

# ROBUST DESIGN USING FUZZY SYSTEM

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**Abstract** : To design high quality products at low cost is one of very important task for engineers Design optimization for performances can be one solution in this task. This is robust design which has been proved effectively in many field of engineering design. In this paper, the concept of robust design is introduced and combined to fuzzy optimization and nonsingleton fuzzy logic system. The optimum parameter set points were obtained by the fuzzy optimization method and nonsingleton fuzzy logic system. These methods are applied to a filter circuit, a part of the audio circuit of mobile radio transceiver. The results are compared each other.

**Keyword** : Robust Design, Fuzzy Optimization, Fuzzy Logic System, Nonsingleton Fuzzy Logic System

## 1. Introduction

Robust design is one method to make manufacturing less sensitive to manufacturing process. Because it reduces variations by reducing the effect of variation sources, not by controlling the variations. Robust design is cost effective technique to improve the quality process. This method uses statistically planned experiments to vary settings of important process control parameters. In the experiment process performance characteristics are measured to consider the effect of manufacturing variations on each process variation combinations. And some simple plots or tables are used in predicting the settings to minimize their effects. To follow up experiment is conducted to examine those predictions. In this paper, the fuzzy optimization and fuzzy logic system are applied to robust design concept. First, a method which uses fuzzy optimization in obtaining optimum settings by measured data from experiments will be presented. Second, fuzzy logic system is made to reduce experiments using experiments results consisted with key control parameter combinations. Then optimum parameter set points are obtained by the described first fuzzy optimization method after predicting the results of each parameter combinations considering each control parameter variations by nonsingleton fuzzy logic system concept.

## 2. Robust Design

Robust design is a cost effective approach to

improve yield. It uses statistically planned experiments to identify process control parameter settings that reduce the process's sensitivity to manufacturing variation. This method is used on the technique of robust design developed by Genichi Taguchi, a Japanese expert on quality.[1,2]

A key component of Taguchi's philosophy is "*reduction of variability*". We often require that each quality characteristic has a target or nominal value. The objective is to reduce variability around this target. Taguchi models the departures that may occur from this value with a *loss function*. The loss refers to the cost that is incurred by society when the consumer uses a product whose quality characteristics differ from the nominal. The concept of societal loss is a departure from traditional thinking. Taguchi imposes a quadratic function form.[1,2] Clearly, this type of function will penalize even small departures of  $y$  from the target  $T$ . Again, this is a departure from traditional western thinking, which usually only attaches penalties to the case where  $y$  is outside of specifications.

Following the robust parameter design methodology, one experimental design is selected for the controllable factors and another for the noise or uncontrollable factors. Taguchi recommends orthogonal array for experimental design. For controllable factors one orthogonal array is designed, which is called inner array. For uncontrollable factors another orthogonal array is designed and it is called outer array. The purpose of the noise factor array is to create noise so that researcher can identify the controllable factor levels that are least sensitive to it. Taguchi said that

for data analysis he recommends analyzing the mean response for each run in the inner array: he also suggests analyzing variation using an appropriately chosen signal-to-noise ratio(SN). These signal-to-noise ratios are derived from the quadratic loss function, and three of them are considered 'standard' and widely applicable.[7]

### 3. Fuzzy Optimization and Fuzzy Logic System

#### 3.1. Fuzzy Optimization

This paper use a data optimization method using fuzzy membership function. Bellman and Zadeh[3] proposed a method for decision-making in fuzzy environment. They used fuzzy membership function to decide some values. They defined a maximizing decision is a point in the space of alternatives at which the membership function of a fuzzy decision attains its maximum value. Generally design problem is represented as follows.

$$\begin{aligned} & \text{minimize} \{ f_1(x), \dots, f_m(x) \} \\ & \text{subject to : } g_i(x) \leq \text{spec}_i, \quad i = 1 \dots m \quad (1) \\ & \quad \quad \quad x_{\min} \leq x \leq x_{\max} \end{aligned}$$

where  $f_i(x)$  are M objective functions to be minimized.  $g_i(x) \leq \text{spec}_i$  are m constraints to be satisfied,  $\text{spec}$  is the limit value of the ith specification, x is the vector of design parameters and  $x_{\min} \leq x \leq x_{\max}$  are bounding conditions on the design parameters.

