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Variable Structure Control System Design Guaranteeing Continuity of Control Signal

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Abstract In this paper, a sliding mode control scheme that guarantees the smoothness of the control signal and the exponential error convergence is proposed for robot manipulators. The proposed method inserts a low pass filter (LPF) in front of the plant, and the *virtual* controller is designed for the *virtual* plant – the combination of the LPF and the robot manipulator. The virtual control signal contains high frequency components because of a switching function. The *real* control signal, however, always shows a smooth curve since it is an output of the LPF. In addition to the smoothness of the control signal is always assured, the overall system is in the sliding mode at all times, that is, its performance is always invariant under the existence of parameter uncertainties and external disturbances. The closed-loop system is shown to be globally exponentially stable.

Keywords Variable Structure Control, Continuous Control Signal, Robust Control, Robot Control

1. INTRODUCTION

The sliding mode control has been widely studied in recent years and started to play an important role in the application of the control theory to practical problems, e.g., robot controls [1]-[3]. Although this kind of control methods shows the invariant property against parameter uncertainties and external disturbances, it is assumed that the control signal can be switched from one value to another infinitely fast. In practical systems, however, it is impossible to achieve the infinitely fast switching control because of finite time delays for the control computation and limitations of physical actuators. In addition, in the steady state, the chattering phenomena appear as a high frequency oscillation near the desired equilibrium point and it may excite the unmodeled high-frequency dynamics of the system [4]. Since it is almost always desirable to avoid the chattering phenomena, lots of studies have been performed to overcome this phenomena [5]-[8].

Zinober *et al.* inserted the first order low pass filter (LPF) between the controller and the plant in order to yield a smoother control signal [5]. However, it was just a trial and no analysis was presented in [5]. Moreover, since the control system was designed for the closed-loop system *without* LPF, it is obvious that the overall system, including LPF, may be unstable.

Espana *et al.* proposed switching *zone* instead of a switching surface [6]. Since the proposed method used a corn shape sliding manifold, the resultant control signal showed a smooth curve. In the steady state, however, it also shows the chattering phenomena because the corn shape sliding surface is the same as that of the conventional one in the vicinity of origin.

Almost all of researchers has used continuation techniques. These techniques are focused on the smoothing of a switching function. Instead of a switching function, lots of continuous functions has been used such as: saturation functions, sigmoid functions, relay functions, and hysteresis-saturation functions [1]. All of these methods, however, suffer from the same difficulty – there is no quantitative rule for assigning the boundary layer thickness of their algorithms. Thus, the corresponding effects of the chattering alleviation may not be guaranteed. In addition, these approaches can not ensure the error convergence to zero, i.e., only the ultimate boundedness of the error within some predetermined boundary layer can be guaranteed.

Chang *et al.* proposed a chattering alleviation control (CAC) scheme [7]. But, as is noted by Shyu [8], this method may lost the invariance property against parameter variations and external disturbances. Furthermore, the method needs a predictive calculation to switch the control law in the hitting or sliding phase, and this increases the difficulty in the implementation [8].

Shyu *et al.* proposed the modified smoothing function [8]. As the Espana's work [6], they proposed two sliding surfaces to define a sliding zone (corn shape) and then make a controller to switch from one sliding surface to the other one. However, the proposed method also shows the chattering phenomena in the steady state. Furthermore, as can be shown in the simulation results in [8], the output performance is not so good/smooth compared to the results when the saturation function is used.

Thus, in this paper, a sliding mode control scheme that guarantees the smoothness of the control signal

and the exponential error convergence to zero is proposed for robot manipulators. The main concept is to consider the combination of the inserted LPF and the robot system as a *virtual* plant, and then design a virtual sliding mode controller u for this virtual plant. Although the virtual control signal u shows the chattering phenomena because of the switching function, the *real* control signal τ shows a smooth curve since it is an output of the LPF whose input is u as shown in Fig. 1. A function augmented sliding surface is also proposed such that the system state is on the surface at the initial time. Due to the proposed function augmented sliding surface, the closed-loop system always shows the invariant property against parameter uncertainties maintaining the smoothness of the *real* control signal without sacrificing the tracking accuracy. The closed-loop system is shown to be globally exponentially stable.

