A Development of Input and Output Interfaces for Fuzzy Hierarchical Analysis

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Abstract

Fuzzy hierarchical analysis(FHA) has the usefulness to allow decision maker's ambiguities when comparing two alternatives. But, for easily appling it to a decision problem, the handling its many data and for decision makers much not knowing fuzzy theory are the obstacles to must be overcomed even if the results of final fuzzy weights can be computed by a personal computer. This paper decribes that FHA is revised, and input/output interfaces are developed to collect input data easily and interprete the fuzzy resutlts. Finally, a fuzzy decision process is suggested with them.

Keywords : Fuzzy Hierarchical Analysis(FHA), Fuzzy Number, Input/Output Interfaces

1. Introduction

Numerous applications of the analytic hierarchy process(AHP) to a wide variety of decision problems, such as economics and planning, policies and allocations of resources, health, project selection, marketing, and so on, have been made since its development by T.L.Saaty in the 1970s[3][5]. It is desirable that fuzzy theory is considered in the ratios of AHP, because these are resulted from the decision maker's qualitative judgments for the pairwise comparisons of two alternatives. J.J.Buckley extended Saaty's AHP to the case where decision makers are allowed to employ fuzzy ratios in place of exact ratios [1]. It, fuzzy hierarchical analysis(FHA), has an usefulness to be able to express a decision maker's feeling or ambiguity when comparing two alternatives. But, when FHA is applied, its process is very hard because of increasing the amount of input data and the complexity of calculations, and considering the fuzzy theory. This paper describes a fuzzy decision process. This process is composed of FHA to be able to analysis incomplete hierarchies and input/output interfaces to collect input data easily and to help to interprete the fuzzy results. And, in the process it is available for multiple experts to participate one decision problem together with. The input and output interfaces are represented graphically.

The input interface serves as a pre-processor for collecting fuzzy ratios. The output interface serves as a post-processor for interpreting the fuzzy results. By this process, a decision problem can be solved easily without too much efforts and time.

2. Fuzzy Hierarchical Analysis[1]

It is difficult for people to always assign exact ratios when comparing two alternatives (issues, candidates,...). Buckley developed Saaty's hierarchical analysis, when the decision makers(experts, judges,...) are allowed to use fuzzy ratios in place of exact ratios. fuzzy numbers are used in the ratios. Fuzzy numbers can express a decision maker's feeling that a ratio is approximately 5 to 1 instead of exactly 5/1, or that a ratio is between 6 to 1 and 8 to 1 instead of exactly 7/1. The fuzzy ratio $\overline{a}_{ij}$ is

$$\overline{a}_{ij}=(\alpha_{ij}/\beta_{ij},\ \gamma_{ij}/\delta_{ij})$$

$$0<\alpha\leq\beta\leq\gamma\leq\delta$$

$$\alpha,\beta,\gamma,\delta\in\{1,2,\ldots,9\}$$

$$\overline{a}_{ji}=(\overline{a}_{ij})^{-1}=(\delta_{ij}^{-1}/\gamma_{ij}^{-1},\ \beta_{ij}^{-1}/\alpha_{ij}^{-1}).$$

Let $\overline{A}=[\overline{a}_{ij}]$ be the m×m fuzzy positive reciprocal matrix of all paired comparisons for the alternatives $A_1, A_2, \ldots, A_m$. A fuzzy positive reciprocal matrix $\overline{A} = [\overline{a}_{ij}]$ is consistent if and only if $\overline{a}_{ik} \otimes \overline{a}_{kj} \approx \overline{a}_{ij}$. To obtain the final set of weights for the alternatives, firstly, the fuzzy weights $\overline{w}_i$ of any fuzzy positive reciprocal matrix $\overline{A}$ are calculated by λ-max procedure. The geometric mean technique for computing the weights is easily extended to fuzzy positive reciprocal matrices $\overline{A}$. For generalize the λ-max method, considering

$$\overline{A} \otimes \overline{w} = \overline{\lambda} \otimes \overline{w},$$

where $\overline{w}^T = (\overline{w}_1, \ldots, \overline{w}_m)$ and the $\overline{w}_i$ and $\overline{\lambda}$ are fuzzy numbers.

Finally, final fuzzy weights, and the final ranking of the alternatives are obtained. Fuzzy positive reciprocal matrix $\overline{A}_k$ of pairwise comparisons can be obtained for each criterion C_K(aspect, characteristic,...), and also a fuzzy positive reciprocal matrix $\overline{E}$ for the pairwise comparisons of the criteria can be obtained. Fuzzy weights $\overline{w}_k = (\overline{w}_{1k}, \ldots, \overline{w}_{mk})$ are computed for each $\overline{A}_k$ and fuzzy weights $\overline{e} = (\overline{e}_1, \ldots, \overline{e}_K)$ are derived from $\overline{E}$. The final fuzzy weight for alternative A_i is

$$\overline{f}_j = (\overline{w}_{j1} \otimes \overline{e}_1) \oplus \cdots \oplus (\overline{w}_{jK} \otimes \overline{e}_K).$$

The membership functions for the $\overline{f}_j$ are easily found from the membership functions for the $\overline{w}_{jk}$ and $\overline{e}_j$. Again, one might multiply each $\overline{f}_j$ by a suitable constant so that all the $\overline{f}_j$ have their support in [0, 1].

