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Benefit of Sag Vertical Curves for Rail Transit Routes

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Benefits of Dipped Vertical Alignments for Rail Transit Routes

Abstract

Dipped track profiles between rail transit stations can significantly reduce propulsive energy, braking energy and travel times. This work quantifies their potential benefits for circumstances reflected in various values for dips, speed and acceleration limits, station spacings, and available power. A deterministic simulation model has been developed to precisely estimate train motions and performance using basic equations for kinematics, resistance, power and braking. For a dip of 1% of station spacing, in which gradients never exceed 4%, our results show savings (compared with level tracks) exceeding 9% for propulsive energy, 15% for braking energy and 5% for travel time between stations.

Introduction

A dipped profile for the vertical track alignment between rail transit stations, as sketched in Fig. 1, takes advantage of gravity for accelerating as well as decelerating trains. In Fig. 1, δ is the maximum dip magnitude while the percent may be defined as $100\ \delta/S$, where S is the station spacing. A dipped profile offers potential benefits in (1) propulsion energy requirements, (2) braking energy requirements, and (3) travel times between stations. The travel time savings can reduce operator as well as user costs. Dipped alignments also allow stations to remain at higher elevation, nearer their surface riders, than a level alignment when crossing under rivers. The main potential problems of such a profile are:

(1) increased construction cost,

(2) centrifugal acceleration in the vertical plane, and

(3) maximum gradient climbing ability of trains.

Problem (1) above may be practically negligible if deep tunnels are used anyway. However, if cut-and-cover construction is otherwise practicable, a dipped profile would definitely increase excavation cost and/or force a switch to deep tunnel construction. Problem (2) constrains the speed and vertical curvature to some combinations satisfying passenger comfort requirements. Problem (3) limits the gradient to values that trains can climb without momentum, i.e., after they have stopped in the dipped section.

This paper develops a model for quantifying the potential benefits of the dipped profile concept. In focusing on benefit estimation, it leaves the estimation of any additional construction costs to future work. It also treats the maximum gradient and vertical acceleration problems as exogenously specifiable constraints rather than as design variables to be optimized. The potential benefits of dipped profiles are shown to be significant enough to justify additional development work.

Our literature review has not identified any published analysis on vertical profiles deliberately dipped between transit stations to exploit gravity. However, this concept was analyzed in a course project at the University of Maryland by several students, including Andreadis(1991). Various researchers, including Vuchic(1981), Uher and Sathi(1983), Holden (1980) and Kikuchi(1991) have analyzed velocity and acceleration profiles offering interesting tradeoffs between travel time and energy consumption. Three such profiles are illustrated in Fig. 2. Uher and Sathi(1983) provide some valuable references to previous studies in rail transit energy management.

It should be noted that dipped profiles have been used out of necessity when railroad tracks had to be tunnelled under rivers. Hence, there is no doubt the concept is practically feasible. The question is whether it is desirable to deliberately dip track alignment in the absence of physical constraints.

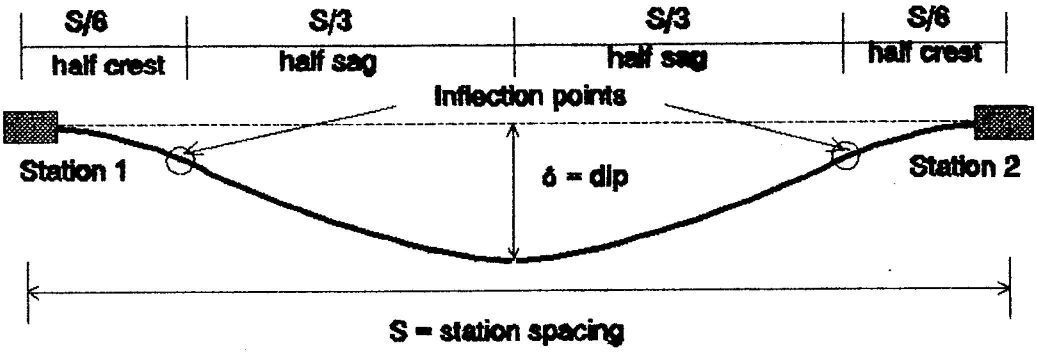


Fig. 1 **Dipped profile concept**

Methodology

The basic methodology in this work relies on deterministic simulation of train movements over a specified vertical alignment. Vehicle kinematics, resistances, tractive effort, power, propulsive energy and braking energy are computed with basic engineering equations. The results are then compared for different cases representing various exogenous conditions and design or control options.7

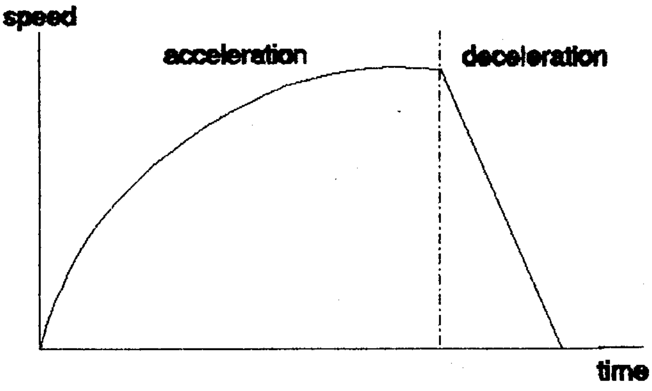
The evaluation focuses on energy consumption and travel time. Braking energy and tractive energy are considered separately since they require different resource expenditures. Furthermore, braking energy may be considerably cheaper than tractive energy to provide (e.g. may simply require some maintenance to compensate for brake wear), and may even be transformed into tractive energy if a regenerative braking system is acquired. After analyzing a baseline case, sensitivity analysis is conducted with respect to several variables, including dip, station spacing, maximum allowable acceleration, train power and speed constraints.