In the attempt to minimize a performance function  $f_i(x)$ , designers often stop the search procedure when  $f_i(x)$  attains acceptable values, even before the strict minimum is reached. Additional searching may be very time consuming with no significant improvement in the objective function. So we can associate with each object function  $f_i(x)$  a fuzzy set that formulates the fuzzy meaning of minimize(or maximize) and what precisely the designer wants to achieve. For each fuzzy objective  $f_i(x)$ , we define a membership function  $\mu_{f_i}(x)$  reflecting the degree of acceptability of that particular performance value, if  $D_{f_i}$  is the interval of possible values of  $f_i(x)$ ,  $\mu_{f_i}$  will be defined as follows:

$$\begin{aligned} \mu_{f_i}(x) : D_{f_i} & \rightarrow [0,1], \\ f_i(x) & \rightarrow \mu_{f_i}(x) \quad (2) \end{aligned}$$

$\mu_{f_i}(x)$  is real number in [0,1] reflecting the degree of fulfillment of the fuzzy objectives associated with the objective function  $f_i$ . The formulation (1) is now replaced as follows:

$$\begin{aligned} & \text{maximize} \{ \mu_{f_1}, \dots, \mu_{f_m} \} \\ & \text{subject to : } g(x) \leq \text{spec}_i, \quad i = 1 \dots m \quad (3) \\ & \quad \quad \quad X_{\min} \leq X \leq X_{\max} \end{aligned}$$

#### 3.2. Fuzzy Logic System (FLS)

Fuzzy Logic system is a name for the system which has a direct relationship with fuzzy concepts(like fuzzy sets, linguistics variables, and so on) and fuzzy logic. In general, FLS is a nonlinear mapping of an input data vector into a scalar output and used in fuzzy logic controllers and signal processing applications. FLS is composed of four components : rules, fuzzifier, inference engine, and defuzzifier. Once the rules have been established, a FLS can be viewed as a mapping from inputs to outputs. Rules may be provided by experts or can be extracted from numerical data. In either case rules are expressed as a collection of IF-THEN statement.[4, 5].

Generally fuzzy logic system use singleton data. However it would be difficult if we use the data that is corrupted by noise. So we use nonsingleton fuzzy logic system. This is suggested by Mendel [6] to identify a system using noise corrupted data. Through this system we can use the noisy data set as input of fuzzy system. In this paper we first we make a fuzzy system using crisp data. And nonsingleton(noncrisp) fuzzy logic system method is used to consider noisy factor of each control parameter set.

## 4. Implementation

#### 4.1. Filter Circuit

A filter circuit, part of the audio-circuit of a mobile radio transceiver[8] was to be studied. The response is the 'gain' which at a certain frequency f is defined in decibels as :

$$G_f = 10 \log_{10}(V_f)$$

where  $V_f$  can be calculated on the basis of the resistance of four resistors R1-R4 and the capacitance of four capacitors C1-C4 which comprise the circuit.

For a proper performance of the circuit, it is required that the gains at frequencies 300Hz and 700Hz should consistently satisfy:

$$\begin{aligned} G_{300} - G_{1000} & \leq -3dB \\ & \text{or equivalently} \end{aligned}$$

$$Y_{300} = (V_{300}/V_{1000}) \leq 0.7079$$

and also

$$G_{700} - G_{1000} \leq |1dB|$$

or equivalently

$$0.891 \leq Y_{700} (V_{700}/V_{1000}) \leq 1.122$$

Since a model relating input(resistors and capacitors) to output(  $Y_{300}$  or  $Y_{700}$  ) is available, it is possible to perform computer simulation experiments using experimental design techniques. By using experimental data we have optimum resistor and capacitor values in given range.

