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EXTENDED IMPEDANCE CONTROL OF REDUNDANT MANIPULATORS

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Abstracts An impedance control approach based on an extended task space formulation is addressed to control the kinematically redundant manipulators. Defining a weighted inner product in joint space, a minimal parametrization of the null space can be achieved and we can visualize the null space motion explicitly. Based on this formulation, we propose a control method called inertially decoupled impedance controller to control the motion of the end-effector as well as the internal motion expanding the conventional impedance control. Some numerical simulations are given to demonstrate the performance of the proposed control methods.

Keywords Extended impedance control, Redundant manipulator, Weighted decomposition of joint space

1. INTRODUCTION

Consideration of task space dynamics is essential for higher performance of manipulators especially when the control of the end-effector motion is combined with that of contact forces[1, 2, 5]. For kinematically redundant manipulators, even if the dynamic behavior of the end-effector can be described using the task space formulation, there is a hidden dynamics which can not be observed in task space[2]. In order to achieve given task, the null motion should be controlled for redundant manipulators as the secondary task.

To control the redundant manipulator utilizing the redundancy, many results are available in literature[3, 4, 6]. One of the most popular method is the configuration control method[3]. Using this approach, many compliant control schemes are proposed[5, 6]. However, they do not consider the null motion or null dynamics. It was firstly noted by Hsu *et al*[4] and T. Tsuji and A. Jazidie[7] proposed an approach to utilize the redundancy within the impedance control framework. But they failed to parametrize the null motion with a minimal set.

In this paper, an extended impedance control methods for kinematically redundant manipulators are presented based on weighted decomposition of joint space. The impedance control approach is employed to control the motion of end-effector and modified resolved acceleration control method is used to control the internal motion. Those results are simulated with a planar three link redundant manipulator.

2. MODELING OF MANIPULATOR

For an n-DOF serial manipulator operating in m-dimensional task space, the kinematic relations can be expressed by

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}, \quad \boldsymbol{J}(\boldsymbol{q}) = \frac{\partial \boldsymbol{k}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{1}$$

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}), \quad \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) \triangleq \dot{\boldsymbol{J}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \tag{2}$$

where $\boldsymbol{J} \in \Re^{m\times n}$ is called the manipulator Jacobian matrix and for kinematically redundant manipulator, $n > m$ and $r = n - m$ is called the degree of redundancy.

Dynamic equations of motion in joint space is described as follows

$$\boldsymbol{\tau} = \boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f}, \tag{3}$$

where $\boldsymbol{\tau} \in \Re^n$ is joint torque vector; $\boldsymbol{H}(\boldsymbol{q}) \in \Re^{n\times n}$ is joint inertia matrix; $\boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \Re^n$ is the nonlinear torque and $\boldsymbol{f} \in \Re^m$ is the contact force exerted by the end-effector on the environment.

When we consider the motion and force control of the manipulator, it is more convenient to express the dynamic equations in the task space form as follows[2]

$$\boldsymbol{f}_c = \boldsymbol{\Lambda}(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{\eta}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{f}, \tag{4}$$

where $\boldsymbol{f}_c \in \Re^m$ is a fictitious force applied to the end-effector of the manipulator, $\boldsymbol{\Lambda}(\boldsymbol{q}) \in \Re^{m\times m}$ is called the pseudo-inertia matrix[2] and $\boldsymbol{\eta} \in \Re^m$ is the nonlinear force vector in the task space. Eq. (4) can be used to describe the motion of end-effector, however, it is not complete to describe the manipulator's configuration.

3. NEW EXTENDED TASK SPACE FORMULATION

In this section, we present a new extended task space formulation of redundant manipulators based on joint space decomposition.

