DESIGN OF AN H_∞ CONTROLLER WITH PREVIEW

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Abstracts An optimal preview controller based on the discrete-time H_∞ control is presented. The preview controller is synthesized by considering the bounded unknown disturbances as well as previewable commands and disturbances. The controller derivation procedure is analogous to the LQ-based scheme. The designed preview gain matrix has a similar structure as the LQ-based one. As the infinity norm γ of the transfer function matrix tends to ∞, the preview gains obtained by H_∞ control method approach to the gains by the LQR. The LQ-based preview gains are verified to be subsets of the H_∞-based preview gains.

Keywords H_∞, preview, disturbances

1. INTRODUCTION

Most of control systems are designed in the form of feedback structure depending on the error signals in the current instant without considering future information, such as tracking commands and measurable disturbances into the systems. But if we know the future input informations and these are fully reflected on a control law, it is anticipated that the resulting control system may have better performances. Thus, several researches have been performed on the preview control since 1960's, but most of them only illustrate its importance with simplified examples.

Tomizuka[1] presented the general optimal preview controller taking account of measurement noises as well as stochastic informations. He established the seminal researches on the region of the preview control through the successive contributions.

This paper presents a design method of the preview controller based on discrete-time state feedback H_∞ control theory for the case where some plant disturbances are measured and/or even previewable and others are unmeasurable. The preview and feedback controller are designed to minimize the worst case RMS value of the regulated variables when the bounded unknown disturbances and the previewable disturbances hit the dynamical plants. Namely, both the preview and feedback controller are simultaneously optimized so that the infinity norm of the transfer function matrix from both the previewable and unmeasurable disturbances to the regulated variables is minimized. The full-information(FI) H_∞controller for the continuous-time case uses only the state signal $\boldsymbol{x}(t)$, if $\boldsymbol{D}_{11} = \boldsymbol{0}$ in the generalized plant. But the discrete-time FI H_∞ controller uses both $x(k)$ and $w(k)$ even in the case $\boldsymbol{D}_{11} = \boldsymbol{0}$[1]. So, a state feedback controller and the related preview controller are derived in this design , even though problem formulation and solving an algebraic Riccati equation are based on the FI controller design scheme. The controller derivation procedure is analogous to the LQ-based scheme, but is more complicated because the discrete algebraic Riccati equation for the full-information H_∞ controller takes a sophisticated form. The designed preview gain matrix has a similar structure as the LQ-based one. The gain matrix is sensitively determined by the infinity norm of the transfer function from the disturbance to the regulated output.

2. CONTROLLER DESIGN

A stabilizing central full-state feedback controller $\boldsymbol{K}$ that achieves the objective $\|\mathcal{F}_l(\boldsymbol{G},\boldsymbol{K})\|_\infty < \gamma$ for a plant $\boldsymbol{G}$ is given by[3]

$$\boldsymbol{K} = -(\boldsymbol{R}_3 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{R}_2')^{-1}(\boldsymbol{L}_2 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{L}_1). \quad (1)$$

An H_∞ controller with preview compensation will be synthesized in the analogous method to Tomizuka in this section. Consider a discrete-time linear time-invariant dynamical system with a previewable disturbance and an unknown bounded disturbance. The state-space equations are given by

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi x}(k) + \boldsymbol{\Gamma}_1\boldsymbol{w}(k) + \boldsymbol{\Gamma}_2\boldsymbol{u}(k) + \boldsymbol{\Gamma}_d\boldsymbol{v}(k) \quad (2)$$

$$\boldsymbol{y}(k) = \boldsymbol{Cx}(k) \quad (3)$$

where $\boldsymbol{v}(k)$ and $\boldsymbol{w}(k)$ are the q-dimensional previewable disturbance and the l-dimensional unmeasurable disturbance, respectively. It is assumed that N time step preview measurement of $\boldsymbol{v}$ is possible, which means that $\boldsymbol{v}(k), \boldsymbol{v}(k+1), \cdots, \boldsymbol{v}(k+N-1)$ are known in advance. It is supposed that future values of $\boldsymbol{v}$ beyond the time $k+N$ are the same as that of the $\boldsymbol{v}(k+N-1)$, since the effects of the previewable disturbances have time correlations. Actually, the disturbances in the distant future hardly have effects on the system at the current time. A sequence of these known signals can make up measurable states of a dynamic system without the loss of generality.