Table 1. Nominal and Tolerance Level

Circuit Parameters as Controllable Factors	Nominal levels			Tolerance levels(% from nominal)		
	1	2	3	1	2	3
R1(k $\Omega$ )	10	12	15	-2	0	+2
R2(k $\Omega$ )	100	120	150	-2	0	+2
R3(k $\Omega$ )	4.7	5.6	6.8	-2	0	+2
R4(k $\Omega$ )	10	15	22	-2	0	+2
C1(k $\Omega$ )	0.01	0.015	0.022	-10	0	+10
C2(k $\Omega$ )	0.01	0.015	0.022	-10	0	+10
C3(k $\Omega$ )	0.01	0.015	0.022	-10	0	+10
C4(k $\Omega$ )	0.022	0.033	0.047	-10	0	+10

#### 4.2. Parameter Design of Robust Method

Orthogonal array is used for design of experiments. From [7] inner array is designed 27 runs for 8 parameters and outer array is composed of 18 runs or 3 noise levels for each parameters. So 468(27 $\times$ 18) experiments are needed for each gain. In this example we use PSPICE for circuit simulation. After getting results ANOVA table is used for data analysis.

#### 4.3. Data Analysis using Fuzzy Optimization Method

In robust design method data is analyzed by ANOVA table. But in fuzzy decision method we use fuzzy membership function to decide an optimum parameter set for object functions. This example object functions are  $G_{300}$  and  $G_{700}$ . So we can consider two membership functions.

This method use the same data that robust design method used. So for each gain 468(27 $\times$ 18) data are used for parameter decision. From 3, The fuzzy optimization problem can be defined as follows:

$$\begin{aligned} & \text{maximize} \{ \mu_{fG300}(x), \mu_{fG700}(x) \} \quad X = \{ x_1 \cdots x_8 \} \\ & \text{subject to: } x_{imin} \leq x_i \leq x_{imax} \\ & i = 1 \cdots 8 \quad (R_1, R_2, R_3, R_4, C_1, C_2, C_3, C_4) \end{aligned} \quad (4)$$

Using robust method, each parameters are selected that have marginal maximum membership function

values. The new formulation is obtained as follows:

$$\begin{aligned} \mu_D(X') = & \\ & \frac{1}{n} [ \sum_{i=1}^N \mu_{G300}(G_{300}(X')) + \sum_{i=1}^N \mu_{G700}(G_{700}(X')) ] \end{aligned} \quad (5)$$

(  $X'$  is a parameter nominal value, n is the number of experiment results for that parameter value)

One of the parameter that satisfies  $\mu_D(X) = 1$  means that it is satisfying all design objective and constraints. To decide the optimum parameter value for each parameter, the parameter value is selected that maximize  $\mu_D(X')$ .

#### 4.4. Fuzzy Optimization using Nonsingleton Fuzzy Logic System

Generally fuzzy logic system use singleton (crisp) data. However it would be difficult if the data corrupted by noise is used. Therefore, the nonsingleton fuzzy logic system is used in this paper. Then, the noisy data set can be used as input of fuzzy system. It is similar to outer array of robust design method. In robust design method an outer array is suggested for each controllable parameter set. Also in this nonsingleton fuzzy logic system, the same controllable test set as in robust design method is used and for each controllable test set noisy data set is applied for each controllable test set. This system consists of as follows.

A fuzzy rule base consists of a collection of fuzzy IF-THEN rules in the following form:

$$R^{(1)} : \text{IF } R1 \text{ is } F1 \dots R4 \text{ is } F4 \text{ and } C1 \text{ is } F5 \text{ and } \dots C4 \text{ is } F8 \text{ then } y \text{ is } G1 \quad (6)$$

where  $R1 \dots R4$ ,  $C1 \dots C4$  and  $F1 \dots F8$  are fuzzy sets and  $G1$  is a constant.

