Proceedings of the 11th
KACC, October 1996

LINEARIZATION OF NONLINEAR SYSTEM BY USE OF VOLTERRA KERNEL

Eiji NISHIYAMA* and Hiroshi KASHIWAGI**
*Kumamto National College of Technology
Tel:+82-96-242-6068;Fax:+82-96-242-6067;E-mail:enishi@tc.knct.ac.jp
**Faculty of Engineering,Kumamoto Univsersity
Tel:+82-96-342-3742;Fax:+82-96-342-3730;E-mail:kashiwa@gpo.kumamoto-u.ac.jp

Abstract: In this paper, the authors propose a new method for linearizing a nonlinear dynamical system by use of Volterra kernel of the nonlinear system. The authors have recently obtained a new method for measuring Volterra kernels of nonlinear contorol systems by use of a pseudo-random M-sequence and correlation technique. In this method, an M-sequence is applied to the nonlinear system and the crosscorrelation function between the input and the output gives us every crosssection of Volterra kernels up to 3rd order. Once we can get Volterra kernels of nonlinear system, we can construct a linearization method of the nonlinear system. Simulation results show good agreement between the observed results and the theoretical considerations.

1. Introduction

The actual control systems are considered to be nonlinear essentially, although there exist many cases where the nonlinearity in the system is negligibly small (in this case the system is said to be linear). Therefore the problem of identification of a nonlinear system is quite important. Nevertheless, there are not many methods for nonlinear identification, because the nonlinear systems are in general very complex to be identified.

In this paper, the authors propose a new method for linearizing a nonlinear dynamical system by use of Volterra kernels of the nonlinear system. The authors have recently obtained a new method for measuring Volterra kernels of nonlinear contorol systems by use of a pseudo-random M-sequence and correlation technique[1)-5)]. In this method, an M-sequence is applied to the nonlinear system and the crosscorrelation function between the input and the output gives us every crosssection of Volterra kernels up to 3rd order.

Once we can get Volterra kernels of nonlinear system, we can construct a linearization method of nonlinear system. When the effects of higher order Volterra kernels on the output are subtracted from the system output, we can linearize a nonlinear system easily. Simulation results show good agreement between the observed results and the theoretical considerations.

2. Princeple of identification of Volterra kernels

Let us denote the system input as $u(t)$ and the output $y(t)$. Then the output of a nonlinear system can be written in general as follows:

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \ldots, \tau_i) \times u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i)d\tau_1 d\tau_2 \ldots d\tau_i. \quad (1)$$

Here we call $g_i(\tau_1, \tau_2, \ldots, \tau_i)$ i-th oder Volterra kernels. When we take the crosscorrelation function between $u(t)$ and $y(t)$, we have,

$$\begin{aligned} \Phi uy(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \ldots, \tau_i) \\ &\quad \times \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i)} \\ &\quad \times d\tau_1 d\tau_2 \ldots d\tau_i. \end{aligned} \quad (2)$$

Here $\overline{\quad}$ denotes time average. When we use a pseudo-random M-sequence as input $u(t)$, we get Eq.(3) by use of so-called "shift and add property" of the M-sequence. That is, there exists one and only one interger number k_{ii}^j (mod N) which satisfies the following equation.

$$\begin{aligned} &u(t)u(t+k_{i1}^j\Delta t)u(t+k_{i2}^j\Delta t)\ldots u(t+k_{ii-1}^j\Delta t) \\ &= u(t+k_{ii}^j\Delta t) \end{aligned} \quad (3)$$

Then, Eq.(2) becomes the next equation.

$$\begin{aligned} \Phi uy(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i \\ &\times \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^j\Delta t, \ldots, \tau - k_{ii}^j\Delta t) \end{aligned} \quad (4)$$

where $F(\tau)$ is a function of τ.

If $k_{ii}^{(j)}$'s of $g_i(\tau, \ldots, \tau_i)$ are apart from each other sufficiently (say, 20 to 30 Δt apart) ,then we can obtain Volterra kernels separately from Eq.(4).

