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Force Tracking Position-Based Impedance Control of Robot Manipulator with Unknown Environment Stiffness

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Abstract

In impedance control for contact force tracking it is well known that the reference trajectory of the robot is calculated from known environment stiffness. The accuracy of estimating the environment stiffness determines the performance of the resulting force tracking. Here we present a simple technique, called the trajectory modification technique(TMT), of determining the reference trajectory under the condition that the environment stiffness is unknown. Computer simulation studies have shown that force tracking using the proposed technique is excellent for unknown environment with time varying stiffness.

Keywords Impedance control, Robot manipulator, environment

I Introduction

Impedance control proposed by Hogan has been one of the fundamental concepts for force tracking of robot manipulators. Unlike hybrid position and force control approach [1] impedance control regulates the contact force of a manipulator with the environment by the target impedance relationship between position and contact force rather than by specifying the desired contact force directly. Thus the desired contact force is indirectly controlled by prespecifying a robot positional reference trajectory which is dependent on the stiffness and location of the environment. The major practical problem with this technique is that the environment stiffness is usually not known precisely so that accurate reference trajectory can not be designed to achieve accurate contact force control.

There have been many attempts made to solve the problem such as by employing separate trajectory modification control loop using integral control of the difference between the desired force and the measured force, or by generating reference position using adaptive control. Although these techniques do work, they tend to increase the complexity of the system dynamics which requires special attention to the system stability.

Here we propose a simple technique for reference trajectory design without using any informations of environment stiffness and without introducing complex dynamics to the system. The proposed technique is purely implemented based on algebraic calculations by replacing the unknown environment stiffness with the measured force and the position which are available at any instance. Thus it is able to provide robust force tracking for arbitrarily varying unknown environment stiffness. The stability analysis is provided to show that the robot system is stable with the TMT.

II Position-Based Impedance Force Control

We have the robot dynamic equation model including disturbance force in Cartesian space as

$$D^* \ddot{X} + h^* + F_f^* = F - F_e \qquad (1)$$

where D^* is the n x n symmetric positive definite Cartesian inertia matrix; h^* is the n x 1 vector of Cartesian Coriolis and centrifugal torques; F_f^* is the n x 1 vector of friction forces; F is the n x 1 vector of Force; F_e is n x 1 vector of exerted force.

In position-based impedance force control [2], the impedance control is added as an additional control loop around the position controlled manipulator as shown in Figure 1 while the impedance relationship is implemented inside of the control loop in the torque based impedance control. Thus the advantage of the position based impedance control over the torque based impedance control is that the position controlled robot, it is true that most industrial robots are accurate position controlled, can easily be transformed into the force controlled robot without modifying internal control structure. Let us restate the essence of the position based impedance control here. The reference trajectory X_r is adjusted by X_a which is calculated based on sensed force F_e from a force sensor. The desired impedance can be written as

$$F_e = M\ddot{X}_a + B\dot{X}_a + KX_a \tag{2}$$

where X_a is the position adjustment vector, B, K are damping and stiffness matrices. Vector X_a is calculated by filtering the sensed force, F_e through the impedance relationship. In frequency domain X_a is represented as

$$X_a(s) = [Ms^2 + Bs + K]^{-1}F_e(s) \tag{3}$$

The vector X_a is used to modify the reference trajectory command X_r to generate the new position command X_c so that

$$X_c = X_r - X_a \tag{4}$$

The control law F is

$$F = \hat{D}^*U + \hat{h}^* + F_e \tag{5}$$

And U is given by

$$U = \ddot{X}_c + K_D(\dot{X}_c - \dot{X}) + K_P(X_c - X) \tag{6}$$

where K_D and K_P are n x n symmetric positive definite desired damping and stiffness gain matrices, respectively. Combining (1),(5), and (6) yields the closed loop tracking error dynamic equation

$$\begin{aligned} \ddot{E} + K_D\dot{E} \quad &+ \quad K_PE = \hat{D^*}^{-1}[\Delta D^*\ddot{X} + \Delta h^* \\ &+ \quad F_f^*] + \ddot{X}_a + K_D\dot{X}_a + K_PX_a \end{aligned} \tag{7}$$