Expressing a sequence of previewable disturbances as states, these can be expressed as

$$\boldsymbol{x}_d(k+1) = \boldsymbol{\Phi}_d \boldsymbol{x}_d(k) + \boldsymbol{E}_d \boldsymbol{v}(k+N) \tag{4}$$

where

$$\boldsymbol{x}_d(k) = \begin{bmatrix} \boldsymbol{v}(k) \\ \boldsymbol{v}(k+1) \\ \vdots \\ \boldsymbol{v}(k+N-1) \end{bmatrix}, \boldsymbol{\Phi}_d = \begin{bmatrix} \boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{I} & \cdots & \boldsymbol{0} \\ \vdots\ \vdots & \cdots\cdots & \boldsymbol{I} \\ \boldsymbol{0}\ \boldsymbol{0} \cdots & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{5}$$

and

$$\boldsymbol{E}_d = \begin{bmatrix} \boldsymbol{0} \cdots \boldsymbol{0}\ \boldsymbol{I} \end{bmatrix}'. \tag{6}$$

The augmented state-space equation including the previewable disturbances expressed as Eq.(4) is given by

$$\begin{bmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{x}_d(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}\ \boldsymbol{\Gamma}_{pd} \\ \boldsymbol{0}\ \boldsymbol{\Phi}_d \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}_d(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma}_1 \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{w}(k) + \begin{bmatrix} \boldsymbol{\Gamma}_2 \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{u}(k) + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{E}_d \end{bmatrix} \boldsymbol{v}(k+N) \tag{7}$$

where $\boldsymbol{\Gamma}_{pd} = \begin{bmatrix} \boldsymbol{\Gamma}_d\ \boldsymbol{0} \cdots \boldsymbol{0} \end{bmatrix}$.

Then, the state-space equations are rewritten as

$$\begin{aligned} \bar{\boldsymbol{x}}(k+1) &= \bar{\boldsymbol{\Phi}}\bar{\boldsymbol{x}}(k) + \bar{\boldsymbol{\Gamma}}_1 \boldsymbol{w}(k) + \bar{\boldsymbol{\Gamma}}_2 \boldsymbol{u}(k) + \bar{\boldsymbol{E}}_d \boldsymbol{v}(k+N) \\ \boldsymbol{y}(k) &= \bar{\boldsymbol{C}}\bar{\boldsymbol{x}}(k). \end{aligned} \tag{8}$$

The transfer function matrix for the augmented system is given as:

$$\boldsymbol{G}_a = \left[\begin{array}{c|cc} \bar{\boldsymbol{\Phi}} & \bar{\boldsymbol{\Gamma}}_1 & \bar{\boldsymbol{\Gamma}}_2 \\ \hline \bar{\boldsymbol{C}} & \boldsymbol{0} & \boldsymbol{0} \end{array}\right]$$

where $\boldsymbol{v}(k+N)$ is not included because it is the distant future signal which has little effects on the system at the current time.

The FI control problem for the above descriptions will now be cast into a two-port framework. The objective is to keep the plant output small despite unknown bounded external disturbances acting on the system. The control signal is also included in the objective signal $\boldsymbol{z}$ to prevent actuator saturation. This will insure that the rank condition on $\boldsymbol{D}_{12}$ in the assumptions of the H_∞ control problems should be satisfied. The $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are weighting matrices for the regulated output and the control input, respectively. The $\boldsymbol{W}_1$ is positive semi-definite and the $\boldsymbol{W}_2$ is positive definite. These also bound the output energy. Obtaining the regulated outputs whose average RMS power is to be minimized in the formulation,

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{W}_1^{1/2} \boldsymbol{G}_a \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{u} \end{bmatrix} \\ \boldsymbol{W}_2^{1/2} \boldsymbol{u} \end{bmatrix} \tag{9}$$