3. Linearization method

When the output of a linear system is written as in Eq.(1), the first term in Eq.(1)

$$\int_0^\infty g_1(\tau_1)u(t-\tau_1)d\tau_1$$

is considered to be the output of the linear sytem. Therefore,when we can obtain those Volterra kernels $g_i(\tau_1, \tau_2, \ldots, \tau_i)$ beforehand we can linearize the nonlinear sytem by subtracting those effects caused by $g_2(\tau_1, \tau_2)$, $g_3(\tau_1, \tau_2, \tau_3)$ and so on from the system output. That is

$$\begin{aligned} \hat{y}(t) &= y(t) - \sum_{i=2}^{\infty} \int_0^\infty \cdots \int_0^\infty g_i(\tau_1, \tau_2, \ldots, \tau_i) \\ &\quad \times u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i) \\ &\quad \times d\tau_1 d\tau_2 \ldots d\tau_i \end{aligned} \tag{5}$$

where $\hat{y}(t)$ is the output of the linearized system.

Fig.1 is the block diagram showing the linearization method by use of identified Volterra kernels. We suppose that the nonlinear system considered here is composed of linear part (denoted as $g(t)$) plus non-linear element (Wiener type nonlinear sytem). We also suppose here that the nonlinear system is composed of up to 3rd order Volterra kernels for explanation purpose. The input $u(t)$ is fed to delay element τ_1, τ_2, producing the signal $u(t-\tau_1)$ and $u(t-\tau_2)$. Then from Eq.(6) we obtain the effects of the second Volterra kernel on the output as follows.

$$\sum_{\tau_1}\sum_{\tau_2} g_2(\tau_1, \tau_2)u(t-\tau_1)u(t-\tau_2). \tag{6}$$

And the effect of third Volterra kernel is also obtained in the same way.

These effect of 2nd and 3rd order Volterra kernels are subtracted from the system output as is shown in Fig.1, and the output of the linearized system $\hat{y}$ is obtained.

3. Simulation

We have applied this method of obtaining Volterra kernels of nonlinear system and it's linearization to three nonlinear control systems.

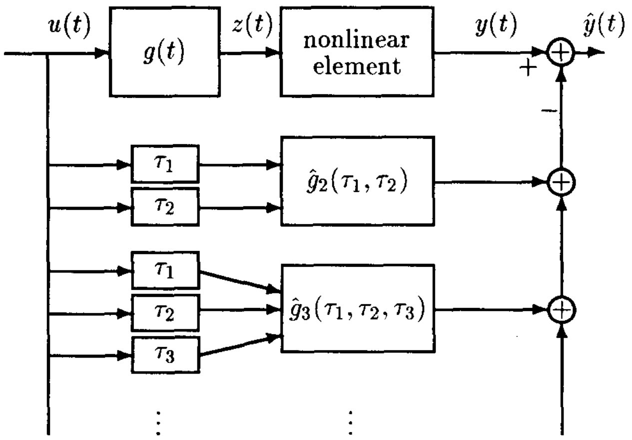

Fig.1 Linearlization of nonlinear system

(1) Polynomial-type nonlinear system : The nonlinear system having the following charactaristics as shown in Fig.2:

$$y(t) = z(t) + 0.5z^2(t) + 0.5z^3(t) \tag{7}$$

where z(t) is the output of the linear part $g(t)$ when the input to $g(t)$ is $u(t)$. The measured Volterra kernels $g_1(\tau_1), g_2(\tau_1, \tau_2)$ and $g_3(\tau_1, \tau_2, \tau_3)$ are shown in Fig.3 ,Fig.4, and Fig.5 ,respectively. In Fig 5, only one crosssection of $g_3(\tau_1, \tau_2, \tau_3)$ when $\tau_1 = 2$ is shown. Fig.6 shows one of the simulation results. The bold solid line in Fig.6 shows the output of the nonlinear system $y(t)$, and the solid line shows the system output when only the linear term $z(t)$ exists. The square marks show $\hat{y}(t)$, the output of the linearized system, showing good agreement with the linear system. In this case, it is noted that the nonlinear system has Volterra kernels only up to 3rd order, so the linearization is quite satisfactarily carried out.

(2) Saturation-type nonlinear system: The next example of nonlinear system is shown in Fig. 7, having saturation nonlinearity put in cascade with a linear element $g(t)$. The measured Volterra kernels in this case are shown in Fig.8 and Fig.9. The results of simulation are shown in Fig.10, where the output of the saturation type nonlinear system is shown in a bold solid line, and the output of the imaginary linear system(supposing the saturation element is linear) is shown in a solid line. The square marks show the output $\hat{y}(t)$ after the linearization is carried out as in Fig.1.

From this simulation, it is shown that the saturation element can be linearized to a certain extent.