3.1 Kinematic Decomposition

Let us assume that $\dot{\boldsymbol{q}}$ and $\dot{\boldsymbol{x}}$ belong to a vector space $\mathcal{Q} \subset \Re^n$ and $\mathcal{X} \subset \Re^m$, respectively. According to the type of manipulators, each component of $\dot{\boldsymbol{q}}$ may have different physical dimensions and even if the components are physically consistent, the joint limits may be different from each joint. To resolve this, the inner product in $\mathcal{Q}$ is defined by a metric $\boldsymbol{W}$ as follows:

$$< \dot{\boldsymbol{q}}_1, \dot{\boldsymbol{q}}_2 >_W \triangleq \dot{\boldsymbol{q}}_1^T \boldsymbol{W} \dot{\boldsymbol{q}}_2, \quad \forall\, \dot{\boldsymbol{q}}_1, \dot{\boldsymbol{q}}_2 \in \mathcal{Q} \tag{5}$$

where $W \in \Re^{n\times n}$ is a symmetric positive definite matrix.

Based on the above, general solution of Eq. (1) can be obtained as

$$\dot{q} = \dot{q}_p + \dot{q}_h = J_W^+ \dot{x} + \left(I_n - J_W^+ J\right)\xi, \tag{6}$$

where J_W^+ is a weighted generalized inverse defined by

$$J_W^+ = W^{-1}J^T\left(JW^{-1}J^T\right)^{-1} \tag{7}$$

and $\xi \in \Re^n$ is an arbitrary vector in $\mathcal{Q}$. Commonly, $\dot{q}_p$, $\dot{q}_h$ are called the particular and homogeneous solutions of Eq. (1), respectively. The particular solution $\dot{q}_p$ is related with the task space motion and the homogeneous solution $\dot{q}_h$ is related with the null space motion. However, the homogeneous solution is not a minimal set to specify the null space motion. Since

$$\mathcal{Q} = \mathcal{R}\left(J^T\right) \oplus \mathcal{N}(J)$$

and $\mathcal{R}(J^T)$ can be characterized by $\dot{x}$, only r–dimensional vectors are necessary to specify the null space of J. Let $V(q)$ be a full column rank matrix in $\mathcal{N}(J)$, *i.e.*,

$$J(q)V(q) = 0. \tag{8}$$

Since $\dot{q}_h \in \mathcal{N}(J)$, it is possible to define r velocities $\dot{x}_N = [\dot{x}_{N,1}, \dots, \dot{x}_{N,r}]^T \in \mathcal{X}_N$ such that

$$\dot{q}_h = \left(I_n - J_W^+ J\right)\xi = V(q)\dot{x}_N = \sum_{i=1}^{r} \dot{x}_{N,i} v_i. \tag{9}$$

It means that the homogeneous velocity $\dot{q}_h$ can be represented by a linear combination of v_i with a magnitude of $\dot{x}_{N,i}$. Using Eqs. (6) and (9), we obtain $\dot{x}_N \in \mathcal{X}_N$ as follows:

$$\dot{x}_N = \left(V^T W V\right)^{-1} V^T W \dot{q} \triangleq J_N(q)\dot{q}, \tag{10}$$

where the linear mapping $J_N : \mathcal{Q} \to \mathcal{X}_N$ is defined as the null space Jacobian matrix. Since x_N is not available generally, the control of null space motion can be regarded as the velocity tracking problem.

Now, define an extented task space $\dot{x}_E$ as follows:

$$\dot{x}_E^T = \left[\dot{x}^T \ \dot{x}_N^T\right] \tag{11}$$

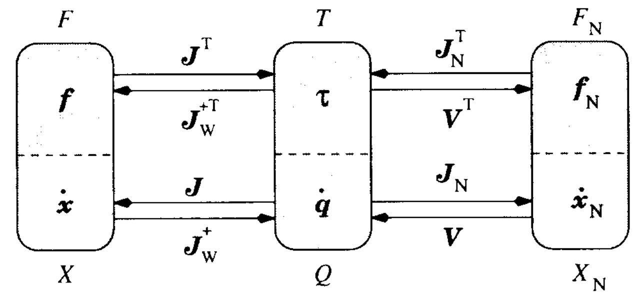
Then the extended task space kinematics can be written using proposed minimal representation as

$$\dot{x}_E = J_E(q)\dot{q} \ \text{ and } J_E = \begin{bmatrix} J \\ J_N \end{bmatrix}. \tag{12}$$

If the manipulator is not in singular configuration, the extended Jacobian matrix J_E is always invertible.