It is assumed that $\boldsymbol{W}_1^{1/2} = (\boldsymbol{W}_1^{1/2})'$ and $\boldsymbol{W}_2^{1/2} = (\boldsymbol{W}_2^{1/2})'$. The relevant state-space systems for designing an H_∞ controller are written as

$$\begin{bmatrix} \boldsymbol{A}_s\ \boldsymbol{B}_s \\ \boldsymbol{C}_s\ \boldsymbol{D}_s \end{bmatrix} = \left[\begin{array}{c|cc} \bar{\boldsymbol{\Phi}} & \bar{\boldsymbol{\Gamma}}_1 & \bar{\boldsymbol{\Gamma}}_2 \\ \hline \boldsymbol{W}_1^{1/2}\bar{\boldsymbol{C}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{W}_2^{1/2} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \end{array}\right]. \tag{10}$$

The algebraic Riccati equation(ARE) for a discrete-time FI H_∞ controller is

$$\boldsymbol{X}_\infty = \boldsymbol{C}_s'\boldsymbol{J}\boldsymbol{C}_s + \boldsymbol{A}_s'\boldsymbol{X}_\infty\boldsymbol{A}_s - \boldsymbol{L}'\boldsymbol{R}^{-1}\boldsymbol{L}. \tag{11}$$

Calculating the terms composing of the ARE,

$$\boldsymbol{R} = \boldsymbol{D}_s'\boldsymbol{J}\boldsymbol{D}_s + \boldsymbol{B}_s'\boldsymbol{X}_\infty\boldsymbol{B}_s \tag{12}$$

$$\boldsymbol{L} = \boldsymbol{B}_s'\boldsymbol{X}_\infty\boldsymbol{A}_s = \begin{bmatrix} \bar{\boldsymbol{\Gamma}}_1' \\ \bar{\boldsymbol{\Gamma}}_2' \end{bmatrix} \boldsymbol{X}_\infty \bar{\boldsymbol{\Phi}} = \begin{bmatrix} \bar{\boldsymbol{\Gamma}}_1'\boldsymbol{X}_\infty\bar{\boldsymbol{\Phi}} \\ \bar{\boldsymbol{\Gamma}}_2'\boldsymbol{X}_\infty\bar{\boldsymbol{\Phi}} \end{bmatrix}, \tag{13}$$

$$\boldsymbol{C}_s'\boldsymbol{J}\boldsymbol{C}_s = \bar{\boldsymbol{C}}'\boldsymbol{W}_1\bar{\boldsymbol{C}}. \tag{14}$$

Applying the above results to the ARE,

$$\begin{aligned} \boldsymbol{X}_\infty = \boldsymbol{A}_s'[\boldsymbol{X}_\infty - \boldsymbol{X}_\infty'\boldsymbol{B}_s\boldsymbol{R}^{-1}\boldsymbol{B}_s'\boldsymbol{X}_\infty]\boldsymbol{A}_s \\ + \bar{\boldsymbol{C}}'\boldsymbol{W}_1\bar{\boldsymbol{C}}. \end{aligned} \tag{15}$$

The ARE is often rewritten as

$$\boldsymbol{X}_\infty = \boldsymbol{A}_s'\boldsymbol{M}_\infty\boldsymbol{A}_s + \bar{\boldsymbol{C}}'\boldsymbol{W}_1\bar{\boldsymbol{C}} \tag{16}$$

where

$$\boldsymbol{M}_\infty = \boldsymbol{X}_\infty - \boldsymbol{X}_\infty'\boldsymbol{B}_s\boldsymbol{R}^{-1}\boldsymbol{B}_s'\boldsymbol{X}_\infty. \tag{17}$$

Partitioning the matrices $\boldsymbol{X}_\infty$ and $\boldsymbol{M}_\infty$ conformably with $\boldsymbol{x}(k)$ and $\boldsymbol{x}_d(k)$,

$$\boldsymbol{X}_\infty = \begin{bmatrix} \boldsymbol{X}_{xx}\ \boldsymbol{X}_{xd} \\ \boldsymbol{X}_{xd}'\ \boldsymbol{X}_{dd} \end{bmatrix}, \text{ and } \boldsymbol{M}_\infty = \begin{bmatrix} \boldsymbol{M}_{xx}\ \boldsymbol{M}_{xd} \\ \boldsymbol{M}_{xd}'\ \boldsymbol{M}_{dd} \end{bmatrix}.$$