(3) Nonlinear chemical process: The third example of nonlinear system is shown in Eq.(9).

$$\frac{dy(t)}{dt} = -ky^2(t) + \frac{1}{V}(d(t) - y(t))u(t) \qquad (8)$$

This system is a certain chemical process showing nonlinearity. $u(t)$ is the volumetric flow rate of feed stream (l/h). $y(t)$ is the output of this reactor indicating (mol/l) concentration of outlet stream. k is the rate of reaction $(1/mol/l \cdot h)$. V is the rector volume. d is concentration of inlet stream. The measured Volterra kernels of this nonlinear reactor are shown in Fig.11-12. Fig.13 shows the comparison of the output for nonlinear and the linearized system, indicating the effects of nonlinearity in this case are relatively small.

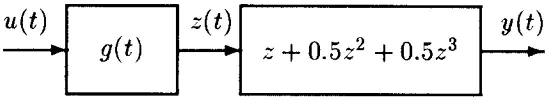

Fig.2 Nonlinear system having up to 3rd order Volterra kernels

4.Conclusion

A nonlinear system can be expressed in Volterra series expansin as in Eq.(1). In this case, if those Volterra kernels can be obtained beforehand, we can linearize the nonlinear system simply by subtracting the effects of high order Volterra kernels on the system output.

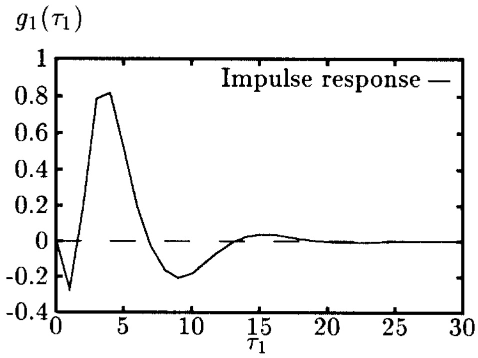

Fig.3 Impulse response of polynomial-type nonlinear system

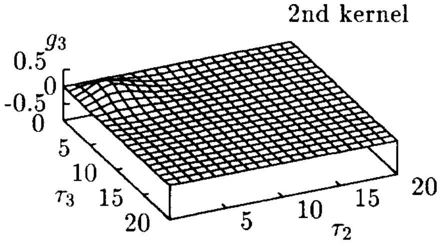

Fig.4 2nd kernels of polynomial-type nonlinear system

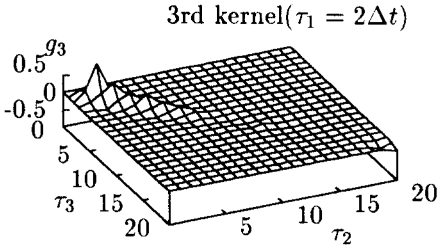

Fig.5 3rd kernels of polynomilal-type nonlinear system (when $\tau_1 = 2\Delta t$)