3.2 Dynamic Decomposition

For redundant manipulator, n–joint torques are applied to the manipulator. Since for m kinematic equations it is necessary to supply only m independent constraints forces f_c and f_c are related with $\tau = J^T f_c$. However, if the redundant manipulator is not held at static equilibrium, it can not be satisfied because of the internal motion. Let us define $f_N \in \mathcal{F}_N$ as a null space force vector which gives rise to null motion $\dot{x}_N$ without producing any work along



Task space Joint space Null space

Fig. 1 Motion and force decomposition of redundant manipulator

$\dot{x}$. Augmenting the task space force f_c and null space force f_N, define f_{Ec} as follows:

$$f_{Ec}^T = \left[f_c^T \ \ f_N^T\right]. \tag{13}$$

Then we obtain the following relation from the virtual work principle:

$$\tau = J_E^T f_{Ec} = J^T f_c + J_N^T f_N. \tag{14}$$

Pre–multiply the above equation by V^T, we get f_N

$$f_N = V^T(q)\tau. \tag{15}$$

From the above analysis, we can decompose the joint velocity vector $\dot{q}$ into the task space velocity and null velocity with minimal representation, respectively. Similarly, the joint torque vector τ can be decomposed into the task space force and the null space force as shown in Fig. 1, where the vector spaces $\mathcal{X}_N$, $\mathcal{F}_N \subset \Re^r$ and $\mathcal{X}$, $\mathcal{F} \subset \Re^m$.

Now, we can reformulate the equations of motion of the redundant manipulator in terms of the task space and null space variables, explicitly. It can be easily shown that

$$f_{Ec} = \Lambda_E(q)\ddot{x}_E + \eta_E(q,\dot{q}) + f_E, \tag{16}$$

where

$$\Lambda_E = J_E^{-T} H J_E^{-1} = \begin{bmatrix} J_W^{+T} H J_W^+ & J_W^{+T} H V \\ V^T H J_W^+ & V^T H V \end{bmatrix} \tag{17}$$

$$\eta_E = J_E^{-T} n(q,\dot{q}) - \Lambda_E(q) h_E(q,\dot{q}) \tag{18}$$

$$h_E(q,\dot{q}) = \dot{J}_E \dot{q} = \begin{bmatrix} h(q,\dot{q}) \\ h_N(q,\dot{q}) \end{bmatrix} \ \text{ and } f_E = \begin{bmatrix} f \\ 0 \end{bmatrix}. \tag{19}$$

It should be noted that clever choice of W can reduce the inertia matrix Λ_E to the block diagonal form, *i.e.*, inertial decoupling of the task space dynamics from the null space motion.

4. EXTENDED IMPEDANCE CONTROLLER

Typically the term impedance control refers to a control approach that implements some desired dynamical relation consisting of some inertial, damping and stiffness parameters. Consider the desired impedance relation in extended task space as follows:

$$\alpha f_E = M_{Ed}\ddot{e}_E + B_{Ed}\dot{e}_E + K_{Ed}e, \tag{20}$$

where $\boldsymbol{M}_{Ed}$, $\boldsymbol{B}_{Ed} \in \Re^{n\times n}$ and $\boldsymbol{K}_{Ed} \in \Re^{n\times m}$ denote the desired mass, damping coefficient and stiffness matrices in the extended task space, respectively, and α is a force scaling factor which will be described later. Since $\boldsymbol{x}_N$ is not defined in general, $\boldsymbol{K}_{Ed}$ is not a square matrix. $\dot{\boldsymbol{e}}_E^T = [\dot{\boldsymbol{e}}^T \ \dot{\boldsymbol{e}}_N^T]$, $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$ and $\dot{\boldsymbol{e}}_N = \dot{\boldsymbol{x}}_{Nd} - \dot{\boldsymbol{x}}_N$, $\boldsymbol{B}_{Ed}$ and $\boldsymbol{K}_{Ed}$ are given by

$$\boldsymbol{B}_{Ed} = \begin{bmatrix} \boldsymbol{B}_d & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_N \end{bmatrix} \quad \text{and} \quad \boldsymbol{K}_{Ed} = \begin{bmatrix} \boldsymbol{K}_d \\ \boldsymbol{0} \end{bmatrix}, \tag{21}$$

