}u(k) \quad (i = 1, 2, \cdots, n) \qquad (13)$$

Many types of FIR digital filters can be applied. Assume that the sampling period is $T(\omega_{dc} \leq \pi/T)$, we can represent Q_F as the Fourier expansion:

$$Q_F(\omega) = \sum_{m=-\infty}^{\infty} h_d(m)e^{-jm\omega T} \qquad (14)$$

From the theory of Fourier series we can choose Fourier coefficients of $h_d(m)$ for $-M \leq m \leq M$

$$\begin{aligned} h_d(m) &= \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} H_d(\omega)e^{jm\omega T}d\omega \\ &= \frac{\sin(m\omega_{dc}T)}{m\pi} \end{aligned} \qquad (15)$$

To reduce the effect of truncation, we apply a window function. In this paper we use the Hamming window, given by

$$w_m = \begin{cases} 0.54 + 0.46\cos(m\pi/M), & |m| \leq M \\ 0, & otherwise \end{cases} \qquad (16)$$

Then we have the windowed transfer function

$$Q'_F(w) = \sum_{m=-M}^{M} c'_m z^{-m} \quad (c'_m = h_d(m)w_m) \qquad (17)$$

Finally, the FIR digital filter for example of length M may be employed

$$Q_F(q) = \sum_{m=0}^{2M} q_m z^{-m} \quad (q_m = c'_{m-M}) \qquad (18)$$

3.3 IIR filter approach

There are many design methods for IIR digital filters. One of the most popular formulations is to digitize traditional continuous-time or analogue filters such as the Butterworth filter, Chebyshev filter, etc. In this paper we choose an mth$(m \geq n)$ order Butterworth filter $H_I(p)$

$$\begin{aligned} |H_I(j\omega)|^2 &= H_I(p)H_I(-p)|_{p=j\omega} \\ &= \frac{1}{1+(\omega/\omega_c)^{2m}} \end{aligned} \tag{19}$$

where ω_c is the cut-off frequency. The poles of the filter satisfies the equation

$$1+(-jp/\omega_c)^{2m}=0$$

obtained by setting $\omega = p/j = -jp$. Only the left half-plane roots are kept. Therefore the Butterworth filter is described to be

$$H_I(p) = \prod_{i=1}^{m} \frac{1}{(p/\omega_c - \lambda_i)} \tag{20}$$

For example, a second-order Butterworth filter is given by

$$H_{I2}(p) = \frac{1}{(p/\omega_c)^2 + \sqrt{2}(p/\omega_c) + 1} \tag{21}$$

Multiplying both sides of the system (2) by the predesigned $H_I(p)$, we have

$$H_I(p)p^n x(t) + \sum_{i=1}^{n} a_i H_I(p) p^{n-i} x(t) = \sum_{i=1}^{n} b_i H_I(p) p^{n-i} u(t) \tag{22}$$

Discretizing by the bilinear transformation yields

$$\xi_{I0x}(k) + \sum_{i=1}^{n} a_i \xi_{Iix}(k) = \sum_{i=1}^{n} b_i \xi_{Iiu}(k)$$

$$\xi_{Iix}(k) = Q_I(z^{-1}) \left(\frac{T}{2}\right)^i (1+z^{-1})^i (1-z^{-1})^{n-i} x(k) \quad (i = 0, 1, \cdots, n)$$

$$\xi_{Iiu}(k) = Q_I(z^{-1}) \left(\frac{T}{2}\right)^i (1+z^{-1})^i (1-z^{-1})^{n-i} u(k) \quad (i = 1, 2, \cdots, n) \tag{23}$$

where

$$Q_I(z^{-1}) = \frac{\left(\frac{T}{2}\right)^{m-n} (1+z^{-1})^{m-n}}{\left(\frac{1-z^{-1}}{\omega_c}\right) + \sum_{i=1}^{m} c_i \left(\frac{1-z^{-1}}{\omega_c}\right)^{m-i} \left(\frac{T}{2}\right)^i (1+z^{-1})^i}$$

4 Identification algorithm

Eqn.(10) may be presented in a more suitable form for estimation by the regression equation,

$$y_k = \phi_k^T \theta + error \tag{24}$$

where the regression and parameter vectors, respectively, are given by,

$$\phi_k^T = \left[\frac{w^{n-1}}{F(w)} y_k, \cdots, \frac{1}{F(w)} y_k, \frac{w^{n-1}}{F(w)} u_k, \cdots, \frac{1}{F(w)} u_k\right]$$

$$\begin{aligned} \theta^T = [&f_{n-1} - a_{n-1}, f_{n-2} - a_{n-2}, \cdots, f_0 - a_0, \\ &b_{n-1}, b_{n-2}, \cdots, b_0] \end{aligned}$$

The IV vector is given by

$$z_k^T = \left[\frac{w^{n-1}}{F(w)} x_k, \cdots, \frac{1}{F(w)} x_k, \frac{w^{n-1}}{F(w)} u_k, \cdots, \frac{1}{F(w)} u_k\right]$$

The recursive estimation algorithms can be described by the following form.