With the partitioned forms of the $\boldsymbol{X}_\infty$ and $\boldsymbol{M}_\infty$,

$$\begin{bmatrix} \boldsymbol{X}_{xx}\ \boldsymbol{X}_{xd} \\ \boldsymbol{X}_{xd}'\ \boldsymbol{X}_{dd} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}' & \boldsymbol{0} \\ \boldsymbol{\Gamma}_{pd}' & \boldsymbol{\Phi}_d' \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_{xx}\ \boldsymbol{M}_{xd} \\ \boldsymbol{M}_{xd}'\ \boldsymbol{M}_{dd} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Phi}\ \boldsymbol{\Gamma}_{pd} \\ \boldsymbol{0}\ \boldsymbol{\Phi}_d \end{bmatrix} + \begin{bmatrix} \boldsymbol{C}'\boldsymbol{W}_1\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{18}$$

where $\boldsymbol{M}_p = \boldsymbol{\Gamma}_{pd}'\boldsymbol{M}_{xx} + \boldsymbol{\Phi}_d'\boldsymbol{M}_{xd} + (\boldsymbol{\Gamma}_{pd}'\boldsymbol{M}_{xd} + \boldsymbol{\Phi}_d'\boldsymbol{M}_{dd})\boldsymbol{\Phi}_d$.
From (18), it results that

$$\boldsymbol{X}_{xx} = \boldsymbol{\Phi}'\boldsymbol{M}_{xx}\boldsymbol{\Phi} + \boldsymbol{C}'\boldsymbol{W}_1\boldsymbol{C}, \tag{19}$$

$$\boldsymbol{X}_{xd} = \boldsymbol{\Phi}'\boldsymbol{M}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{\Phi}'\boldsymbol{M}_{xd}\boldsymbol{\Phi}_d. \tag{20}$$

Define a new matrix described by:

$$\begin{bmatrix} \bar{\boldsymbol{X}}_1 \\ \bar{\boldsymbol{X}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_1\ \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_2 \\ \boldsymbol{X}_{xd}'\boldsymbol{\Gamma}_1\ \boldsymbol{X}_{xd}'\boldsymbol{\Gamma}_2 \end{bmatrix}. \tag{21}$$

Then, the matrix M_∞ is given as a more concise form.

$$\begin{bmatrix} M_{xx} & M_{xd} \\ M'_{xd} & M_{dd} \end{bmatrix} = \begin{bmatrix} X_{xx} & X_{xd} \\ X'_{xd} & X_{dd} \end{bmatrix} - \begin{bmatrix} \bar{X}_1 R^{-1} \bar{X}_1{}' & \bar{X}_1 R^{-1} \bar{X}_2{}' \\ \bar{X}_2 R^{-1} \bar{X}_1{}' & \bar{X}_2 R^{-1} \bar{X}_2{}' \end{bmatrix}. \quad (22)$$

There exist the matrices M_{xx} and M_{xd} of the matrix M_∞ such that

$$M_{xx} = X_{xx} - \bar{X}_1 R^{-1} \bar{X}_1{}'$$
$$M_{xd} = X_{xd} - \bar{X}_1 R^{-1} \bar{X}_2{}'. \quad (23)$$

Proposition 1: A closed-loop system matrix $\Phi_{c_{xx}}$ of the H_∞ control system for the states $x(k)$ excluding $x_d(k)$ is

$$\Phi_{c_{xx}} = (I - \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix} R^{-1} \begin{bmatrix} \Gamma'_1 \\ \Gamma'_2 \end{bmatrix} X_{xx})\Phi. \quad (24)$$

Proof: The closed-loop system matrix Φ_c of the FI H_∞ control system is given as $A_s - B_s R^{-1} L$. Rearranging Φ_c,

$$\Phi_c = \begin{bmatrix} \Phi - \bar{X}_3 R^{-1} \bar{X}_4 & \Gamma_{pd} - \bar{X}_3 R^{-1} \bar{X}_5 \\ 0 & \Phi_d \end{bmatrix} \quad (25)$$

where

$$\bar{X}_3 = [\Gamma_1 \quad \Gamma_2], \bar{X}_4 = \begin{bmatrix} \Gamma'_1 X_{xx} \Phi \\ \Gamma'_2 X_{xx} \Phi \end{bmatrix},$$
$$\bar{X}_5 = \begin{bmatrix} \Gamma'_1 (X_{xx} \Gamma_{pd} + X_{xd} \Phi_d) \\ \Gamma'_2 (X_{xx} \Gamma_{pd} + X_{xd} \Phi_d) \end{bmatrix}.$$