where $\Delta D^* = D^* - \hat{D}^*, \Delta h = h^* - \hat{h}^*$, and $E = (X_r - X)$. The term $\hat{D^*}^{-1}[\Delta D^*\ddot{X} + \Delta h^* + F_f^*]$ is called robot dynamic uncertainty. In the ideal case where $\Delta D^* = \Delta h^* = 0$, and $F_f^* = 0$, the closed loop robot behavior satisfies the target impedance relationships

$$\ddot{X}_a + K_D\dot{X}_a + K_PX_a = \ddot{E} + K_D\dot{E} + K_PE \tag{8}$$

$$F_e = M\ddot{X}_a + B\dot{X}_a + KX_a = M\ddot{E} + K'_D\dot{E} + K'_PE \tag{9}$$

where $B = K'_D = MK_D$ and $K = K'_P = MK_P$ which turns out to be same as torque-based impedance control [3].

III A Simple Reference Trajectory Design Technique

Consider that the robot manipulator is working in an n dimensional task space. When the robot is in contact with an environment, a contact force F_e is exerted on the environment which can be determined as $F_e = K_e(X - X_e)$ where K_e is an n x n environment stiffness matrices, X is the n x 1 end effector position, and X_e is the n x 1 environment position. The problem of force control is to regulate F_e while the robot is moving along a trajectory on the surface of the environment. Impedance control approach calls for the specification of a desired target impedance function

$$M\ddot{E} + B\dot{E} + KE = F_e \tag{10}$$

where M, B, K are n x n positive definite diagonal matrices representing inertia, damping, and stiffness respectively, $E = X_r - X$, and X_r is the n x 1 reference trajectory the robot is commanded to track.

Regulating F_e to track a desired contact force F_d, X_r must be appropriately designed based on F_d, X_e, K, and K_e as follows :

$$x_r = x_e + \Delta x_r \qquad \Delta x_r = \frac{f_d}{k_{eff}} \tag{11}$$

where x_r, x_e, f_d represents an element of X_r, X_e, F_d respectively, and k_{eff} is

$$k_{eff} = \frac{kk_e}{k + k_e} \tag{12}$$

in which k and k_e are the corresponding elements of K and K_e. So when the environment stiffness uncertainty k_e is known exactly, it is easy to calculate the reference trajectory x_r from (11) and (12). In practice, k_e can not be accurately measurable so that x_r is also inaccurate resulting in poor force control performance.

Our proposed technique is to replace the environment stiffness k_e with f_e through the relationship

$$f_e = k_e(x - x_e) \tag{13}$$

Solving k_e yields

$$k_e = \frac{f_e}{x - x_e} \tag{14}$$

where f_e is the measured contact force obtained from a wrist force sensor. In practice (14) is only a close estimate of k_e as f_e may be subjected to measurement error. Substituting (14) into (12) yields

$$k_{eff} = \frac{k\frac{f_e}{(x-x_e)}}{k + \frac{f_e}{(x-x_e)}} = \frac{kf_e}{k(x - x_e) + f_e} \tag{15}$$

Combining (15) with (11) yields

$$x_r = x_e + f_d[\frac{k(x - x_e) + f_e}{kf_e}] \qquad (16)$$

This is the desired results of designing x_r, we see that the reference trajectory x_r is modified from x_e based on f_d, f_e, k, x, and x_e, all of which are assumed to be known at all times. We see that the trajectory modification term Δx

$$\Delta x = f_d \frac{k(x - x_e) + f_e}{kf_e} \qquad (17)$$

is indefinite at $f_e = 0$ when $x - x_e = 0$. To take care of this case, we determine the limits of Δx for $f_e = 0$ at $x - x_e = 0$ using $L'Hopital$'s rule which yields $\Delta x = \frac{f_d}{k}$. Therefore (16) is modified as

$$x_d = \begin{cases} x_e + \frac{f_d}{k} & \text{if } f_e = 0 \\ x_e + f_d[\frac{k(x-x_e)+f_e}{kf_e}] & \text{if } f_e \neq 0 \end{cases} \qquad (18)$$

It appears that this result is also applicable to robot position control during the non contact phase.

