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Decentralized H_∞ Controller Design - reduced order observers approach

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Abstract In this paper, we consider the decentralized reduced-order H_∞ controller for the general plant. Simplifying method is suggested for the general plant with the decentralized controller structure. When the controller is reconstructed for the original system, the decentralizability of the controller for the transformed system is generally destroyed with the older method. We solve this problem. For the simplified system, the structure of the decentralized controller is suggested.

Keywords H_∞ control, Decentralization, reduced order observer

1 Introduction

In this research, we consider the decentralized reduced-order H_∞ controller for the general plant like a boiler system. Individual system has a reduced order observer, and with these, estimates of states or worst case exogeneous inputs of a channel are obtained. Then we can design the controller minimizing the H_∞ norm of the transfer function matrix from exogeneous inputs to the controlled outputs.

The assumptions of the standard condition is modified for the decentralized controller design. General plant includes a direct feedthrough term from input to the output and controlled output has a direct feed through term from exogeneous input. Hence, method of simplifying to standard plant assumption is not usable to the decentralized controller structure. Reduced-order observers are used for the real attraction.

2 Preliminaries

Firstly, we consider the system having the D_{11} and D_{22} for the general plant.

$$P(s) \cong \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases} \tag{1}$$

We can rewrite above as a following matrix form :

$$\begin{bmatrix} \dot{x} \\ \hline z \\ y_1 \\ y_2 \end{bmatrix} = \left[\begin{array}{c|ccc} A & B_1 & B_2^1 & B_2^2 \\ \hline C_1 & D_{11} & D_{12} & D_{12} \\ C_2^1 & D_{21}^1 & D_{22}^1 & D_{22}^2 \\ C_2^1 & D_{21}^2 & D_{22}^3 & D_{22}^4 \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u_1 \\ u_2 \end{bmatrix} \tag{2}$$

We want to design the decentralized controller $K(s)$ for the system $P(s)$.

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} K_1(s) & 0 \\ 0 & K_2(s) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{3}$$

2.1 Assumptions

The above system must satisfy the following assumptions. Singular problem is not considered in this paper yet.
A1. There are no unstable decentralized fixed modes.
A2. D_{12} and D_{21} has full row/column rank.
A3. $(A \quad B_2 \quad C_2)$ is stabilizable and detectable.
A4. The system $(A,\ B_2,\ C_1,\ D_{12})$ and $(A,\ B_1,\ C_2,\ D_{21})$ have no invariant zeros in the $j\omega$-axis.

2.2 Simplifying Assumptions

For the simplicity, we can remove the terms of D_{11} and D_{22}. For this, general procedure for the decentralized controller design is suggested by modifying the general simplying procedure.

Step 1 : Decentralized norm minimization problem

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} \tag{4}$$

where $F = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}$. By applying above transformation, we can obtain following LFT :

$$\begin{bmatrix} \dot{x} \\ \hline z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} \bar{A} & \bar{B}_1 & \bar{B}_2 \\ \hline \bar{C}_1 & \bar{D}_{11} & \bar{D}_{12} \\ \bar{C}_2 & \bar{D}_{21} & \bar{D}_{22} \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u \end{bmatrix} \tag{5}$$

where

$$\begin{aligned}
\bar{A} &= A + B_2 F (I - D_{22} F)^{-1} C_2 \\
\bar{B}_1 &= B_1 + B_2 F (I - D_{22} F)^{-1} D_{21} \\
\bar{B}_2 &= B_2 (I - D_{22} F)^{-1} \\
\bar{C}_1 &= C_1 + D_{12} F (I - D_{22} F)^{-1} C_2 \\
\bar{C}_2 &= (I - D_{22} F)^{-1} C_2 \\
\bar{D}_{11} &= D_{11} + D_{12} F (I - D_{22} F)^{-1} D_{21} \\
\bar{D}_{12} &= D_{12} (I - D_{22} F)^{-1} \\
\bar{D}_{21} &= (I - D_{22} F)^{-1} D_{21} \\
\bar{D}_{22} &= (I - D_{22} F)^{-1} D_{22}
\end{aligned}$$

Step 2 : Making $D_{11} = 0$

In real situation, it is generally not satisfied by the frequency weighted function for the closed loop shaping.