Since Φ_c is a block matrix, the characteristic polynomial of Φ_c is just the product of the characteristic polynomial of $\Phi - \bar{X}_3 R^{-1} \bar{X}_4$ and the characteristic polynomial of the Φ_d. Therefore, $\Phi - \bar{X}_3 R^{-1} \bar{X}_4$ is a closed-loop system matrix $\Phi_{c_{xx}}$ for the state $x(k)$.

$$\Phi_{c_{xx}} = \Phi - \bar{X}_3 R^{-1} \bar{X}_4$$
$$= (I - \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix} R^{-1} \begin{bmatrix} \Gamma'_1 \\ \Gamma'_2 \end{bmatrix} X_{xx})\Phi \quad (26)$$

□

Note that the closed-loop system matrix $\Phi_{c_{xx}}$ is determined by the state-space description of only the $x(k)$.

The matrix R can be written in a simple form

$$R = \begin{bmatrix} -\gamma^2 I_l + \Gamma'_1 X_{xx} \Gamma_1 & \Gamma'_1 X_{xx} \Gamma_2 \\ \Gamma'_2 X_{xx} \Gamma_1 & W_2 + \Gamma'_2 X_{xx} \Gamma_2 \end{bmatrix}. \quad (27)$$

Proposition 2: The solutions of partitioned matrices X_{xx} and X_{xd} of the ARE are given as

$$X_{xx} = \Phi'_{c_{xx}} X_{xx} \Phi + C' W_1 C \quad (28)$$

and

$$X_{xd}(0) = \Phi'_{c_{xx}} X_{xx} \Gamma_d \quad (29)$$
$$X_{xd}(1) = \Phi'_{c_{xx}} X_{xd}(0) = (\Phi_{c_{xx}})^2 X_{xx} \Gamma_d \quad (30)$$
$$\vdots$$
$$X_{xd}(N-1) = (\Phi_{c_{xx}})^N X_{xx} \Gamma_d \quad (31)$$

where $X_{xd} = \begin{bmatrix} X_{xd}(0) & X_{xd}(1) & \cdots & X_{xd}(N-1) \end{bmatrix}$.

Proof: X_{xx} and X_{xd} are obtained as follows:

$$X_{xx} = \Phi'(X_{xx} - \bar{X}_1 R^{-1} \bar{X}_1{}')\Phi + C' W_1 C$$
$$= \Phi'_{c_{xx}} X_{xx} \Phi + C' W_1 C, \quad (32)$$

$$X_{xd} = \Phi'(I - X_{xx} \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix} R^{-1} \begin{bmatrix} \Gamma'_1 \\ \Gamma'_2 \end{bmatrix}) X_{xx} \Gamma_{pd}$$
$$+ \Phi'(I - X_{xx} \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix} R^{-1} \begin{bmatrix} \Gamma'_1 \\ \Gamma'_2 \end{bmatrix}) X_{xd} \Phi_d \quad (33)$$

Recalling the closed-loop system matrix $\Phi_{c_{xx}}$, the matrix X_{xd} becomes

$$X_{xd} = \Phi'_{c_{xx}} X_{xx} \Gamma_{pd} + \Phi'_{c_{xx}} X_{xd} \Phi_d \quad (34)$$

Since the Φ_d has a form of a companion matrix and Γ_{pd} has a simple structure, X_{xd} can have a partitioned form.

$$X_{xd} = \begin{bmatrix} X_{xd}(0) & X_{xd}(1) & \cdots & X_{xd}(N-1) \end{bmatrix} \quad (35)$$