2. CONTROL SYSTEM DESIGN

The dynamic equation of an n degree-of-freedom robot manipulator can be derived as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta = \tau \qquad (1)$$

where $M(q)$ is an $n \times n$ inertia matrix, $C(q,\dot{q})$ is an $n \times n$ matrix corresponding to Coriolis and centrifugal factors, $G(q)$ is an $n \times 1$ vector of gravitational torques, q is an $n \times 1$ vector of joint angular positions, τ is an $n \times 1$ input torque vector, θ is a constant p-dimensional vector of inertia parameters and Y is an $n \times p$ matrix of known functions of the generalized coordinates and their higher derivatives. It can be assumed that the parameter vector θ is uncertain but there exists known values $\theta_0 \in \Re^p$ and $\rho \in \Re_+$ such that [2]

$$\|\tilde{\theta}\| \equiv \|\theta - \theta_0\| \leq \rho.$$

From the input-output relation of the LPF, the following equation can be obtained (see Fig. 2).

$$\dot{\tau} + \Lambda\tau = \Lambda u \qquad (2)$$

where $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\lambda_i > 0$, and $i = 1, 2, \ldots, n$. Substituting (1) into (2), we can obtain the dynamic equation of the virtual plant as

$$M\dddot{q} + \dot{M}\ddot{q} + \Lambda M\ddot{q} + C\ddot{q} + \dot{C}\dot{q} + \Lambda C\dot{q} + \dot{G} + \Lambda G = \Lambda u. \qquad (3)$$

Then, the virtual sliding mode control law u can be derived based on the above dynamic equation (3). Let us define the tracking error as

$$e(t) = q(t) - q_d(t)$$

where $q_d(t) \in \Re^n$ represents the desired trajectory, and propose a function augmented sliding surface.

$$s = \ddot{e} + \Lambda_1\dot{e} + \Lambda_2 e - F(t) \qquad (4)$$

where $\Lambda_i = diag(\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in})$, $\lambda_{ij} > 0$, $i = 1, 2$, $j = 1, 2, \ldots, n$, $F(t) \in \Re^n$, and the following assumption holds for each $F_j(t)$.

Assumption 1 The augmented function $F_i(t)$ is a differentiable continuous function defined on $t \in [0, \infty)$, $F_i(0) = \ddot{e}(0) + \Lambda_1\dot{e}(0) + \Lambda_2 e(0)$, and there exists positive constants A and B, such that

$$|F_i(t)| \leq Ae^{-Bt}.$$

Remark 1 Since the augmented function $F_i(t)$ can be designed arbitrarily, the above Assumption 1 always holds. Therefore, it is obvious that the proposed function augmented sliding surface (4) ensures $s(0) = 0$.

Let us define the following positive-definite function as a Lyapunov function candidate:

$$V = \frac{1}{2}s^T M s. \qquad (5)$$

Differentiating (5) with respect to time and adopting the skew-symmetricity of $\dot{M}(q) - 2C(q,\dot{q})$, we have

$$\begin{aligned}\dot{V} &= s^T M\dot{s} + s^T Cs \\ &= s^T\left(M\dot{s} + Cs\right) \\ &= s^T(\Lambda u + M(\Lambda_1\dddot{e} + \Lambda_2\ddot{e} - \ddot{F} - \dddot{q}_d) \\ &\quad + C(\Lambda_1\ddot{e} + \Lambda_2\dot{e} - \dot{F} - \ddot{q}_d) - (\dot{M} + \Lambda M)\ddot{q} \\ &\quad -(\dot{C} + \Lambda C)\dot{q} - (\dot{G} + \Lambda G)). \end{aligned} \qquad (6)$$

Here, let us define a vector H as following:

$$\begin{aligned}H &= M(\Lambda_1\dddot{e} + \Lambda_2\ddot{e} - \ddot{F} - \dddot{q}_d) + C(\Lambda_1\ddot{e} + \Lambda_2\dot{e} - \dot{F} - \ddot{q}_d) \\ &\quad -(\dot{M} + \Lambda M)\ddot{q} - (\dot{C} + \Lambda C)\dot{q} - (\dot{G} + \Lambda G). \end{aligned} \qquad (7)$$

Since the Lagrangian dynamic equation of a robot is linearly parameterizable as (1), it is obvious that H is also linearly parameterizable as

$$H = \Gamma(t, q, \dot{q}, \ddot{q}, q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d)\phi \qquad (8)$$

where ϕ is a constant r-dimensional vector of inertia parameters and Γ is an $n \times r$ matrix of known functions of the generalized coordinates, desired trajectories, and their higher derivatives. It can be also assumed that the parameter vector ϕ is uncertain but there exists known values $\phi_0 \in \Re^r$ and $\xi \in \Re^r$ such that

$$|\tilde{\phi}_i| \equiv |\phi_i - \phi_{0i}| \leq \xi_i$$

where $\xi_i > 0$ and $i = 1, 2, \ldots, n$.