In singleton fuzzy logic system the input data is crisp value. But in nonsingleton fuzzy logic system input data is membership function. In the case of Gaussian membership functions

$$\mu_{X_r}(X_r) = \exp \left[ -\frac{1}{2} \left( \frac{x_r - m_{x_r}}{\sigma_{x_r}} \right)^2 \right] \quad (7)$$

$$\mu_{F_r}(X_r) = \exp \left[ -\frac{1}{2} \left( \frac{x_r - m_{F_r}}{\sigma_{F_r}} \right)^2 \right] \quad (8)$$

Then, for product inference and modified height defuzzification

$$\begin{aligned} f_{ns}(x) = & \sum_{i=1}^M \bar{y}^i p^i(x) = \\ & \frac{\sum_{i=1}^M \bar{y}^i \prod_{k=1}^n \mu_{Q_k} \left( \frac{\sigma_{x_k}^2 m_{F_k} + \sigma_{F_k}^2 x_k}{\sigma_{x_k}^2 + \sigma_{F_k}^2} \right) / (\sigma^i)^2}{\sum_{i=1}^M \prod_{k=1}^n \mu_{Q_k} \left( \frac{\sigma_{x_k}^2 m_{F_k} + \sigma_{F_k}^2 x_k}{\sigma_{x_k}^2 + \sigma_{F_k}^2} \right) / (\sigma^i)^2} \end{aligned} \quad (9)$$

where  $\mu_{Q_i} = \mu_{F_i} \cdot \mu_{x_k}$ ,  $mX_k = x_k$  and  $\sigma_{x_k}$  are the mean and standard deviation of the  $k^{th}$  input, and,  $m_{F_i}$  and  $\sigma_{F_i}$  are the mean and standard deviation of the  $k^{th}$  antecedent of the  $1^{th}$  rule,  $R^1$ , respectively.

In this example, first, 27 simulation results of inner array parameter set are obtained. It is the same parameter set used in the robust design method. After simulation a fuzzy logic system is made using those data. 24(8×3) test sets are created for each parameter value (ex R1:10k,12k,15k). Each test set consists of 27 parameter sets. When this set is used for fuzzy logic system each parameter value is converted to a gaussian membership function as (7). The mean is each parameter value and variance is calculated for each parameter considering each parameter tolerance level. So 27 simulation data is got for each tolerance level of each parameter. After getting simulation data we used the same fuzzy optimization method as 4.3.

#### 4.5. Compare Each Method Result

As follows, different optimum parameter set for each method were determined in Table 2.

Table 2 Each Optimum Parameter Set

Parameter Name	Robust Design Method	Fuzzy Opt. Method	Fuzzy System Method
R1	15k	12k	15k
R2	100k	120k	100k
R3	5.6k	4.7k	6.8k
R4	10k	15k	10k
C1	0.022 $\mu$ F	0.01 $\mu$ F	0.022 $\mu$ F
C2	0.022 $\mu$ F	0.01 $\mu$ F	0.022 $\mu$ F
C3	0.015 $\mu$ F	0.01 $\mu$ F	0.01 $\mu$ F
C4	0.022 $\mu$ F	0.033 $\mu$ F	0.022 $\mu$ F

Confirmation trials at the optimal level settings using the associated outer array for this settings are as follows. This outer array design technique is same as robust design method. Table 3 and Table 4 are the mean and standard deviation of conformational trials

Table 3 Gain of 300 Hz

	Mean	Standard Deviation
Robust Design Method	0.5825	0.0388
Fuzzy Opt. Method	0.4977	0.0611
Fuzzy System Method	0.4982	0.0400

Table 4 Gain of 700 Hz

	Mean	Standard Deviation
Robust Design Method	0.9380	0.0166
Fuzzy Opt. Method	1.0348	0.0290
Fuzzy System Method	0.9462	0.0190

## 5 . Conclusion

In this paper, the fuzzy optimization and fuzzy logic system are applied to robust design concept. These methods are applied to a filter circuit, a part of the audio circuit of mobile radio transceiver.

Some results are drawn from computer simulation as follows:

- Fuzzy optimization method mean and fuzzy logic system method mean are better than original robust design method mean at 300Hz.
- Fuzzy optimization method mean and fuzzy logic system method mean are better than original robust design method mean at 700Hz.
- Fuzzy optimization method better than has more safety margin than original robust design method.
- Fuzzy logic system method is better than original robust design method.
- If fuzzy logic system method is applied to expensive experimental cost process or product, it would be very effective.

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