Model Formulation

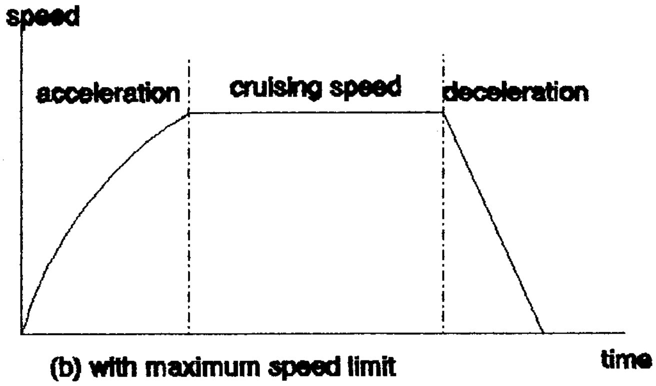
The model developed here is designed to analyze train travel time and energy consumption over a dipped profile, such as that shown in Fig. 1. The basic question examined is whether dipped profiles have significant advantages, rather than finding the exact optimal geometry and speed profile for all conceivable situations. Hence, the following assumptions are considered acceptable:

1. The vertical profile is symmetric about a central axis
2. AREA (American Railway Engineering Association) guidelines for vertical curvature are applicable.
3. Horizontal curvature is negligible.
4. Trains use maximum allowable power at any time (for baseline case).
5. The full weight of the train contributes to initial tractive effort.

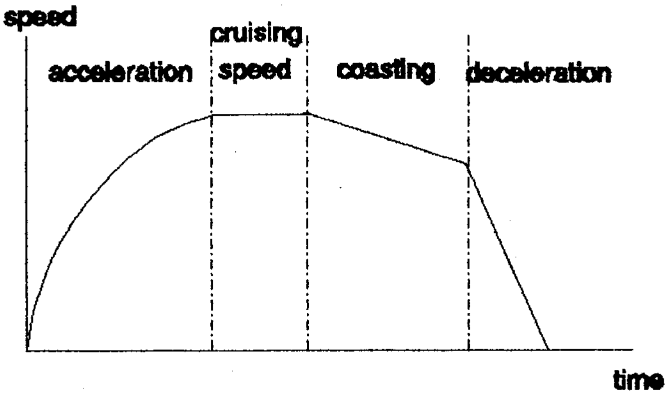
Assumption 1 reflects what the optimal profile would be with homogeneous tunneling conditions and end stations with equal elevations. Adjustments for different elevations of the end stations (station 1 and station 2 in Fig. 1) can be easily made by shifting the low point of



(a) without maximum speed limit



(b) with maximum speed limit



(c) with coasting

Fig. 2 Speed profile for different operating characteristics the dip toward the lower station and re-applying Assumption 2. However, non-uniform tunneling conditions may present very many vertical alignment alternatives for investigation.

Assumption 2 means that parabolic vertical curves may be used, with the sag curves being twice as long as the crest curves as shown in Fig.1. The rate of change of grade (r) should not exceed 0.10 on crests and 0.05 on sags for main lines according to AREA(1981). However, it seems probable that at speeds above 130 km/h the r values should be even lower to reduce centrifugal acceleration in the vertical plane.

Assumption 3 is reasonable within the scope of this work, especially since horizontal curvature is often zero between stations and also because horizontal curvature may be analyzed separately from vertical curvature.

Assumption 4 means that trains are assumed to accelerate as quickly as possible to the highest allowable speed. That acceleration may be limited by passenger comfort considerations (4.3 ft/sec^2 is used here as a baseline acceleration limit) and by a maximum cruising speed (which may reflect safety and/or energy considerations). Trains are assumed to maintain maximum allowable acceleration or cruising speed until reaching a point where braking at the maximum allowable normal deceleration rate is required to reach zero speed at station 2, as in Figs. 2a and 2b. Thus, "coasting" (i.e., zero power) or partial power cruising are not considered in this paper. Future work on coasting and partial power operation may identify desirable tradeoffs between travel time and operating costs.

Assumption 5 means that the adhesive force that limits tractive effort at low speeds is generated by the full weight of vehicles. This is realistic since in modern transit systems all trains axles are usually powered.