Obtaining the elements of X_{xd},

$$X_{xd}(0) = \Phi'_{c_{xx}} X_{xx} \Gamma_d \quad (36)$$
$$X_{xd}(1) = \Phi'_{c_{xx}} X_{xd}(0) = (\Phi_{c_{xx}})^2 X_{xx} \Gamma_d \quad (37)$$
$$\vdots$$
$$X_{xd}(N-1) = (\Phi_{c_{xx}})^N X_{xx} \Gamma_d. \quad (38)$$

□

The control gain matrix K of the augmented system can also be partitioned as K_x and K_d conformably with $x(k)$ and $x_d(k)$ and the control input for the system is a sum of the control input by K_x and the control input by K_d. Therefore,

$$u(k) = K \bar{x}(k) = \begin{bmatrix} K_x & K_d \end{bmatrix} \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix}. \quad (39)$$

Theorem 3: Suppose that following conditions hold.

a) $[\Phi, \Gamma_2]$ is stabilizable.

b) $rank \begin{bmatrix} \Phi - e^{j\theta} I & \Gamma_2 \\ \begin{bmatrix} W_1^{1/2} C \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ W_2^{1/2} \end{bmatrix} \end{bmatrix} = n + m$, for all $\theta \in (-\pi, \pi]$.

Then, the discrete-time stabilizing state feedback and preview control gains satisfying $\|\mathcal{F}_l(G_g, K)\|_\infty < \gamma$ for both previewable and unmeasurable disturbances are given as

$$K_x = R_I \Omega X_{xx} \Phi \quad (40)$$
$$K_d(0) = R_I \Omega X_{xx} \Gamma_d \quad (41)$$
$$K_d(1) = R_I \Omega (\Phi'_{c_{xx}}) X_{xx} \Gamma_d \quad (42)$$
$$\vdots$$
$$K_d(N) = R_I \Omega (\Phi'_{c_{xx}})^N X_{xx} \Gamma_d. \quad (43)$$

where $\boldsymbol{K}_d = \left[\boldsymbol{K}_d(0)\ \boldsymbol{K}_d(1) \cdots \boldsymbol{K}_d(N-1)\right]$ consistently with the number of the preview time step.

Proof:

A stabilizing state feedback controller is given by

$$\boldsymbol{K} = (\boldsymbol{R}_3 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{R}_2')^{-1}(\boldsymbol{L}_2 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{L}_1). \quad (44)$$

Define $\boldsymbol{R}_I$ for convenience:

$$\boldsymbol{R}_I = (\boldsymbol{R}_3 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{R}_2')^{-1}. \quad (45)$$

Expressing $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ as simple forms,

$$\begin{aligned}\boldsymbol{L}_1 &= \bar{\boldsymbol{\Gamma}}_1{}'\boldsymbol{X}_\infty\bar{\boldsymbol{\Phi}} \\ &= \boldsymbol{\Gamma}_1'\left[\boldsymbol{X}_{xx}\boldsymbol{\Phi}\ \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{X}_{xd}\boldsymbol{\Phi}_d\right] \quad (46)\end{aligned}$$

and

$$\boldsymbol{L}_2 = \bar{\boldsymbol{\Gamma}}_2{}'\boldsymbol{X}_\infty\bar{\boldsymbol{\Phi}} = \boldsymbol{\Gamma}_2'\left[\boldsymbol{X}_{xx}\boldsymbol{\Phi}\ \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{X}_{xd}\boldsymbol{\Phi}_d\right] (47)$$

For simplicity, a new matrix is defined as:

$$\boldsymbol{\Omega} = \boldsymbol{\Gamma}_2'(\boldsymbol{I} - \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_1(-\gamma^2\boldsymbol{I}_l + \boldsymbol{\Gamma}_1'\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_1)^{-1}\boldsymbol{\Gamma}_1'). \quad (48)$$

Then,

$$\begin{aligned}&\boldsymbol{L}_2 - \boldsymbol{R}_2\boldsymbol{R}_1^{-1}\boldsymbol{L}_1 = \\ &\boldsymbol{\Omega}\left[\boldsymbol{X}_{xx}\boldsymbol{\Phi}\ \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{X}_{xd}\boldsymbol{\Phi}_d\right]. \quad (49)\end{aligned}$$