IV Stability Analysis

In this section we prove that the closed loop system is stable with the TMT. Rewriting (10) in a element of vector or matrix yields

$$m\ddot{e} + b\dot{e} + ke = k_e(x - x_e) \qquad (19)$$

Using $\ddot{x}_r = \ddot{x}_e, \dot{x}_r = \dot{x}_e$ substituting (16) into (19) yields

$$m\ddot{\varepsilon} + b\dot{\varepsilon} + k(\varepsilon + f_d(\frac{k(x - x_e) + f_e}{kf_e}) = k_e(x - x_e) \qquad (20)$$

where $\varepsilon = x_e - x$.

$$m\ddot{\varepsilon} + b\dot{\varepsilon} + (k + k_e)\varepsilon = -kf_d(\frac{k(x - x_e) + f_e}{kf_e}) \qquad (21)$$

Substituting $(x - x_e) = \frac{f_e}{k_e}$ into (21) yields

$$m\ddot{\varepsilon} + b\dot{\varepsilon} + (k + k_e)\varepsilon = -f_d(\frac{k + k_e}{k_e}) \qquad (22)$$

Since the f_d, k and k_e are positive constant the closed loop system (22) is BIBO stable.

At steady state (22) becomes

$$(k + k_e)(x_e - x) = -f_d(\frac{k + k_e}{k_e}) \qquad (23)$$

Dividing both sides by $k + k_e$ yields

$$f_d = k_e(x - x_e) = f_e \qquad (24)$$

Specially, if k_e is known the damping b for critically-damped case or over-damped case can be found as

$$b \geq 2 * \sqrt{m(k + k_e)} \qquad (25)$$

In most case k_e is not known exactly so the approximated k_e can give approximated damping b.

V Computer Simulation Study

A three link rotary robot representing the first three links of the puma 560 arm is used for simulation studies. The circular trajectory is tilted 45 degrees as shown in Figure 2(the first link of the robot manipulator is not shown). The robot is required to move on different environment stiffnesses with a desired contact force $f_d = 10N$. The impedance parameters are $M = I, B = K'_D = diag[60, 20, 20]$, and $K = K_P = diag[100, 100, 100]$.

In order to show the versatility of design technique, we test the environment stiffnesses to change with time as shown in Figure 3. The environment stiffnesses are changed after each completion of circular trajectory performed by the robot manipulator. The cycle of a circle trajectory is 4 secs.

Starting with $x = x_e$, the force tracking performance using (18) is plotted in Figure 4. As expected, the impedance controller is able to track effectively the desired contact force for a abruptly time varying stiffness profile of Figure 3. We observe that the sharp force overshoot occurs when the environment stiffness is abruptly changing. If the environment stiffness is continuous and smooth, then the resulting force overshoot is minimized and smooth. The corresponding position tracking in xyz-axis is shown in Figure 5. We can see that the position tracking performance is also excellent.

VI Concluding Remarks

The trajectory modification technique developed above gives a simple solution to the practical problem of imprecisely known environment stiffness in impedance control. The new technique for reference trajectory design does not require knowledge of the environment stiffness and it is very simple to apply. The great advantage of the TMT is not requiring any cost or complexity that might effect the stability of the robot system. Since the TMT is developed purely