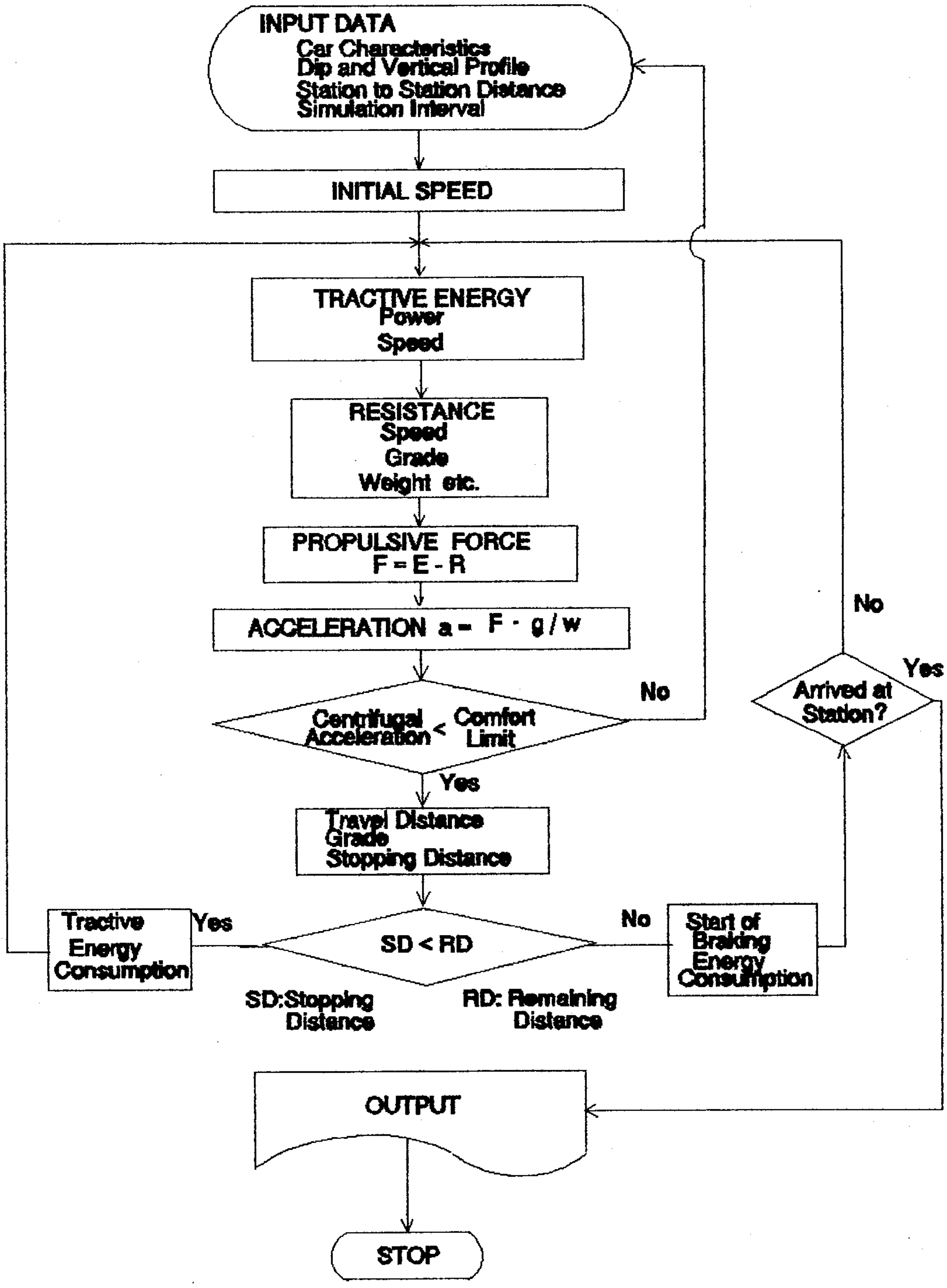


Fig. 3 **Model Flow Chart**

The main relations used to determine the location, speed and acceleration of a vehicle at any point is the acceleration (a) equation.

$$a = \frac{F}{\rho m} = \frac{F \cdot g}{\rho W} = \frac{(E - R)g}{\rho W} \qquad (1)$$

where F is force, E is tractive effort, m is mass of car, g is gravity force, W is weight of car and ρ is a coefficient for rotating masses (including wheels, shafts, and axles).

In Eq. (1) the net accelerating force is the difference between tractive effort (E) and resistance (R). The resistance includes the effect of gravity (which may produce a positive or negative resistance) and the effect of brakes. The acceleration should not exceed the maximum value a_{max} based on the comfort of standees.

$$a = \min \{ a_{\max} , \frac{(E-R)g}{\rho W} \} \qquad (2)$$

The maximum tractive effort (E_{max}) is limited by available propulsive force (F_p) at high speeds and by adhesive force (F_a) at low speeds. Hence it is the minimum of these two forces:

$$E_{\max} = \min \{ F_p , F_a \} \qquad (3)$$

The propulsive force is:

$$F_p = \frac{375 \, \eta \, P}{V} \qquad (4)$$

where P is the power, V is the speed and η is transmission efficiency coefficient. For a typically used η value of 0.82(Hay), Eq. 4 becomes

$$F_p = \frac{308 \cdot P}{V} \tag{5}$$

The adhesive force is:

$$F_a = \mu\, W \cos\theta \tag{6}$$

where μ the is coefficient of friction and θ is the gradient angle. Following Vuchic (Fig. 3.22, p155), with appropriate unit transformations, and assuming a linear change in μ between 0 and 80 kph, $\mu = 0.30 - ((0.30-0.18)/80)\ V\ (1.609)$ where the unit of V is mph. It may be noted that $\cos\theta$ approaches 1.0 for the small gradients of rail transit systems.

Since trains do not always use full power, the actual tractive effort needed at each step to overcome resistance and accelerate can be derived by re-arranging Eq. (1):

$$E = a\, \rho\, \frac{W}{g} + R \tag{7}$$

Resistance is calculated here using the well known Davis equation (Hay,1982). The unit resistance (in lb of force per ton of vehicle weight) is

$$R_u = 1.3 + \frac{29}{w} + bV + \frac{CAV^2}{wn} + 20G + 0.8D \tag{8}$$

where w is weight per axle, n is number of axles per car, b is a flange friction coefficient, A is the vehicle cross-sectional area, G is the gradient in percent and D is the degree of curvature. The total resistance of a vehicle, R_v, is the unit resistance R_u multiplied by the vehicle weight W (in tons):

$$R_v = R_u\, W = R_u\, w\, n \tag{9}$$

The total resistance of a train, R_T, is obtained by summing the resistances of the vehicles in the train or, if the vehicles have similar resistances, by multiplying the vehicle resistance by the number of vehicles.

$$R_T = \sum_{all\ v} R_v \tag{10}$$

In general, at least the leading car will have a different resistance, since it encounters greater aerodynamic drag. As previously stated tractive effort, and hence acceleration, are limited by adhesive force as well as available power. Additional factors limiting the tractive effort that should be applied at any instant are policy limits on acceleration and speed. The acceleration limit (typically plus or minus 0.15 g) is usually specified for passenger comfort, since passengers have difficulty standing at higher accelerations. The speed limit may be imposed for safety reasons or to reduce energy and maintenance costs.

The energy required to either propel or to brake a train is

$$e_i = \frac{P_i\, \Delta t}{3600\ (1.341)} \tag{11}$$

where e_i is energy use during step i (kwh), P_i is the power used during step i and Δ t is simulated time increment.

The actual braking force F_b used to decelerate a train is the minimum of two forces:

$$F_b = \min \{ F_{bc} , F_{ba} \} \tag{12}$$

In Eq.12, F_{bc} is a comfort-limited braking force which avoids exceeding acceleration limits for standees, while F_{ba} is an adhesion-limited braking force. F_{bc} can be obtained by rearranging Eq.1 as

$$F_{bc} = a_{\max} \frac{W}{g} \rho + R \tag{13}$$

where a_{max} is the maximum allowable acceleration. F_{ba} is the product of the friction coefficient μ and the normal force (W cosθ)

$$F_{ba} = \mu \; W \cos\theta \tag{14}$$

The rate of change (r) in grade in percent per 100 ft is defined as

$$r = \frac{G_1 - G_2}{L} \tag{15}$$

where G_1, G_2 are the percent grade of the two intersecting tangents, and L is curve length in 100 ft stations.