where $\boldsymbol{B}_d$, $\boldsymbol{K}_d \in \Re^{m\times m}$ and $\boldsymbol{B}_N \in \Re^{r\times r}$. From Eq. (20), we obtain the following command input force in extended task space:

$$\boldsymbol{f}_{Ec} = \boldsymbol{\Lambda}_E(\boldsymbol{q})\left\{\ddot{\boldsymbol{x}}_{Ed} + \boldsymbol{M}_{Ed}^{-1}[\boldsymbol{B}_{Ed}\dot{\boldsymbol{e}}_E + \boldsymbol{K}_{Ed}\boldsymbol{e} - \alpha\boldsymbol{f}_E]\right\} + \boldsymbol{\eta}_E(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{f}_E. \tag{22}$$

In early impedance control works, it was assumed that an arbitrary desired impedance could be emulated. Recently, however, Newman[5] demonstrated that it is impossible to emulate an arbitrary set of target impedance and achievable dynamic behavior has a limiting performance. His stability condition in impedance control approaches uses the desired inertia matrix as the real inertia matrix, *i.e.*, $\boldsymbol{M}_{Ed} = \boldsymbol{\Lambda}_E(\boldsymbol{q})$. Although this choice of $\boldsymbol{M}_{Ed}$ gives an inertial coupling in Cartesian motion, it can reduce the computational burden[8] and it is necessary to satisfy the passivity condition[5]. In this case, however, the force feedback term disappears in conventional impedance controller form and it becomes an equivalent stiffness controller[5, 7, 9]. The force scaling factor α is inserted for this reason. As mentioned in [5, 9], for stable contact with environment, the force scaling factor α should be less than 1. With this choice of $\boldsymbol{M}_{Ed}$, we can restate the command force $\boldsymbol{f}_{Ec}$ as

$$\boldsymbol{f}_{Ec} = \boldsymbol{\Lambda}_E(\boldsymbol{q})\left\{\ddot{\boldsymbol{x}}_{Ed} - \boldsymbol{h}_E(\boldsymbol{q},\dot{\boldsymbol{q}})\right\} + \boldsymbol{B}_{Ed}\dot{\boldsymbol{e}}_E + \boldsymbol{K}_{Ed}\boldsymbol{e} + (1-\alpha)\boldsymbol{f}_E + \boldsymbol{J}_E^{-T}\boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}). \tag{23}$$

In real implementation, the joint torque vector can be partitioned into three terms as shown in below:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_P + \boldsymbol{\tau}_N + \boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}) \tag{24}$$

where $\boldsymbol{\tau}_P$ and $\boldsymbol{\tau}_N$ depend upon the weighting matrix $\boldsymbol{W}$.

4.1 Inertially Decoupled Impedance Controller (IDIC)

With $\boldsymbol{W} = \boldsymbol{H}(\boldsymbol{q})$, the weighted pseudoinverse gives the inertia-weighted pseudoinverse, $\boldsymbol{J}_W^+ = \boldsymbol{J}_H^+$. Since $\boldsymbol{J}_W^{+T}\boldsymbol{H}\boldsymbol{V} = \boldsymbol{0}$ in this case, the inertia matrix $\boldsymbol{\Lambda}_E$ has the following form:

$$\boldsymbol{\Lambda}_E(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{\Lambda}(\boldsymbol{q}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Lambda}_N(\boldsymbol{q}) \end{bmatrix}, \tag{25}$$

where $\boldsymbol{\Lambda}(\boldsymbol{q}) = \left(\boldsymbol{J}\boldsymbol{H}^{-1}\boldsymbol{J}^T\right)^{-1}$ and we define $\boldsymbol{\Lambda}_N \triangleq \boldsymbol{V}^T\boldsymbol{H}\boldsymbol{V}$ as the null-inertia matrix.