$$\begin{aligned} e_k &= y_k - \phi_k^T \widehat{\theta}_{k-1} \\ \widehat{\theta}_k &= \widehat{\theta}_{k-1} + \frac{P_{k-1}\phi'_k e_k}{\lambda_k + \phi_k^T P_{k-1} \phi'_k} \\ P_k &= \frac{1}{\lambda_k}\left[P_{k-1} - \frac{P_{k-1}\phi'_k \phi_k^T P_{k-1}}{\lambda_k + \phi_k^T P_{k-1}\phi'_k}\right] \end{aligned} \tag{25}$$

The LS method is obtained by setting

$$\phi'_k = \phi_k$$

and the IV method is obtained by setting

$$\phi'_k = z_k$$

When the digital low-pass filters are designed, we have the approximated discrete-time estimation models, either (13) for the FIR filtering approach or (23) for the IIR filtering approach. Both can be written in regression form as

$$\xi_{0y}(k) = \Psi_k^T h + error \tag{26}$$

where the regression and parameter vectors are defined respectively as

$$\begin{aligned} \Psi_k^T &= [\xi_{1y}(k), \cdots, \xi_{ny}(k), \xi_{1u}(k), \cdots, \xi_{nu}(k)] \\ h^T &= [a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n] \end{aligned}$$

where

$$\xi_{iy}(k) = \xi_{Fiy}(k), \xi_{iu}(k) = \xi_{Fiu}(k) \qquad (FIR filter)$$

or

$$\xi_{iy}(k) = \xi_{Iiy}(k), \xi_{iu}(k) = \xi_{Iiu}(k) \qquad (IIR filter)$$

The IV vector is constructed to be

$$m_k^T = [\xi_{1x}(k), \cdots, \xi_{nx}(k), \xi_{1u}(k), \cdots, \xi_{nu}(k)]$$

The recursive estimation algorithms are given by

$$\begin{aligned} \varepsilon_k &= \xi_{0y}(k) - \Psi_k^T \widehat{h}_{k-1} \\ \widehat{h}_k &= \widehat{h}_{k-1} + \frac{P_{k-1}\Psi'_k \varepsilon_k}{\lambda_k + \Psi_k^T P_{k-1}\Psi'_k} \\ P_k &= \frac{1}{\lambda_k}\left[P_{k-1} - \frac{P_{k-1}\Psi'_k \Psi_k^T P_{k-1}}{\lambda_k + \Psi_k^T P_{k-1}\Psi'_k}\right] \end{aligned} \tag{27}$$

where λ_k is the forgetting factor and in this paper it is chosen to be

$$\lambda_k = (1 - 0.01)\lambda_{k-1} + 0.001, \qquad \lambda(0) = 0.95$$

The LS method is obtained by setting

$$\Psi'_k = \Psi_k$$

and the IV method is obtained by setting

$$\Psi'_k = m_k$$

If the $F(w)$ filter polynomial and $H(w)$ filter are related by

$$H(w) = 1/F(w),$$

then both estimation algorithms are mathematically equivalent. We write

$$\begin{aligned} \varepsilon_k &= \Psi_{0y}(k) - \Psi_k^T \hat{h}_{k-1} \\ &= \frac{\hat{A}(w)}{F(w)} y_k - \frac{\hat{B}(w)}{F(w)} u_k \\ &= y_k - \frac{F(w) - \hat{A}(w)}{F(w)} y_k - \frac{\hat{B}(w)}{F(w)} u_k \\ &= e_k \end{aligned}$$

Therefore the regression vectors are equal i.e. $\phi_k^T = \Psi_k^T$. We consider cases when the digital filters are not related by their reciprocal.

5 Example

We consider a second-order system described by

$$G(s) = \frac{4}{s^2 + 3s + 4} \qquad K(s) = 1$$

where controller $K(s)$ is unit feedback. Simulation experiments are carried out under the following conditions Input signal:

$$u(t) = \sin(t) + \sin(1.5t) + 0.5\sin(3t) + 1.5\sin(4.5t) \\ +0.3\sin(5t) + 0.2\sin(7t) + 2.5\sin(7.5t) + 5.0\sin(10.5t)$$

The effects of the filter characteristics on the results of the LS method are investigated with sampling period $T = 0.04$ and a N/S ratio of 20%; 2500 samples are taken, and the RLS estimates are shown in Table 1 and Table 2 for the RIV estimation.

Table 1: LS estimates

	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_2$ (4.0)
FIR filter	2.9725	4.0620	0.0147	-0.0102
IIR filter	3.2051	3.8629	0.0161	-0.0118
F(w) filter	2.8147	3.7293	0.0136	0.1294

Table 2: IV estimates

	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_2$ (4.0)
FIR filter	2.9882	3.8743	0.1624	0.1602
IIR filter	3.1172	3.7624	0.1438	0.1471
F(w) filter	2.9134	3.8196	0.1812	0.2921

6 conclusion

In this paper, we consider the identification of continuous-time systems operating under closed-loop control with an analogue controller. The estimation technique employs either FIR or IIR digital filters to avoid direct differentiation of the system input and output signals. The results of several simulation showed that the estimation method had difficulty inaccurately identifying the numerator parameters of the continuous-time system. It is believed that this is due to the feedback of output measurement noise to the system input.

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