For high speed rail lines this rate should not exceed .05 for sag curves and .1 for crest curves (Hay, 1982). The gradient is needed to calculate grade resistance and vertical centrifugal force. To obtain the gradient in each increment, the vertical curve equations (Eqs. 16-18) are differentiated. The parabolic vertical curve equations can be determined if the station spacing and maximum dip halfway between two stations are specified. Since sag curves should be twice as long as crest curves and crest curves are centered at stations, a station to station profile will include a half-crest from $0 < x < S/6$, a sag when $S/6 < x < 5S/6$ and a half-crest for $5S/6 < x < S$ as shown in Fig. 4. (Short vertical transition curves would also be required at S/6 and 5S/6 to avoid excessive vertical jerk rates.) The profiles are expressed as follows:

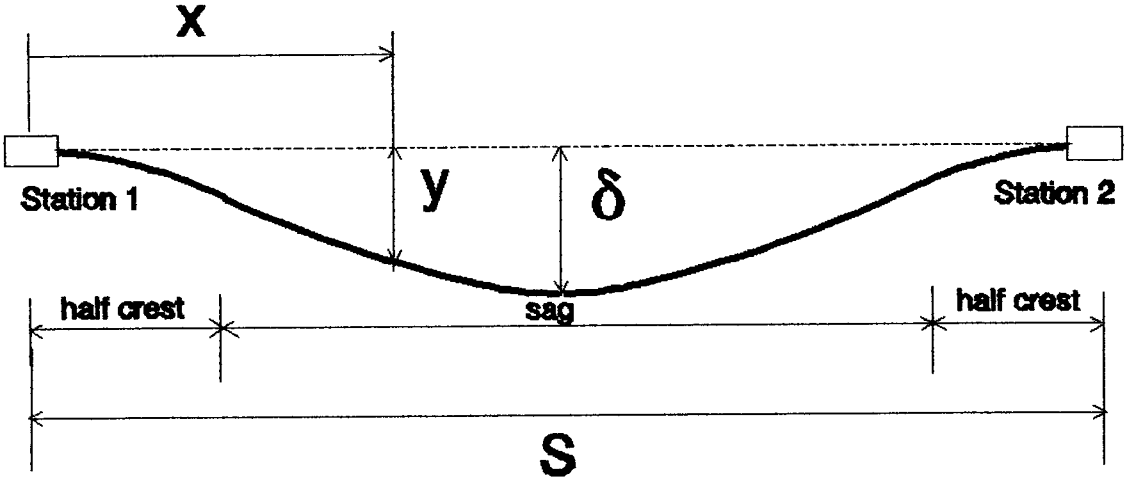


Fig.4 Vertical Railroad Profile with Dip

$$y_1 = -\frac{12\delta}{S^2}x^2 \qquad for \quad 0 \le x \le \frac{S}{6} \qquad (16)$$

$$y_2 = \frac{6\delta}{S^2}x^2 - \frac{6\delta}{S}x + \frac{\delta}{2} \qquad for \quad \frac{S}{6} \le x \le \frac{5S}{6} \qquad (17)$$

$$y_3 = -\frac{12\delta}{S^2}x^2 + \frac{24\delta}{S}x - 12\delta \qquad for \quad \frac{5S}{6} \le x \le S \qquad (18)$$

In Eqs. (16)-(18), y_1, y_2, y_3 is the difference in elevation with respect to that of station 1, x is the horizontal distance from the center of station 1, δ is the maximum dip, halfway between two stations.

The following equation is used to compute vertical acceleration (a_c) and check that it does not exceed passenger comfort limits in each simulation step:

$$a_{c_i} = \frac{V_i^2}{R_i} \tag{19}$$

where V_i is the velocity at step i, R_i is the radius of vertical curvature increment i. R_i is computed by dividing the distance increment (Δd_i) by the angular increment ($\Delta\theta_i$) between simulation steps i-1 and i:

$$R_i = \frac{\Delta d_i}{\Delta\theta_i} \tag{20}$$

The angular increment is computed as

$$\Delta\theta_i = |\ \arctan G_i(x) - \arctan G_{i-1}(x)\ | \tag{21}$$

where $G_i(x)$ is gradient in i-th step.

Differentiating Eqs. (16)-(18), the gradient can be obtained as follows:

$$
\begin{aligned}
G(x) &= y_1'(x) = -\frac{24\delta}{S^2}x && \text{for} \quad 0 \le x \le \frac{S}{6} \\
G(x) &= y_2'(x) = \frac{12\delta}{s^2}x - \frac{6\delta}{S} && \text{for} \quad \frac{S}{6} \le x \le \frac{5S}{6} \\
G(x) &= y_3'(x) = -\frac{24\delta}{s^2}x + \frac{24\delta}{s} && \text{for} \quad \frac{5S}{6} \le x \le S
\end{aligned}
\tag{22}
$$

Results

The simulation is conducted for a baseline case and then repeated with various parameter changes for sensitivity analysis. The baseline case for this paper has following parameter values:

Station spacing	10,000 ft (3,000 m)
Maximum acceleration	4.265 ft/sec^2 (1.3 m/sec^2)
Power per car	4 × 130 kw
Speed constraint	None
Percent dip (100δ/S)	0.5 %

The simulation may be run with various time increments. Smaller time increments increase accuracy but also increase simulation time. Relatively short simulation increments (0.02 sec) were used in this study. The simulation output was printed every 5 seconds (i.e., every 250 increments).