Hence, this preocedure is needed for simplicity. Currently, some loop transformation is used, but it is not satisfactory for the system having decentralized structure. For the decentralized system, following transformation can only be used. If we let $\bar{D}_{11} = D_{11} + D_{12}F(I - D_{22}F)^{-1}D_{21}$, following transformation will remove $\bar{D}_{11}$

1. Find the decentralized form of matrix F such that $\min_F \|\bar{D}_{11}\| = \gamma_o$.

2. Choose $\gamma > \gamma_o$ and performing next transformation.

$$\begin{bmatrix} \tilde{z} \\ w \end{bmatrix} = \begin{bmatrix} \Theta & \Theta \\ \Theta & \Theta \end{bmatrix} \begin{bmatrix} \tilde{w} \\ z \end{bmatrix} = \gamma^{-1} \begin{bmatrix} \gamma^{-1}\bar{D}_{11} & (I - \gamma^{-2}\bar{D}_{11}\bar{D}_{11}^*) \\ (I - \gamma^{-1}\bar{D}_{11}^*\bar{D}_{11} & \gamma^{-1}\bar{D}_{11}^* \end{bmatrix} \begin{bmatrix} \tilde{w} \\ z \end{bmatrix} \quad (6)$$

where $\Theta\Theta^* = \gamma^{-2}I$, $\|\Theta_{22}\|_2 < \gamma^{-2}$, $\forall\gamma > \gamma_o$.

By applying above transformation, we can obtain following LFT :

$$\begin{bmatrix} \dot{x} \\ \tilde{z} \\ y \end{bmatrix} = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_1 & 0 & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \hat{D}_{22} \end{array}\right] \begin{bmatrix} x \\ \tilde{w} \\ \tilde{u} \end{bmatrix} \quad (7)$$

where

$$\begin{aligned}
\hat{A} &= \bar{A} + \bar{B}_1(I - \bar{D}_{11}\Theta_{22})^{-1}\bar{C}_1 \\
\hat{B}_1 &= \bar{B}_1(I - \Theta_{22}\bar{D}_{11})^{-1}\Theta_{21} \\
\tilde{B}_2 &= \bar{B}_2 + \bar{B}_1\Theta_{22}(I - \Theta_{22}\bar{D}_{11})^{-1}\bar{D}_{12} \\
\hat{C}_1 &= \Theta_{12}(I - \Theta_{22}\bar{D}_{11})^{-1}\bar{C}_1 \\
\tilde{C}_2 &= \bar{C}_2 + \bar{D}_{21}(I - \Theta_{22}\bar{D}_{11})^{-1}\bar{C}_1 \\
\tilde{D}_{12} &= \Theta_{12}(I - \Theta_{22}\bar{D}_{11})^{-1}\bar{D}_{12} \\
\tilde{D}_{21} &= \bar{D}_{21}(I - \Theta_{22}\bar{D}_{11})^{-1}\Theta_{21} \\
\hat{D}_{22} &= \bar{D}_{22} + \bar{D}_{21}\Theta_{22}(I - \Theta_{22}\bar{D}_{11})^{-1}\bar{D}_{12}
\end{aligned}$$

Step 3 : Making $D_{22} = 0$

Some feedforward term is added after designing controller. Several consideration is needed.

$$\begin{bmatrix} \dot{x} \\ \tilde{z} \\ y \end{bmatrix} = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hline \hat{C}_1 & 0 & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \hat{D}_{22} \end{array}\right] \begin{bmatrix} x \\ \tilde{w} \\ \tilde{u} \end{bmatrix} \quad (8)$$

We apply following transformation.

$$\begin{aligned} \tilde{y} &= y - \hat{D}_{22}\tilde{u} \\ &= \tilde{C}_2 x + \tilde{D}_{21}\tilde{w} \end{aligned} \quad (9)$$

Then the resulting tranfer system is as this :

$$\begin{bmatrix} \dot{x} \\ \tilde{z} \\ \tilde{y} \end{bmatrix} = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hline \hat{C}_1 & 0 & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & 0 \end{array}\right] \begin{bmatrix} x \\ \tilde{w} \\ \tilde{u} \end{bmatrix} \quad (10)$$

Now, our objective is complete. But one problem remains still unsolved. If you would like to regain the controller for your original plant, other station's inputs are needed in subsystems by the terms $\hat{D}_{22}^2$ and $\hat{D}_{22}^3$ i.e., antidiagonal terms of $\hat{D}_{22}$ on above setting. Hence, following approximation is needed for decentralizing controllers.