Since the above equation has a block diagonal form, we can consider the dynamics of each space as follows:

$$\boldsymbol{f}_c = \boldsymbol{\Lambda}(\boldsymbol{q})\left[\ddot{\boldsymbol{x}} - \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})\right] + \boldsymbol{J}_H^{+T}\boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{f} \tag{26}$$

$$\boldsymbol{f}_N = \boldsymbol{\Lambda}_N(\boldsymbol{q})\left[\ddot{\boldsymbol{x}}_N - \boldsymbol{h}_N(\boldsymbol{q},\dot{\boldsymbol{q}})\right] + \boldsymbol{V}^T\boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}), \tag{27}$$

where $\boldsymbol{h}_N(\boldsymbol{q},\dot{\boldsymbol{q}}) = \dot{\boldsymbol{J}}_N(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$. The first equation shows the conventional operational space dynamics[2], however, the second equation is a new expression of null dynamics based on the proposed minimal parametrization. Note that the external force $\boldsymbol{f}$ has no effect on the null motion. Although there is nonlinear coupling term, we call this type of controller as the inertially decoupled controller.

Based on this formulation, the impedance control approach can be applied to the above two equations. It is named by "inertially decoupled impedance controller". The input torque has the following form:

$$\boldsymbol{\tau}_P = \boldsymbol{J}^T(\boldsymbol{q})\left\{\boldsymbol{\Lambda}(\boldsymbol{q})[\ddot{\boldsymbol{x}}_d - \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})] + \boldsymbol{B}_d\dot{\boldsymbol{e}} + \boldsymbol{K}_d\boldsymbol{e}\right\} + (1-\alpha)\boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f} \tag{28}$$

$$\boldsymbol{\tau}_N = \boldsymbol{H}(\boldsymbol{q})\boldsymbol{V}(\boldsymbol{q})\left[\ddot{\boldsymbol{x}}_{Nd} + \beta_N\dot{\boldsymbol{e}}_N - \boldsymbol{h}_N(\boldsymbol{q},\dot{\boldsymbol{q}})\right] \tag{29}$$

with $\boldsymbol{B}_N = \beta_N\boldsymbol{\Lambda}_N(\boldsymbol{q})$.