The output report shown in Fig. 5 includes the travel time(TIME), speed(SPD), tractive effort(TE), unit resistance(RU), train resistance(RST), difference between tractive effort and resistance(TE-R), acceleration(ACCL), distance travelled during each increment(DIST), cumulative distance(CDIST), grade(GRD), tractive energy(ENGY), cumulative tractive energy(CUME), braking energy(BEGY), and cumulative braking energy(CBEGY). Additional items of interest such as time and distance to reach maximum speed, time and distance to start braking, maximum vertical centrifugal acceleration, and car characteristics are also printed out. The sample output shown in Fig.5 represents the baseline case.

```
 ** REPORT # 1 **  SIMULATION OF VERTICAL RAIL PROFILE WITH DIPS

STATION TO STATION DISTANCE= 10000.0   FT
DIP                        =    50.0   FT
TIME INCREMENT             =     .02   SEC

TIME   SPD   TE      RU   RST  TE-R  ACCL DIST CDIST  GRD  ENGY CUME BEGY CBEGY
(SEC)(FT/S)(TON) (LB/T) (TON)(TON)(FT/S2) (FT)  (FT)  (%)(KWH)(KWH)(KWH)(KWH)
  5.0  26.   37.     4.   0.    37.  4.3   1.    75.  -.1 .016  2.3 .000    .0
 10.0  43.   22.     1.   0.    22.  2.8   1.   249.  -.3 .017  6.6 .000    .0
 15.0  55.   17.    -4.   0.    18.  2.2   1.   496.  -.6 .017 11.0 .000    .0
 20.0  66.   14.   -10.  -1.    16.  2.0   1.   800. -1.0 .017 15.3 .000    .0
 25.0  75.   13.   -17.  -2.    15.  1.9   2.  1153. -1.4 .017 19.6 .000    .0
 30.0  84.   11.   -25.  -3.    14.  1.8   2.  1553. -1.9 .017 23.9 .000    .0
 35.0  93.   10.   -22.  -3.    13.  1.6   2.  1998. -1.8 .017 28.3 .000    .0
 40.0 101.    9.   -15.  -2.    11.  1.4   2.  2484. -1.5 .017 32.6 .000    .0
 45.0 107.    9.    -7.  -1.    10.  1.2   2.  3005. -1.2 .017 36.9 .000    .0
 50.0 113.    8.     1.   0.     8.  1.1   2.  3557.  -.9 .017 41.3 .000    .0
 55.0 118.    8.     9.   1.     7.   .9   2.  4135.  -.5 .017 45.6 .000    .0
 60.0 122.    8.    17.   2.     6.   .7   2.  4735.  -.2 .017 49.9 .000    .0
 65.0 125.    8.    25.   3.     4.   .6   3.  5354.   .2 .017 54.3 .000    .0
 70.0 128.    7.    34.   4.     3.   .4   3.  5987.   .6 .017 58.6 .000    .0
 75.0 130.    7.    42.   5.     2.   .3   3.  6630.  1.0 .017 62.9 .000    .0
 80.0 131.    7.    50.   6.     1.   .2   3.  7281.  1.4 .017 67.3 .000    .0
 85.0 131.    7.    58.   7.     0.   .0   3.  7936.  1.8 .017 71.6 .000    .0
 90.0 111.    0.    53.   1.     0. -4.3   2.  8546.  1.7 .000 71.9 .056  14.1
 95.0  90.    0.    36.   1.     0. -4.3   2.  9049.  1.1 .000 71.9 .048  27.2
100.0  69.    0.    23.   0.     0. -4.3   1.  9446.   .7 .000 71.9 .038  37.9
105.0  47.    0.    14.   0.     0. -4.3   1.  9737.   .3 .000 71.9 .027  46.1
110.0  26.    0.     7.   0.     0. -4.3   1.  9920.   .1 .000 71.9 .015  51.4

TIME TO REACH MAX. SPEED(SEC):   85.4    DIST TO REACH MAX. SPEED(FT) : 7985.9
TIME TO START BRAKING(SEC)   :   85.4    DIST TO START BRAKING(FT)    : 7985.9

MAX. ACCELERATION (FT/S^2)   :  4.265    MAX. DECELERATION (FT/S^2)   : -4.265

NUMBER OF CARS PER TRAIN     :     6     WEIGHT OF CAR IN TON         :    40.
AXLES PER CAR                :     4     HORSEPOWER PER TRAIN         :  4182.
CROSS SECTION AREA IN FT^2   :   113.    FLANGE FRICTION  b           :.03000
AIR DRAG COEFFICIENT  C      :  .00070

VERTICAL CENTRIFUGAL ACCELERATION(FT/S^2):
   MAX:       .002        MIN:      -.003
```

Fig. 5 **Sample Program Output for Baseline Case**

The simulation results for the baseline case with a o.5 % dip and for comparison, additional cases with dips of 0 % and 1.0 % are discussed below.

Tractive energy

Table 1 shows how dipped profiles can reduce energy consumption. A 0.5 % dip reduces tractive energy by 4.9 % while a 1.0 % dip reduces tractive energy by 9.4 % compared with a level profile (0 % dip), Table 2 shows that the cumulative tractive energy consumption is equal during the first 80 seconds regardless of the dip, because trains use full power until maximum speed is reached. However trains travel farther in those 80 seconds if dips increase. As Fig. 8 shows, speeds and distances reached increase faster as dips increase. Since there is no speed constraint in the baseline case, tractive energy consumption stays constant until braking is required as shown in Fig. 6.

Braking energy

Table 1 shows that braking energy consumption also decreases as dip increases. Braking energy consumption ranges here from 72 % to 77 % of tractive energy consumption depending on the dip. As dip increases, trains start to brake earlier in order to decelerate from higher speeds. Compared with a level profile, braking energy savings are 8.4 % for a 0.5 % dip and 15.7 % for a 1.0 % dip. Braking starts and ends earlier and total energy consumption (i.e., the area under the curve) decreases as the dip increases, as shown in Fig. 7.

Travel Time

Travel time also decreases as dip increases. In Table 1, compared to a level profile(0 % dip), we observe a 3.6 % time saving for a 0.5 % dip and a 4.7 % saving for a 1.0 % dip.

Speed profile

Fig. 8 shows that as the dip increases and available power stays the same, a higher speed is reached. Furthermore, the speed approaches its highest value earlier. A speed of 141 km/hr (129 ft/sec) is reached 90 seconds after starting on a level profile while 148 km/hr (135 ft/sec) is reached 55 seconds after starting on a 1.0 % dip.