From (6) and (8), $\dot{V}$ can be rewritten as following:

$$\dot{V} = s^T\left(\Lambda u + \Gamma\phi\right). \qquad (9)$$

Thus, in order to stabilize the overall system, u has to be designed such that the above $\dot{V}$ is negative definite.

Theorem 1 Applying the following virtual control input (10) to the virtual plant (3), the closed-loop system is globally exponentially stable.

$$u = -\Lambda^{-1}\left(\Gamma\phi_0 + \left(\overline{\Gamma}\xi\right) \bullet sgn(s) + \Psi sgn(s)\right) \qquad (10)$$

where $\overline{\Gamma}_{ij} = |\Gamma_{ij}|$, $\Psi = diag(\psi_1, \psi_2, \ldots, \psi_n)$, $\psi_i > 0$, $i = 1, 2, \ldots, n$, $sgn(s)^T = [sgn(s_1), sgn(s_2),$

$\ldots, sgn(s_n)]$, and "$\bullet$" means the element-by-element multiplication of two vectors.

Proof: For the Lyapunov function candidate (5), its time derivative $\dot{V}$ is given as (9). By substituting the proposed control law (10) into (9), a simple calculation shows that $\dot{V}$ is bounded as follows:

$$\begin{aligned}
\dot{V} &= s^T(\Lambda u + \Gamma\phi) \\
&= s^T\left(\Gamma(\phi - \phi_0) - (\overline{\Gamma}\xi) \bullet sgn(s) - \Psi sgn(s)\right) \\
&= s^T\left(\Gamma\tilde{\phi} - (\overline{\Gamma}\xi) \bullet sgn(s) - \Psi sgn(s)\right) \\
&= \sum_{i=1}^{n}\left\{s_i(\Gamma\tilde{\phi})_i - |s_i|\left(\overline{\Gamma}\xi\right)_i\right\} - s^T\Psi sgn(s) \\
&\leq -s^T\Psi sgn(s) = -\sum_{i=1}^{n}(\psi_i|s_i|). \qquad (11)
\end{aligned}$$

From (5) and (11), V is a positive definite function and $\dot{V}$ is a negative definite function, that is, V is a Lyapunov function. From Remark 1, the proposed sliding surface (4) ensures $s(0) = 0$, and it is equivalent to $V(0) = 0$. Hence, from these results, it can be said that V always stays at zero, i.e., $V(t) \equiv 0\ \forall t \geq 0$. This is also equivalent to $s(t) \equiv 0\ \forall t \geq 0$ because V is a positive definite function of s. Therefore, it can be guaranteed that the overall system always shows the invariant property to parameter uncertainties because the proposed control system has no reaching phase. Thus, it is obvious that the tracking error exponentially converges to zero from the definition of the sliding surface (4). ■

Remark 2 Since the virtual control input u given in (10) contains the switching function $sgn(\cdot)$, u produces a high frequency switching signal. The *real* control signal τ, however, does not contain high frequency components because it is made by low pass filtering u (see Fig. 1). Therefore, from the result of the above theorem, even though the *real* control signal does not chatter, the tracking error exponentially converges to zero maintaining the invariant property against system uncertainties at all times.

Although the saturation function has been generally used to get a smooth control signal in the previous works, the proposed scheme guarantees the smoothness of the control signal without sacrificing the tracking accuracy.

3. SIMULATION RESULTS

The simulation has been carried out for a two-link robot manipulator model used by Yeung and Chen [3]. The dynamic equation is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u,$$

where $q = [q_1\ q_2]^T$,

$$\begin{aligned}
M_{11} &= (m_1+m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2\cos q_2 + J_1 \\
M_{12} &= M_{21} = m_2r_2^2 + m_2r_1r_2\cos q_2 \\
M_{22} &= m_2r_2^2 + J_2 \\
C_{11} &= -2m_2r_1r_2\dot{q}_2\sin q_2 \\
C_{12} &= -m_2r_1r_2\dot{q}_2\sin q_2 \\
C_{21} &= m_2r_1r_2\dot{q}_1\sin q_2 \\
C_{22} &= 0 \\
G_1 &= \{(m_1+m_2)r_1\cos q_1 + m_2r_2\cos(q_1+q_2)\}\,g \\
G_2 &= m_2r_2g\cos(q_1+q_2).
\end{aligned}$$

The parameter values are also the same as those of Yeung and Chen as follows:

$$\begin{gathered}
r_1 = 1m, \quad r_2 = 0.8m, \\
J_1 = 5kg\cdot m, \quad J_2 = 5kg\cdot m, \\
m_1 = 0.5kg, \\
0.5kg < m_2 < 6.25kg.
\end{gathered}$$

The simulation results are shown in Figs. 2~4.