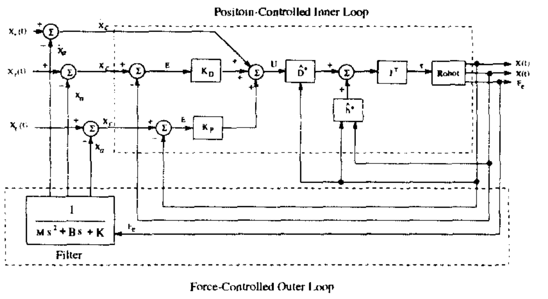

Figure 1: Position Based Impedance Force Control Structure

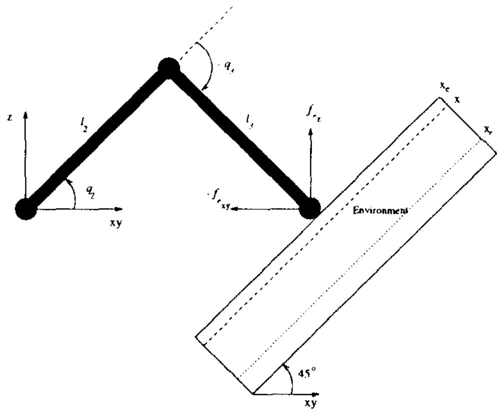

Figure 2: Circular Tracking on 45° Tilted Surface Environment

based on algebraic calculation the stability of the system with the TMT is guaranteed as we proved. Simulation results confirmed that the robot is able to regulate the contact force as desired even though the environment stiffness is time varying.

References

[1] M. H. Raibert and J. J. Craig, "Hybrid position and force control of robot manipulators", *ASME Journal of Dynamic Systems, Measurement, and Control*, pp. 126–133, 1981.

[2] D. A. Lawrence, "Impedance control stability properties in common implementations", *Proc. of the IEEE International Conference on Robotics and Automation*, pp. 1185–1190, 1988.

[3] N. Hogan, "Impedance control : An approach to manipulator, part i, ii, iii", *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 3, pp. 1–24, 1985.

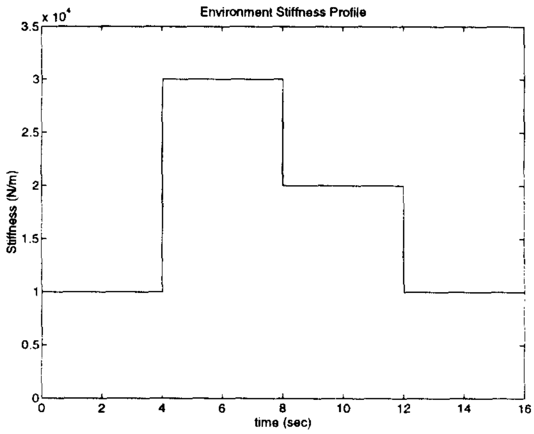

Figure 3: Unknown Environment Stiffness Profile

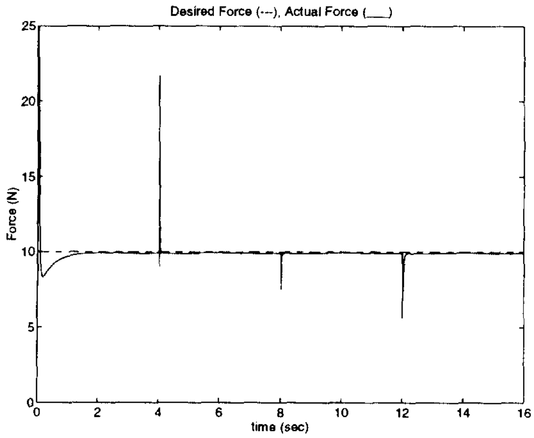

Figure 4: Force Tracking

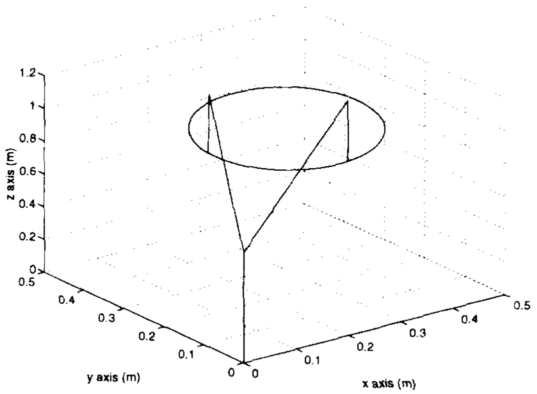

Figure 5: Position Tracking