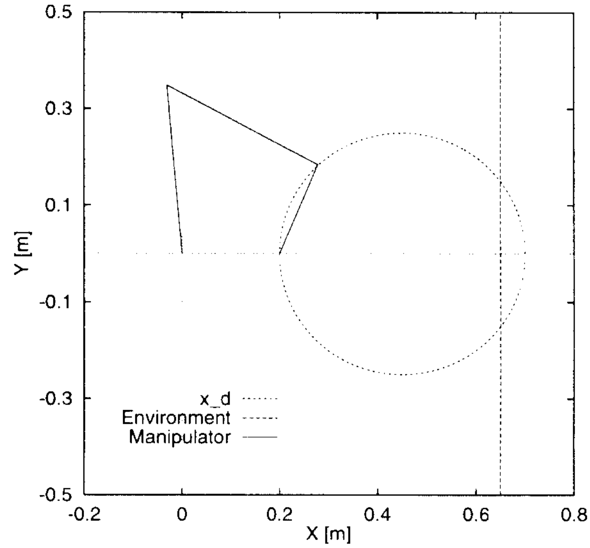


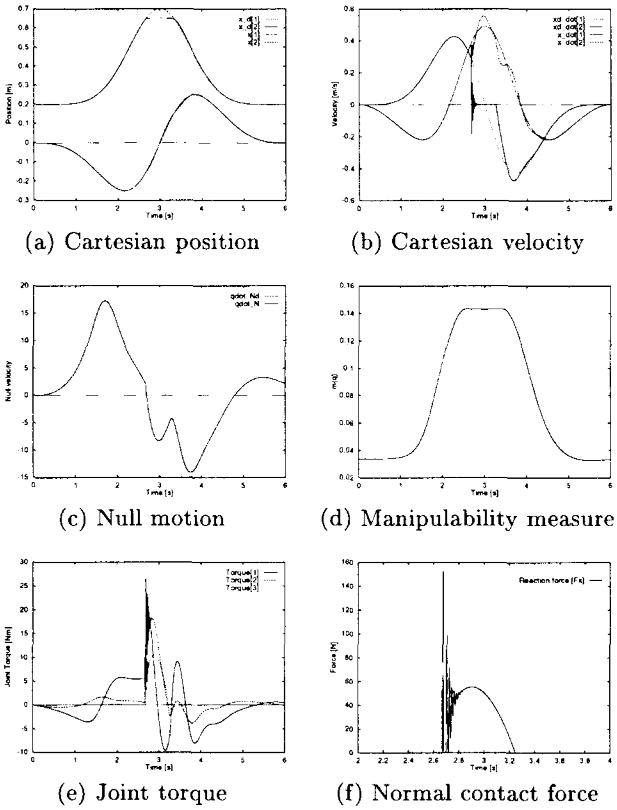
Fig. 2 Desired trajectory in simulation

5. SIMULATION

A three-link planar redundant manipulator is considered in simulation. The kinematic and dynamic parameters are computed from the CAD drawings of POSTECH DDArm-II which is now under construction. The sampling frequency is assumed as 250Hz and integration step is taken 100 times faster than sampling frequency to emulate the continuous system. We investigate the performance of the proposed control methods on the general class of task, *i.e.*, free motion and contact/constrained motion. The primary task of the manipulator is to follow the circular trajectory. Fig. 2 shows the desired Cartesian trajectory and the initial configuration of the manipulator. The desired trajectory is a circular motion which centered at (0.45, 0.0) with radius of 0.25m and it is planned by a fifth order polynomial of time. The total time of execution is given by 6s and the vertical wall which is located at $x = 0.65$m is considered as the environment. For simplicity, the friction force is not included and the linear contact force model is assumed, *i.e.*, $f_x = K_e\delta x$, where the contact stiffness $K_e = 200,000$N/m.

We use the following impedance parameters to control motion of the end-effector

$$\alpha = 0.8, \quad \boldsymbol{B}_d = 60\boldsymbol{I}_2 \quad \text{and} \quad \boldsymbol{K}_d = 900\boldsymbol{I}_2 \tag{30}$$



(a) Cartesian position (b) Cartesian velocity

(c) Null motion (d) Manipulability measure

(e) Joint torque (f) Normal contact force

Fig. 3 Impedance control performance of IDIC

and $\dot{\boldsymbol{x}}_{Nd}$ is given by $\dot{\boldsymbol{x}}_{Nd} = \boldsymbol{\Lambda}_N^{-1}\boldsymbol{V}^T\nabla m(\boldsymbol{q})$.

In addition to this primary task, a secondary task which is to optimize a scalar potential function is assigned. Though we can use the impact minimizing index[8], for convenience, the desired null motion trajectories are given by the manipulability measure as previous case. The initial Cartesian position of the manipulator is $\boldsymbol{x}(0) = (0.2,\ 0.0)$m and the initial joint configuration is chosen by integrating the following optimal condition

$$(\boldsymbol{I}_n - \boldsymbol{J}^+\boldsymbol{J})\,\nabla m(\boldsymbol{q}) = \dot{\boldsymbol{q}}_h$$

until $\dot{\boldsymbol{q}}_h$ becomes zero. The null motion control parameters are given by $\kappa = 100$ and $\beta_N = 20$.

Fig. 3 shows the simulation results. Fig. (a) and (b) show the Cartesian position and velocity profiles. From the figures, we can find very small Cartesian motion tracking error. In Fig. 3(c), the null motion tracking performance of IDIC is depicted. There is no abrupt changes of null motion profile due to the impact as mentioned before. And Fig (d) and (e) show the measure of manipulability and torque profile during the task execution. Finally, Fig. (f) represents the normal contact force.

6. CONCLUSION

To control the end-effector as well as the null motion, an extended task space formulation of kinematically redundant manipulators is considered in this work. Based on the weighted inner product in joint space, a minimal parametrization of the null space can be achieved. Augmenting this and kinematic equation, the extended task space formulation with explicit null motion dynamics is constructed. Expanding the conventional impedance control approach, the inertially decoupled impedance control method was proposed. The effectiveness of the proposed control approach was verified via numerical simulation.

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