Table 1. **Effect of Dip on Energy Requirements and Travel Time**

Item	Level	(%)	Dip(0.5%)	(%)	Dip(1.0%)	(%)
Tractive Energy(kwh)	75.6	0.0*	71.9	-4.9	68.5	-9.3
Braking Energy(kwh)	58.5	0.0	53.6	-8.4	49.3	-15.7
Travel Time(sec)	119.0	0.0	114.7	-3.6	113.5	-4.7

* % difference compared with level profile

Table 2. **Effect of Dip on Cumulative Energy Consumption**

Time (sec)	LEVEL		DIP(0.5%)		DIP(1.0%)	
	T.E. (kwh)	B.E. (kwh)	T.E. (kwh)	B.E. (kwh)	T.E. (kwh)	B.E. (kwh)
5	2.3	0.0	2.3	0.0	2.3	0.0
10	6.6	0.0	6.6	0.0	6.6	0.0
15	11.0	0.0	11.0	0.0	10.9	0.0
20	15.3	0.0	15.3	0.0	15.3	0.0
25	19.6	0.0	19.6	0.0	19.6	0.0
30	24.0	0.0	23.9	0.0	23.9	0.0
35	28.3	0.0	28.3	0.0	28.3	0.0
40	32.6	0.0	32.6	0.0	32.6	0.0
45	37.0	0.0	36.9	0.0	36.9	0.0
50	41.3	0.0	41.3	0.0	41.3	0.0
55	45.6	0.0	45.6	0.0	45.6	0.0
60	49.9	0.0	49.9	0.0	49.9	0.0
65	54.3	0.0	54.3	0.0	54.3	0.0
70	58.6	0.0	58.6	0.0	58.6	0.0
75	62.9	0.0	62.9	0.0	62.9	0.0
80	67.3	0.0	67.3	0.0	67.3	0.0
85	71.6	0.0	71.6	0.0	68.5	9.3
90	75.6	1.4	71.9	14.1	68.5	21.0
95	75.6	18.7	71.9	27.2	68.5	31.4
100	75.6	32.9	71.9	37.9	68.5	39.9
105	75.6	44.0	71.9	46.1	68.5	45.7
110	75.6	51.9	71.9	51.4	68.5	48.8
115	75.6	56.8				

T.E.= Tractive Energy B.E.= Braking Energy

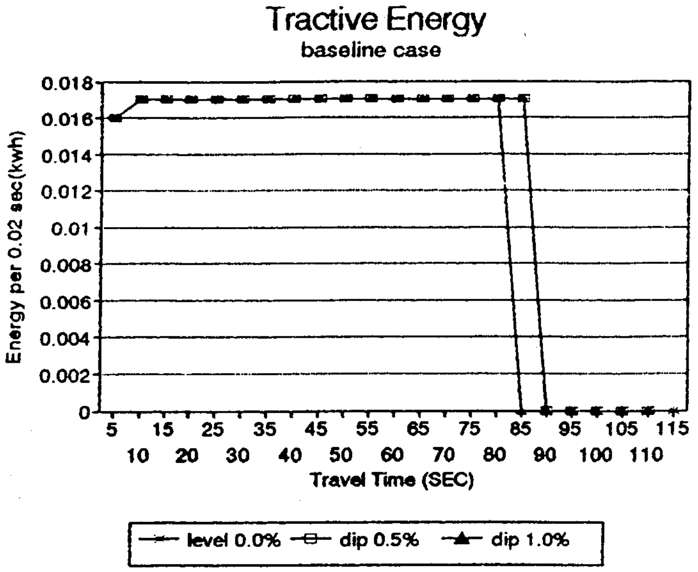


Fig.6 **Tractive Energy Consumption over Travel Time**

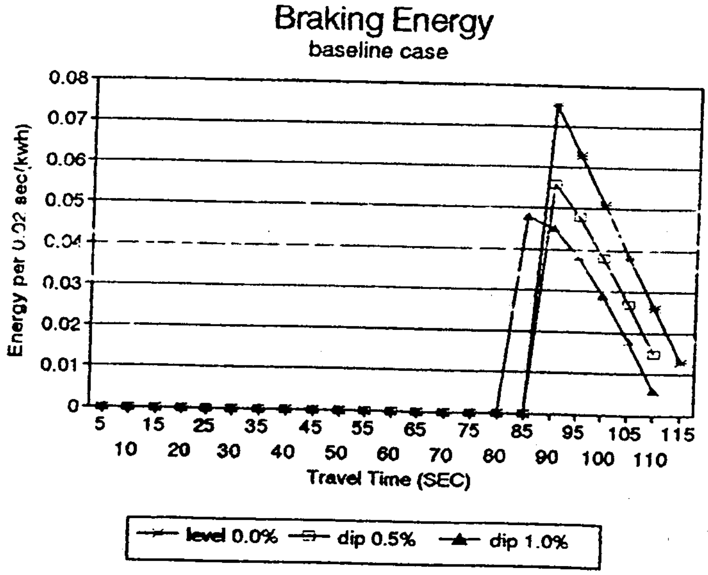


Fig.7 Braking Energy Consumption over Travel Time

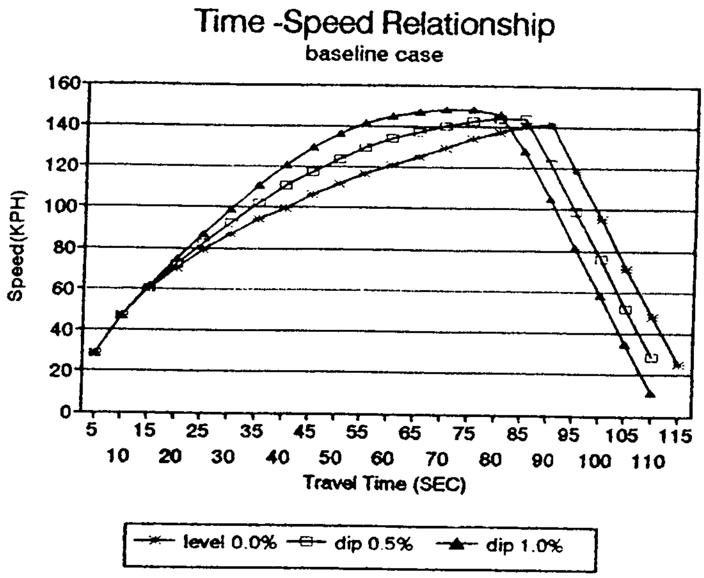


Fig.8 **Travel Speed Versus Travel Time**

Sensitivity Analysis

The sensitivity of the results to 1) station spacing 2) maximum acceleration 3) power per car and 4) speed constraints was analyzed within the following ranges of values:

Station spacing : 600 m to 4,800 m (8 steps)
(2,000 ft to 16,000 ft)
Maximum acceleration: 1.0 m/sec^2 to 1.6 m/sec^2 (3 steps)
(3.28 ft/sec^2 to 5.25 ft/sec^2)
Power of car : 4 × 104 kw to 4 × 156 kw (3 steps)

Speed constraint : Limited to 120 kph (74.6 mph) or unlimited

Maximum dip : 0 %, 0.5 %, 1.0 %

While changing one parameter value as specified above, the other parameters were kept at their baseline values (3,000 m station spacing, a_{max} = 1.3 m/sec^2, 4 × 130 kw per car, unlimited speed, 0.5 % dip). Higher dip values were not considered here to avoid excessive maximum gradients. The results for energy consumption and travel time are discussed below.

Effect of Distance on Energy Consumption

Compared with a level profile (0 % dip), tractive energy savings are about 5 % for a 0.5 % dip, regardless of station spacing and vary from 9.1 % to 9.6 % for a 1.0% dip, depending on distances, as shown in Table 3 and Fig. 9. The braking energy savings are higher than tractive energy savings, although these energies are not directly comparable. Braking energy savings increase as the station spacing increases because the maximum speed increases and the effect of dip on braking energy is greater at higher speed. Braking energy savings range from 5.6 % to 10.6 % for a 0.5 % dip and from 11.1 % to 18.2 % for a 1.0 % dip, as shown in Table 3.

Table 3. **Effect of Dip and Station Spacing on Energy Consumption**

Station Spacing	Level(0%)		Dip(0.5%)		Dip(1.0%)	
	Tractive Energy (kwh)	Braking Energy (kwh)	Tractive Energy (kwh)	Braking Energy (kwh)	Tractive Energy (kwh)	Braking Energy (kwh)
2,000 ft	(0.0)*	(0.0)	(-4.8)	(-5.6)	(-9.4)	(-11.1)
(600 m)	21.9	19.6	20.8	18.5	19.8	17.4
4,000	(0.0)	(0.0)	(-4.8)	(-6.3)	(-9.6)	(-12.3)
(1,200)	37.4	32.3	35.6	30.3	33.8	28.4
6,000	(0.0)	(0.0)	(-4.9)	(-6.9)	(-9.6)	(-13.4)
(1,800)	51.1	42.5	48.6	39.5	46.2	36.8
8,000	(0.0)	(0.0)	(-4.9)	(-7.6)	(-9.5)	(-14.6)
(2,400)	63.7	51.1	60.5	47.2	57.6	43.6
10,000	(0.0)	(0.0)	(-4.9)	(-8.4)	(-9.4)	(-15.7)
(3,000)	75.6	58.5	71.9	53.6	68.5	49.3
12,000	(0.0)	(0.0)	(-4.9)	(-9.1)	(-9.3)	(-16.7)
(3,600)	87.1	65.0	82.8	59.1	78.9	54.2
14,000	(0.0)	(0.0)	(-4.8)	(-9.8)	(-9.2)	(-17.3)
(4,200)	98.1	70.7	93.4	63.8	89.1	58.5
16,000	(0.0)	(0.0)	(-4.8)	-(10.6)	(-9.1)	(-18.2)
(4,800)	108.9	75.9	103.7	67.8	99.0	62.1

* : Number in parentheses is % energy difference compared with level profile

Effect of Maximum Allowable Acceleration on Energy Consumption

Results are compared for the three following acceleration rates without speed limits and for a 10,000 ft (3,000 m) station spacing :

Low acceleration/deceleration : 1.0 m/sec^2 (3.281 ft/sec^2)

Medium acceleration/deceleration : 1.3 m/sec^2 (4.265 ft/sec^2)

High acceleration/deceleration : 1.6 m/sec^2 (5.249 ft/sec^2)

As the acceleration rate increases, the tractive and braking energy consumption increases for all cases. The effect of dip on both tractive and braking energy savings is higher at lower acceleration limits, as shown in Table 4 and Fig. 10.

Table 4. **Effect of Maximum Allowable Acceleration on Energy Consumption**

Maximum acceleration	Level		Dip(0.5%)		Dip(1.0%)	
	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy
Low**	(0.0)* 72.1	(0.0) 56.3	(-5.9) 67.8	(-9.4) 51.1	(-11.5) 63.8	(-18.2) 46.1
Medium	(0.0) 75.6	(0.0) 58.5	(-4.9) 71.9	(-8.4) 53.6	(-9.4) 68.5	(-15.7) 49.3
High	(0.0) 77.9	(0.0) 59.8	(-4.4) 74.5	(-7.8) 55.1	(-8.4) 71.4	(-14.2) 51.3

Unit:kwh/train

* : Energy difference compared with level profile in percent(%)

**: Low;1.0 m/s^2 (3.281 ft/s^2) Medium;1.3 m/s^2 (4.265 ft/s^2) High;1.6 m/s^2 (5.249 ft/s^2)

Effect of Power on Energy Consumption

As vehicle power increases, energy consumption also increases for all dips. The effect of dip on energy consumption is higher for lower-powered cars as shown in Table 5 and Fig. 11. Tractive energy savings from a 1.0 % dip compared with a level profile are 9.9 % for low-powered trains and 9.2 % for high-powered trains, while braking energy savings for the same cases are 16.2 % and 14.9 %, respectively.