The tracking error of the proposed controller is presented in Fig. 2. One can easily know that the tracking error converges to zero.

In Fig. 3, the virtual control signal u is given. Because of the switching function $sgn(\cdot)$, it chatters with the high frequency. However, as can be seen in Fig. 4, the *real* control signal τ applied to the robot(real plant) shows the smooth curve.

4. CONCLUSIONS

In this paper, a sliding mode control method guaranteeing the smoothness of the control signal is proposed. In order to avoid the chattering phenomena, the first order LPF and the concept of a virtual plant/controller has been used. The proposed control scheme always ensures the invariance property against parameter uncertainties maintaining the smoothness of the *real* control signal without sacrificing the tracking accuracy. That is, although the proposed controller generates a smooth control signal τ, the closed-loop system is in the sliding mode *at all times* and the tracking error converges to zero exponentially under the existence of parameter uncertainties and disturbances. The overall system has been shown to be globally exponentially stable.

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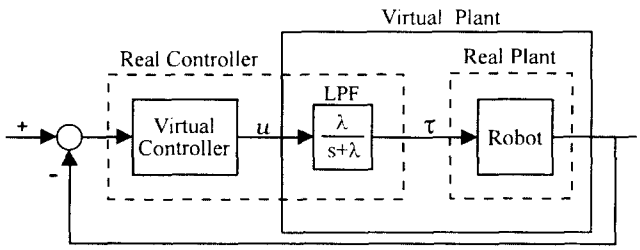

Fig. 1. Block Diagram of the System

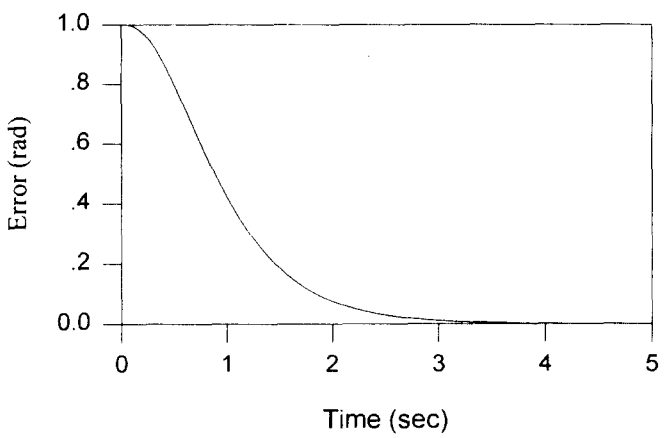

Fig. 2. Tracking Error

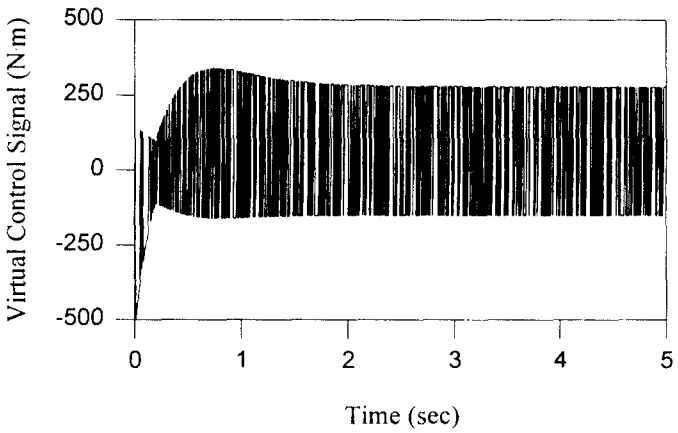

Fig. 3. Virtual Control Signal (u)

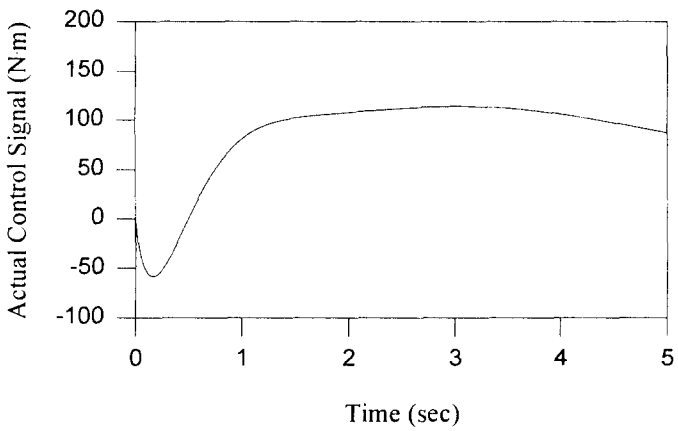

Fig. 4. Actual Control Signal (τ)