$$\begin{aligned} \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} &= \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} \hat{D}_{22}^1 & \hat{D}_{22}^2 \\ \hat{D}_{22}^3 & \hat{D}_{22}^4 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} \\ &\approx \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} \hat{D}_{22}^1 & 0 \\ 0 & \hat{D}_{22}^4 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} - \begin{bmatrix} 0 & \hat{D}_{22}^2 \\ \hat{D}_{22}^3 & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}_1^e \\ \tilde{u}_2^e \end{bmatrix} \end{aligned} \quad (11)$$

Here, $\tilde{u}_i^e$ is the estimate of the other system's input. It can be estimated by some state estimates and replacement. Reduced-order observers gives some estimates of the system state, and from this, we can estimate the other station's inputs. Control strtrategy which is used by full state feedback system can be used, and make a good result. These method is used in the reference [1]. These method is good for the sytems without exchanging information each other. Hence, the assumption $\hat{D}_{22} = 0$ is generalized.

We can construct the simplified procedure for the general system with the above assumption without loss of generality. After now on, we only consider the decentralized reduced order H_∞ controller for the simplified system. Lemmas of the following section will justify our transformations.

2.3 Reconstruction Lemma

Now, we must show that the loop transformation does not destroy the representation of the decentralized controllers and that the order of the designed H_∞ controller is not higher than before transformation. Note that the structure of the controller as Figure 1. Let the state space realizations as this :

$$\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2^e \end{bmatrix} = \tilde{K}_1(s)\tilde{y}_1, \quad \begin{bmatrix} \tilde{u}_2 \\ \tilde{u}_1^e \end{bmatrix} = \tilde{K}_2(s)\tilde{y}_2 \quad (12)$$

where

$$\tilde{K}_1(s) = \left[\begin{array}{c|c} \tilde{A}_{k1} & \tilde{B}_{k1} \\ \hline \tilde{C}_{k1} & \tilde{D}_{k1} \\ \tilde{C}_{k2}^e & \tilde{D}_{k2}^e \end{array}\right], \quad \tilde{K}_2(s) = \left[\begin{array}{c|c} \tilde{A}_{k2} & \tilde{B}_{k2} \\ \hline \tilde{C}_{k2} & \tilde{D}_{k2} \\ \tilde{C}_{k1}^e & \tilde{D}_{k1}^e \end{array}\right] \quad (13)$$

Let the realization for the controller $K_1(s)$ and $K_2(s)$ as this :

$$u_1 = K_1(s)y_1, \quad u_2 = K_2(s)y_2 \quad (14)$$

where

$$K_1(s) = \left[\begin{array}{c|c} A_{k1} & B_{k1} \\ \hline C_{k1} & D_{k1} \end{array}\right], \quad K_2(s) = \left[\begin{array}{c|c} A_{k2} & B_{k2} \\ \hline C_{k2} & D_{k2} \end{array}\right] \quad (15)$$

With above and equation (3) and (10), we can conclude as this lemma.

Lemma 1 (Reconstruction) *If we can design the decentralized controller $\tilde{K}(s)$ for the systems for $\tilde{P}(s)$, then we can reconstruct the controller $K(s)$ for the system $P(s)$ having no higher oreder realization.*

(Proof)

If we can design the controller $\tilde{K}_1(s)$ and $\tilde{K}_2(s)$, we can obtain the $K_1(s)$ and $K_2(s)$ by the manipulation of the matrix equations. Let the equations as this :

$$\begin{aligned}
\tilde{u}_1 &= A_1(s)\tilde{y}_1 \\
\tilde{u}_2^e &= A_2(s)\tilde{y}_1 \\
\tilde{u}_1 &= u_1 - F_1 y_1 \\
\tilde{y}_1 &= y_1 - [\,\hat{D}_{22}^1 \;\; \hat{D}_{22}^2\,] \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2^e \end{bmatrix}
\end{aligned} \quad (16)$$

$$\begin{aligned}\tilde{u}_1^e &= A_3(s)\tilde{y}_2\\ \tilde{u}_2 &= A_4(s)\tilde{y}_2\\ \tilde{u}_2 &= u_2 - F_2 y_2\\ \tilde{y}_2 &= y_2 - [\,\hat{D}_{22}^3 \quad \hat{D}_{22}^4\,]\begin{bmatrix}\tilde{u}_1^e\\ \tilde{u}_2\end{bmatrix}\end{aligned} \tag{17}$$