3. Development of A Fuzzy Decision Process

The purpose of this paper is the development of The fuzzy decision process can be applied easily in decision problems.

3.1 Design of input/output interfaces for FHA

In collecting fuzzy ratios, It may be very cumbersome and much time-spended to handle data because of too many data and not knowing fuzzy theory. In FHA, the input data are more than in Saaty's AHP. The number of input data in Saaty's AHP is

$$\frac{1}{2}\sum_{i=2}^{N} n_i(n_i - 1)n_{i-1},$$

where N is the number of levels, and n_i is the number of alternatives in level i. It is the number of the upper-right elements in the all of matrices excepting diagonal elements. The number of input data in FHA simply is four times to Saaty's because of using a fuzzy number having four values α, β, γ, and δ for each ratio. In the process developed, the diagonal elements in all of matrices are considered to be able to analyze an incomplete hierarchy. Then, the number of input data is

$$2\sum_{i=2}^{N} n_i(n_i + 1)n_{i-1}.$$

For the example having three hierarchical levels and 1,7,4 alternatives for each level, respectively. The difference of the number of input data between the process developed and Saaty's AHP is 329 by the above equations. Therefore, to use easier FHA, first of all, a method of easily handling and collecting the input data must be suggested. Figure 1 is an input interface to collect data easily. Designing the hierarchical structure of a decision problem, and generating fuzzy ratios

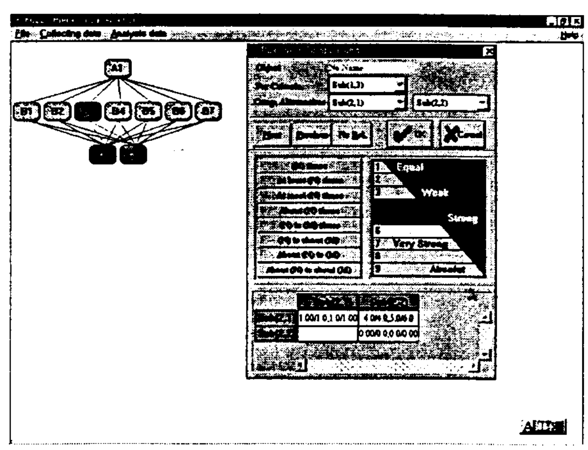

Figure 1. An Input Interface for the Fuzzy Decision Process

and formulating its matrices are available by this input interface. Finally, it generates an input file to proceed to compute fuzzy weights. The results of FHA applied fuzzy theory are also fuzzy. Then, an output interface is suggested to help to interprete the results for decision makers being unfamiliar with fuzzy theory. The output interface graphically represents all of the final fuzzy weights on a ginven hierarchical structure, and the membership function graphs of the final fuzzy weights and its α-cut resluts for the last level of the hierarchy. Figure 2 is an output interface developed.

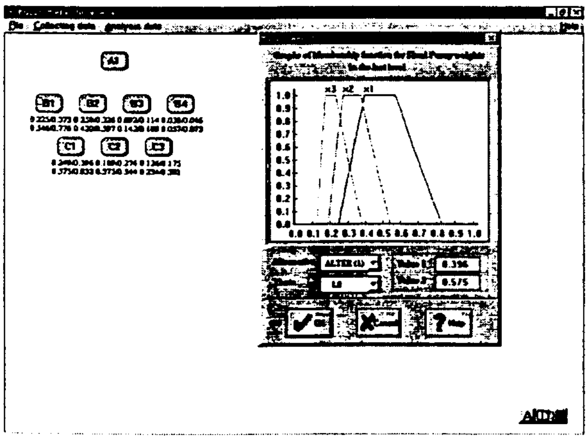

Figure 2. An Output Interface for the Fuzzy Decision Process

3.2 General steps for the fuzzy decision process

Using AHP in solving a decision problem involves four steps[2][5]. The process having input/output interfaces generally adapts these four steps.

Step 1 : Setting up the decision hierarchy.
Breaking down a given decision problem into a hierarchy of interrelated decision elements. Make sure to the number of levels and alternatives for each level, interrelationship between criteria and alternatives for each level, and the hierarchy being complete or incomplete.

Step 2 : Input data related the hierarchical structure and collecting input data by input interface.
On the input interface, input the data in step 1, and generating fuzzy ratios of pairwise comparisons of alternatives can be made by the selecting several icons of the scales of the relative importance(weights, or brightness, etc.) presented step-by-step automatically. If $\alpha=\beta=\gamma=\delta$ for the all fuzzy ratios, the hierarchical analysis is not fuzzy case such as Saaty's AHP. If multiple experts participate to a decision problem, the step 2 is repeated as many as experts. After each repeat, an individual input file is generated by the input interface. Before computing fuzzy weights, every input files must be added to a delegative data. The grouping is processed by arithmetic mean, geometric mean, or weighted mean techniques. If fuzzy consistant ratios can be obtained, one method of allocating weights is to use the consistant ratios of every matrices like as Shen's method[4].