With $\boldsymbol{\Omega}$ and $\boldsymbol{R}_I$, the control gain matrix is

$$\begin{aligned}\boldsymbol{K} &= \left[\boldsymbol{K}_x\ \boldsymbol{K}_d\right] \\ &= \boldsymbol{R}_I\boldsymbol{\Omega}\left[\boldsymbol{X}_{xx}\boldsymbol{\Phi}\ \boldsymbol{X}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{X}_{xd}\boldsymbol{\Phi}_d\right]. \quad (50)\end{aligned}$$

The resulting feedback gain matrix and the preview gain matrix are

$$\boldsymbol{K}_x = \boldsymbol{R}_I\boldsymbol{\Omega}\boldsymbol{X}_{xx}\boldsymbol{\Phi}, \quad (51)$$

$$\boldsymbol{K}_d = \boldsymbol{R}_I\boldsymbol{\Omega}(\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_{pd} + \boldsymbol{X}_{xd}\boldsymbol{\Phi}_d). \quad (52)$$

Since $\boldsymbol{X}_{xd} = \left[\boldsymbol{X}_{xd}(0)\ \boldsymbol{X}_{xd}(1) \cdots \boldsymbol{X}_{xd}(N-1)\right]$, $\boldsymbol{K}_d$ can be rewritten as

$$\boldsymbol{K}_d(0) = \boldsymbol{R}_I\boldsymbol{\Omega}\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_d \quad (53)$$

$$\boldsymbol{K}_d(1) = \boldsymbol{R}_I\boldsymbol{\Omega}(\boldsymbol{\Phi}_{c_{xx}}')\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_d \quad (54)$$

$$\vdots$$

$$\boldsymbol{K}_d(N) = \boldsymbol{R}_I\boldsymbol{\Omega}(\boldsymbol{\Phi}_{c_{xx}}')^N\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_d. \quad (55)$$

□

The preview controller is completely determined with $\boldsymbol{X}_{xx}$ and $\boldsymbol{\Phi}_{c_{xx}}$ of the original dynamical system. The feedback gains depend on the infinity norm and the selected weighting matrices. Note that the preview length doesn't have effects on them. The preview gains are inherited from the feedback gains. The previewable disturbances are compensated by preview action before they hit the system as shown in Fig. 1.

It is interesting to note that as γ tends to ∞ in (55), the H_∞ preview controller actually approaches the LQ preview controller. As $\gamma \to \infty$, $\boldsymbol{\Omega} = \boldsymbol{\Gamma}_2'$ and $\boldsymbol{R}_I = \boldsymbol{W}_2 + \boldsymbol{\Gamma}_2'\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_2$.

Rewriting the gains,

$$\boldsymbol{K}_d(N) = (\boldsymbol{W}_2 + \boldsymbol{\Gamma}_2'\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_2)\boldsymbol{\Gamma}_2'(\boldsymbol{\Phi}_{c_{xx}}')^N\boldsymbol{X}_{xx}\boldsymbol{\Gamma}_d (56)$$

where $\boldsymbol{W}_2$ is equivalent to the control weighting matrix in the LQ cost functional. Eq. (56) is exactly the same as LQ-based preview gain. This is similar to the relationships between H_2 and H_∞ controller.

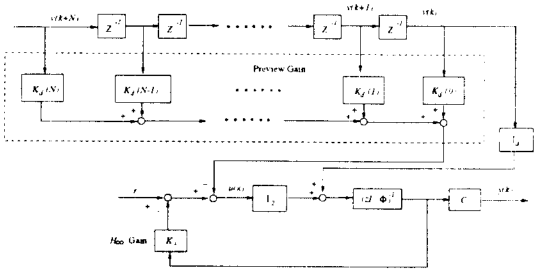

Fig. 1. The closed-loop control system with preview

3. CONCLUSION

A new preview control law is presented based on the discrete-time full-information H_∞ control scheme. The derivation procedure is analogous to the scheme by LQR. The synthesized control law also considers the unknown bounded disturbances as well as the measurable ones. The preview and feedback controller are designed by simultaneous optimization so that the infinity norm of the transfer function matrix from both the previewable and unmeasurable disturbances to the regulated variables is minimized. As the infinity norm of the transfer function tends to ∞, the feedback gain and preview gain approach to LQ-based ones, which is the similar relationship of H_2 and H_∞.

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