If we preform the following operation and manipulation, we can obtain the controller realization for the system $P(s)$.

$$\begin{aligned}\tilde{y}_1 &= y_1 - \hat{D}_{22}^2\tilde{u}_1 - \hat{D}_{22}^2 A_2(s)\tilde{u}_1\\ \tilde{y}_1 &= (I+\hat{D}_{22}^2 A_2(s))^{-1}(y_1 - \hat{D}_{22}^1\tilde{u}_1)\\ \tilde{u}_1 &= A_2(s)\tilde{y}_1\\ &= A_1(s)(I+\hat{D}_{22}^2 A_2(s))^{-1}(y_1 - \hat{D}_{22}^1\tilde{u}_1)\\ \tilde{u}_1 &= \left[I + A_1(s)(I+\hat{D}_{22}^2 A_2(s))^{-1}D_{22}^1\right]^{-1}\\ &\quad \left[A_1(s)(I+\hat{D}_{22}^2 A_2(s))^{-1}\right] y_1\end{aligned} \tag{18}$$

Hence, we can obtain following results :

$$u_1 = \left[F_1 + \left\{I + A_1(s)(I+\hat{D}_{22}^2 A_2(s))^{-1}\hat{D}_{22}^1\right\}^{-1} \left\{A_1(s)(I+\hat{D}_{22}^2 A_2(s))^{-1}\right\}\right] y_1 \tag{19}$$

Similarly, for the $K_2(s)$, we can obtain

$$u_2 = \left[F_2 + \left\{I + A_3(s)(I+\hat{D}_{22}^3 A_4(s))^{-1}\hat{D}_{22}^4\right\}^{-1} \left\{A_3(s)(I+\hat{D}_{22}^4 A_4(s))^{-1}\right\}\right] y_2 \tag{20}$$

We can reconstruct the controllers as $K_i(s)$ having same order realization with $\tilde{K}_i(s)$ from the above equations. Q.E.D

3 Main Results

Now after, we assume that D_{11} and D_{22} equal zero without loss of generality.

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u\\ z &= C_1 x + D_{12} u\\ y &= C_2 x + D_{21} w\end{aligned} \tag{21}$$

We can rewrite above as a following matrix form :

$$\begin{bmatrix}\dot{x}\\ \hline z\\ y_1\\ y_2\end{bmatrix} = \left[\begin{array}{c|ccc} A & B_1 & B_2^1 & B_2^2\\ \hline C_1 & 0 & D_{12}^1 & D_{12}^2\\ C_2^1 & D_{21}^1 & 0 & 0\\ C_2^2 & D_{22}^2 & 0 & 0\end{array}\right]\begin{bmatrix}x\\ \hline w\\ u_1\\ u_2\end{bmatrix} \tag{22}$$

For above system, we would like to design the controller having fixed structure :

$$u = \begin{bmatrix}u_1\\ u_2\end{bmatrix} = \begin{bmatrix}K_1(s) & 0\\ 0 & K_2(s)\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} \tag{23}$$

We can write above equation to following forms :

$$K_1(s) \equiv \begin{cases}\dot{\xi}_1 = F_1\xi_1 + G_1 y_1 + [\,H_{11} \quad H_{12}\,]\begin{bmatrix}u_1\\ u_2^e\end{bmatrix}\\ u_1 = L_1\xi_1 + M_1 y_1\\ u_2^e = L_2^e\xi_1 + M_2^e y_1\end{cases} \tag{24}$$

$$K_2(s) \equiv \begin{cases}\dot{\xi}_2 = F_1\xi_2 + G_2 y_2 + [\,H_{21} \quad H_{22}\,]\begin{bmatrix}u_1^e\\ u_2\end{bmatrix}\\ u_1^e = L_1^e\xi_2 + M_1^e y_2\\ u_2 = L_2\xi_2 + M_2 y_2\end{cases} \tag{25}$$

3.1 Full Information Result

If we have the full state information, we can design the H_∞ state feedback controller by following lemma.