Step 3 : Computing final fuzzy weights
By Buckley's FHA, the final fuzzy weights are computed by an input file generated in step 2. But, in this computing, the $\alpha=\beta=\gamma=\delta=0$ for some fuzzy ratios including the diagonal elements of matrices can be available, and the following arithmeic mean is used for affording these zeros in place of the geometric mean technique mentioned in Buckley's FHA.

$$\bar{r}_i = (\bar{a}_{i1} \oplus \cdots \oplus \bar{a}_{im})/m \quad \text{and}$$

$$\bar{w}_i = \bar{r}_i \otimes (\bar{r}_1 \oplus \cdots \oplus \bar{r}_m)^{-1}.$$

By this method, an incomplete hierarchy can be studied as a complete hierarchy. Using this arithmetic mean, like as the geometric mean, also result to the good estimates[3].

Step 4 : Representing and interpreting the fuzzy results by output interface.
The output interface represents graphically a hierarchical structure with final fuzzy weights, the membership function graphs of the final fuzzy weights for the last level in the hierarchy, and its α-cut results.
Figure 3 is a fuzzy decision process developed.

4. An Example

The decision problem is which type of interface is the most usable for users. Consistency, functionality, effectiveness, learnability, information feedback, error prevention, and

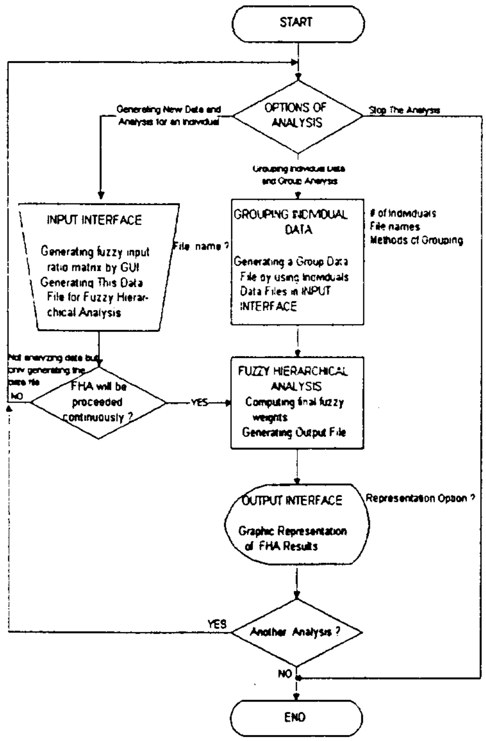

Figure 3. A Fuzzy Decision Process

user's satisfaction are contributed to the usability of interface, and these factors are dependent on the types of interfaces:menu-keyboard, icon-mouse, icon-trackball, and command-keyboard input.

Step 1 :The structure of decision hierarchy is composed of three levels, and has one, seven, and four alternatives for each level, respectively. The fist level is an overall objective, the usability of interface. The alternatives of the second level are consistency,..., and user's satisfaction. The alternatives of last third level are the types of interfaces.

Step 2 : Firstly, input the number of levels and alternatives for each level: 3, 1, 7, and 4. Secondly, by the input interface, the pairwise comparisons are made. There are 98 comparisons computed by the equation mentioned previously.

Step 3 : Computing final fuzzy weights.

Step 4 : The final fuzzy weights for each level and alternative elements are represented with the hierarchical structure as Figure 2. The membership function graphs of membership functions for 4 final fuzzy weights in last level 3, and its α-cut results are represented. In the resul s, The menu-keyboard input type of interface is the most usable for user. The decision making for this example excepting the step 1 of setting up the hierarchy was completed within about 25 minutes by the process developed.

5. Discussions

Fuzzy hierarchical analysis has the usefulness to allow decision makers' ambiguity when comparing two alternatives. But, for appling it easily to a decision problem, the handling its many data and for decision maker much not knowing fuzzy theory are the obstacles to must be overcomed even if the results of final fuzzy weights can be computed by a personal computer. This paper developed the fuzzy decision process having mainly output/input interfaces to make trouble-shooting about these problems. But, as mentioned Buckley, more research is needed on fuzzy eignvalues and vectors of fuzzy positive reciprocal matrices[1]. Also, the computing procedure of the consistency for fuzzy ratios will must be developed.

References

1. Buckley, J. J., 1985, Fuzzy Hierarchical Analysis, Fuzzy Set and Systems 17:233-247, Elsevier Science Publishers B.V., North-Halland.
2. Johnson, C. R., 1980, Constructive critique of a hierarchical prioritization scheme employing paired comparisons, Proceedings of the International Conference of Cybernetics and Society of the IEEE:373-378.
3. Saaty, T. L., 1980, The Analytic Hierarchy Process, McGraw-Hill, Inc., USA.
4. Shen, R., Meng, X., and Yan, Y., 1990, Analytic Hierarchy process applied to synthetically evalute the labour intensity of jobs, Ergonomics 33(7):867-874.
5. Zahedi, F., 1986, The Analytic Hierarchy Process-A Survey of the Method and its Applications, The Institute of Management Sciences, Interfaces 16(4):96-108.