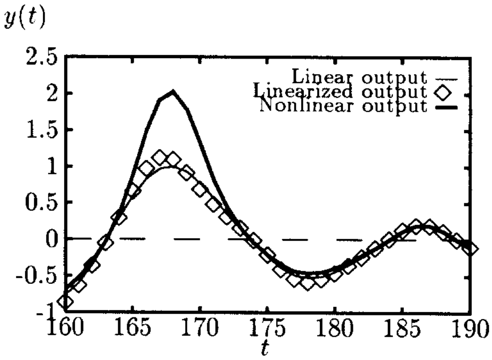

Fig.6 Simulation result of polynomial-type nonlinear system

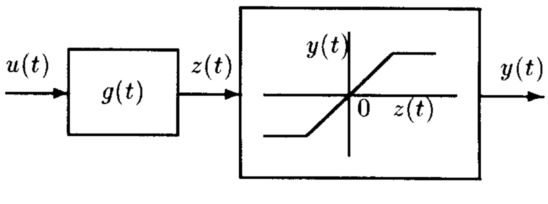

Fig.7 Nonlinear system having saturation element

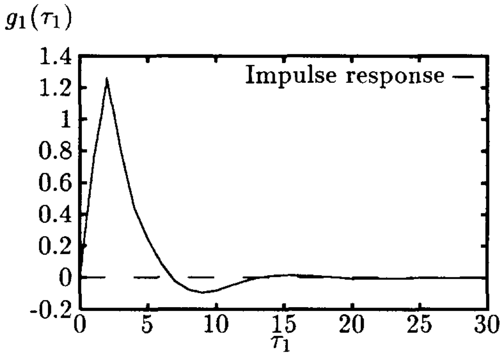

Fig.8 Impulse response of saturation-type nonlinear system

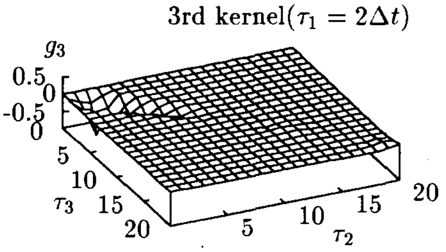

Fig.9 3rd kernel of saturation-type nonlinear system (when $\tau_1 = 2\Delta t$)

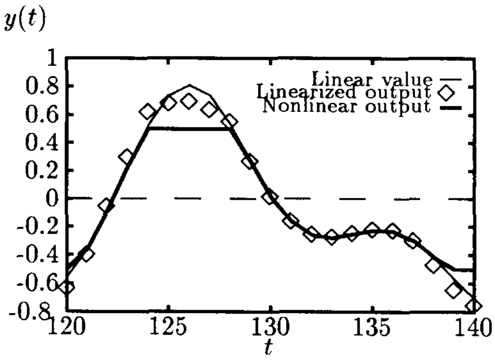

Fig.10 Simulation result of saturation-type nonlinar system

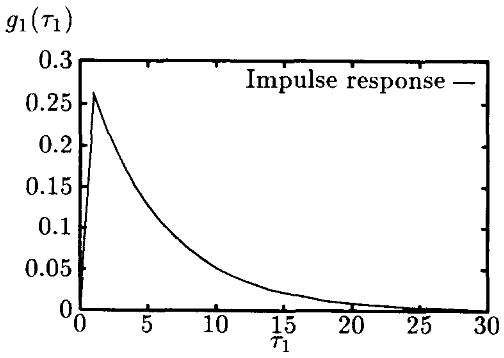

Fig.11 Impulse response of a certain chemical system

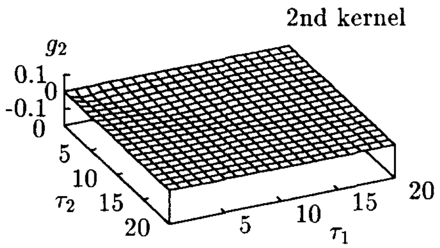

Fig.12 2nd kernel of a certain chemical system (when $\tau_1 = 2\Delta t$)

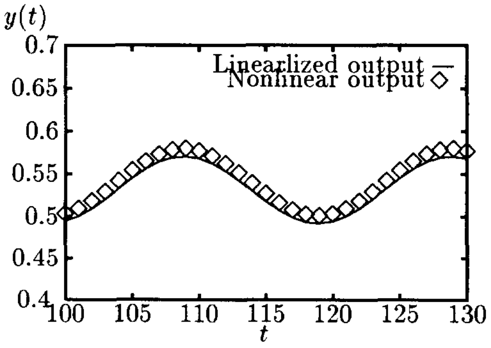

Fig.13 Simulation result of a certain chemical system

The authors applied the method of obtaining Volterra kernels by use of M-sequence and the linearization method to three nonlinear systems; polynomial-type, saturation-type and nonlinear chemical process. The simulation results show that our method of linearization technique of nonlinear system is quite useful in practical cases.

Reference

(1) H.Kashiwagi and Y.P.Sun: A Method for Identifying Volterra Kernels of Nonlinear System: Tran.SICE, **31**-8,1054/1060(1995)

(2) H.Kashiwagi, Y.P.Sun and E. Nishiyama: Identification of Volterra Kernels of Nonlinear Systems by use of M-sequence, Proc.'93 KACC, Seoul,Korea, 150/154 (1993)

(3) H.Kashiwagi, Y.P.Sun and E.Nishiyama: Identification of 2nd and 3rd Volterra Kernels of Nonlinear Systems, Proc.'93 APCCM, Kunming, China,15/19 (1993)

(4) Y.P.Sun and H.Kashiwagi: Comparison of Volterra Kernel Method with Describing Function for Nonlinear System Identification, Proc. APCCM'95, Wuhan, China 28/31(1995)

(5) Y.P.Sun and H.Kashiwagi: A Real Time Method for Nonlinear System Identificatin: Proc.ICAUTO'95, Indore, India, 209/212(1995)