Lemma 2 (FI) *For the above system, if we know full information of states x and exogeneous inputs w, followings are equivalent.*

(1) There exists a $P \geq 0$ such that following Riccati equation is satisfied and $A + \gamma^{-2}B_1B_1^TP - B_2(D_{12}^TD_{12})^{-1}(B_2^TP + D_{12}^TC_1)$ is stable.

$$A^TP + PA + C_1^TC_1 + \gamma^{-2}PB_1B_1^TP - (PB_2 + C_1^TD_{12})(D_{12}^TD_{12})(B_2^TP + D_{12}^TC_1) = 0 \tag{26}$$

(2) $||T_zw(s)|| < \gamma$
where $u^* = -\bar{K}x$, $\quad w^* = \gamma^{-2}B_1^TPx$.

Above lemma follows by bounded real lemma. Now, we will design the reduced-order state observer with above state feedback input assumption. By inserting w^* to the system equations, we obtains

$$\begin{aligned}\dot{x} &= Ax + B_2 u\\ &= Ax + [\,B_2^1 \quad B_2^2\,]\begin{bmatrix}u_1\\ u_2\end{bmatrix}\\ \begin{bmatrix}y_1\\ y_2\end{bmatrix} &= \bar{C}_2 x = \begin{bmatrix}\bar{C}_2^1\\ \bar{C}_2^2\end{bmatrix} x\end{aligned} \tag{27}$$

where $\bar{A} = A + \gamma^{-2}B_1B_1^TP$ and $\bar{C}_2 = C_2 + \gamma^{-2}D_{21}B_1^TB_1P$.

Applying the theory of the observer,

$$\begin{bmatrix}e_1\\ e_2\end{bmatrix} = \begin{bmatrix}x_{k1}\\ x_{k2}\end{bmatrix} - \begin{bmatrix}V_1\\ V_2\end{bmatrix} x \tag{28}$$

Here, V_i is the mapping from states to the reduced-order states space. We can construct the following equations by solving and inserting the related equations,

$$\begin{bmatrix}L_1 & 0\\ 0 & L_2\end{bmatrix}\begin{bmatrix}\bar{C}_2^1\\ \bar{C}_2^2\end{bmatrix} + \begin{bmatrix}M_1 & 0\\ 0 & M_2\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix} = \begin{bmatrix}\bar{K}_1\\ \bar{K}_2\end{bmatrix} \tag{29}$$

$$\begin{bmatrix}L_2^e & 0\\ 0 & L_1^e\end{bmatrix}\begin{bmatrix}\bar{C}_2^1\\ \bar{C}_2^2\end{bmatrix} + \begin{bmatrix}M_2^e & 0\\ 0 & M_1^e\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix} = \begin{bmatrix}\bar{K}_2\\ \bar{K}_1\end{bmatrix} \tag{30}$$

Then, we can obtain the approximate inputs for the original plant,

$$\begin{aligned}u &= \begin{bmatrix}u_1\\ u_2\end{bmatrix}\\ &= \begin{bmatrix}M_1 & 0\\ 0 & M_2\end{bmatrix}\left(\begin{bmatrix}e_1\\ e_2\end{bmatrix} + \begin{bmatrix}V_1\\ V_2\end{bmatrix}x\right) + \begin{bmatrix}L_1 & 0\\ 0 & L_2\end{bmatrix}\begin{bmatrix}\bar{C}_2^1\\ \bar{C}_2^2\end{bmatrix}\\ &= \begin{bmatrix}M_1 & 0\\ 0 & M_2\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix} + \begin{bmatrix}\bar{K}_1\\ \bar{K}_2\end{bmatrix}x\end{aligned} \tag{31}$$

$$\begin{aligned}u^e &= \begin{bmatrix}u_1^e\\ u_2^e\end{bmatrix}\\ &= \begin{bmatrix}M_1^e & 0\\ 0 & M_2^e\end{bmatrix}\left(\begin{bmatrix}e_2\\ e_1\end{bmatrix} + \begin{bmatrix}V_2\\ V_1\end{bmatrix}x\right) + \begin{bmatrix}L_1^e & 0\\ 0 & L_2^e\end{bmatrix}\begin{bmatrix}\bar{C}_2^2\\ \bar{C}_2^1\end{bmatrix}\\ &= \begin{bmatrix}M_1^e & 0\\ 0 & M_2^e\end{bmatrix}\begin{bmatrix}e_2\\ e_1\end{bmatrix} + \begin{bmatrix}\bar{K}_1\\ \bar{K}_2\end{bmatrix}x\end{aligned} \tag{32}$$

Remark 3 *We need the conditions that*
(1) error signals $e_i(t) \to 0$ *as* $t \to \infty$ *, for i =1,2*
(2) overall closed loop system must satisfy the needed H_∞ *norm bound.*

The remained problem is the construction of reduced-order observers. By [3] and above equations, we will try to connect the decentralization and reduced-order observer design problem. Bounded real lemma will help this links.

4 Conclusion

Our works are still constructing. We show that the simplifying method for the controller having fixed structure can not be applied to design the controller with the fixed structures. Some other simplifying method is needed and we provide it. Hence, the problem in the decentralized controller design was made easy to deal with. Reduced-order consroller design problem can be solved by manipulating the related equations.

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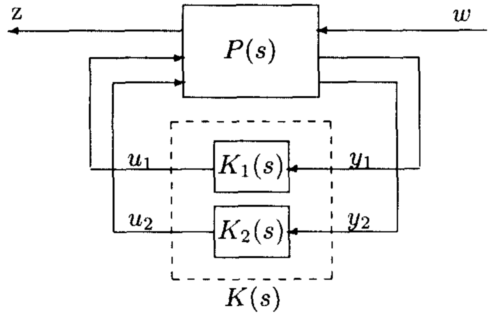

Figure 1: Design Structure

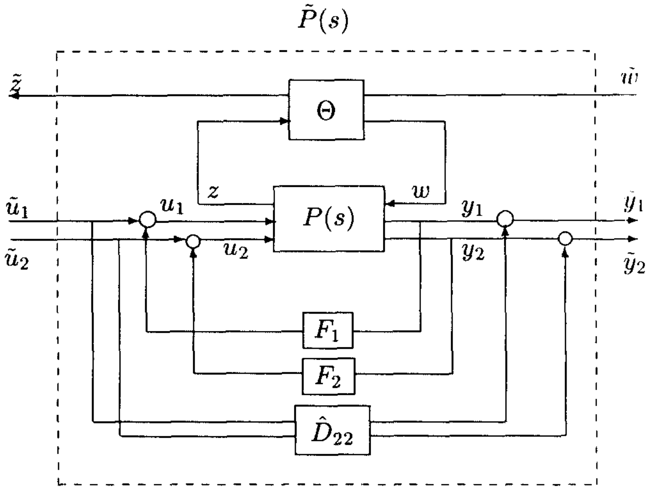

Figure 2: Loop Transformation

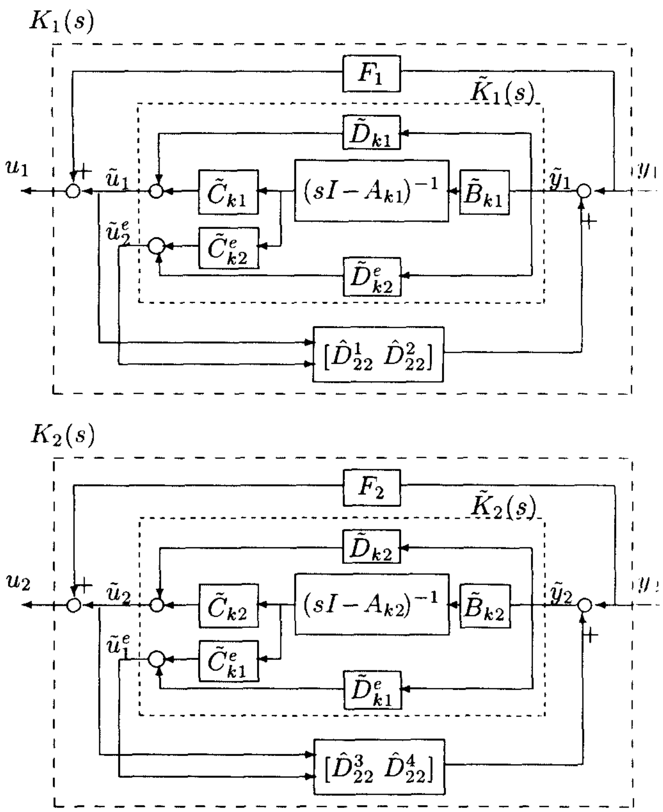

Figure 3: Reconstruction of Decentralized Controller for $K(s)$ with $\tilde{K}(s)$