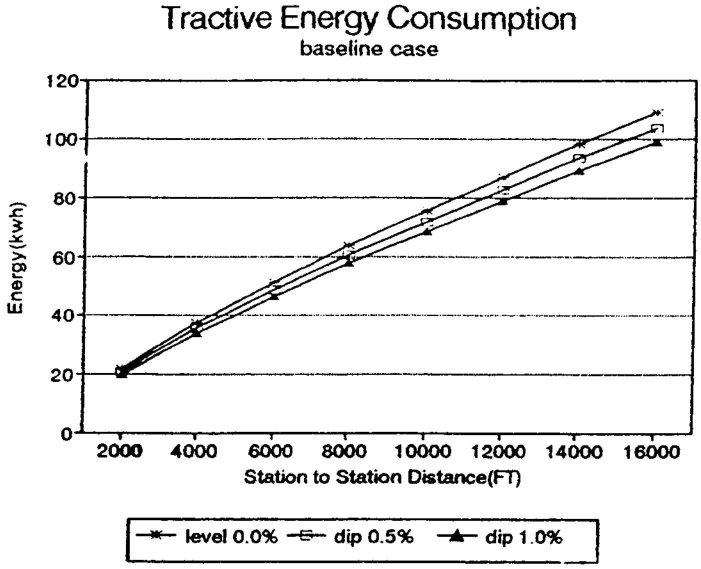


Fig. 9 Effect of Station Spacing On Tractive Energy Consumption

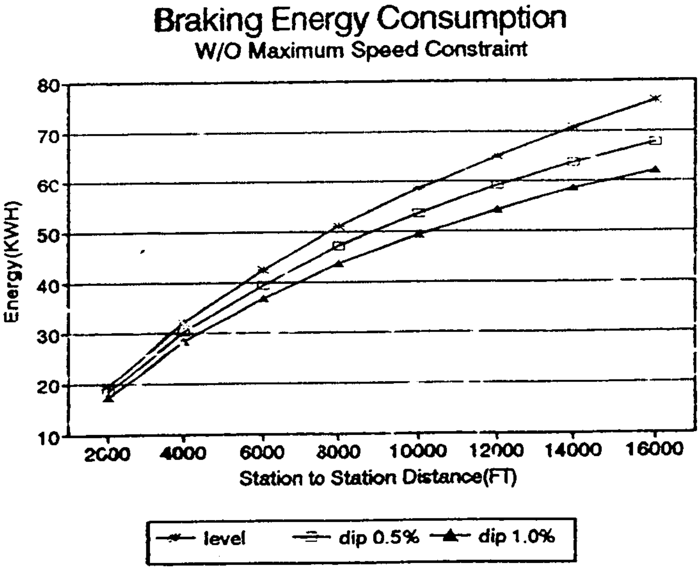


Fig. 10 Effect of Station Spacing on Braking Energy Consumption

Table 5. **Effect of Available Power on Energy Consumption**

	Level		Dip(0.5%)		Dip(1.0%)	
Power (kw/car)	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy
416 Low	(0.0)*	(0.0)	(-5.2)	(-9.1)	(-9.9)	(-16.2)
	66.9	50.9	63.4	46.2	60.2	42.6
520 Medium	(0.0)	(0.0)	(-4.9)	(-8.4)	(-9.4)	(-15.7)
	75.6	58.5	71.9	53.6	68.5	49.3
624 High	(0.0)	(0.0)	(-4.8)	(-7.7)	(-9.2)	(-14.9)
	83.4	65.3	79.4	60.3	75.8	55.5

Unit : kwh/train
* : % energy difference compared with level profile

Effects of Speed Constraint

Table 6 shows that both tractive and braking energy consumption are reduced by a 120 km/h speed constraints for all dip values. In comparing a 1.0 % dip with level profile, a speed constraint reduces tractive energy requirements by 20 % (from 68.5 kwh to 55.1 kwh) and braking energy requirements by 44 % (from 49.3 to 35.2). However the speed constraint reduces the benefits of a 1.0 % dip from 9.4 % to 3.6 % savings in tractive energy and from 15.7 % to 13.8 % in braking energy. Of course, the increased travel time is also a major disbenefit of a speed constraint, as shown in Table 7. Thus, for a 1.0 % dip a speed limit increases travel time between stations by 5.3 %, from 113.5 seconds to 119.5 seconds.

Table 6. **Effect of a 120 km/h Speed Limit on Energy Consumption**

Item	Level		Dip(0.5%)		Dip(1.0%)	
	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy	Tractive Energy	Braking Energy
w/ Speed Constraint	(0.0)*	(0.0)	(-3.7)	(-6.3)	(-3.6)	(-13.8)
	57.1	40.8	55.1	38.3	55.1	35.2
w/o Speed Constraint	(0.0)	(0.0)	(-4.9)	(-8.4)	(-9.4)	(-15.7)
	75.6	58.5	71.9	53.6	68.5	49.3

Unit:kwh/train

* : Energy difference compared with level profile in percent

Sensitivity to Travel Time

Table 7 shows that the effects of acceleration constraints on travel time savings are significant. Travel time saving from a 1.0 % dip are 5.6 % for low acceleration, 4.7 % for medium acceleration and 4.9 % for high acceleration.

The effects of available power on travel time savings are similar to those of acceleration limits. Time savings from lower-power trains are higher for a 0.5 % dip while savings from high-powered trains are higher for a 1.0 % dip, because lower-power trains have more difficulties in climbing steep grades. Time savings from a 1.0 % dip are 3.0 % to 5.1 % depending on the power.

Table 7. Effect of Dip and Operation Characteristics on Travel Time

Item		Level(0%)	Dip(0.5%)	Dip(1.0%)
Acceleration (m/sec^2)	Low (1.0)	(0.0)* 125.7	(-3.4) 121.4	(-5.6) 118.6
	Medium (1.3)	(0.0) 119.0	(-3.6) 114.7	(-4.7) 113.5
	High (1.6)	(0.0) 115.4	(-3.9) 110.9	(-4.9) 109.7
Power (kw)	Low (416)	(0.0) 125.5	(-4.0) 120.4	(-3.0) 121.7
	Medium (520)	(0.0) 119.0	(-3.6) 114.7	(-4.7) 113.5
	High (624)	(0.0) 114.2	(-2.7) 111.1	(-5.1) 108.3
Speed Constraint	120 km/h	(0.0) 122.7	(-1.8) 120.5	(-2.6) 119.5
	none	(0.0) 119.0	(-3.6) 114.7	(-4.7) 113.5

Unit:Seconds between stations

* : Time difference compared with level profile in percent(%)

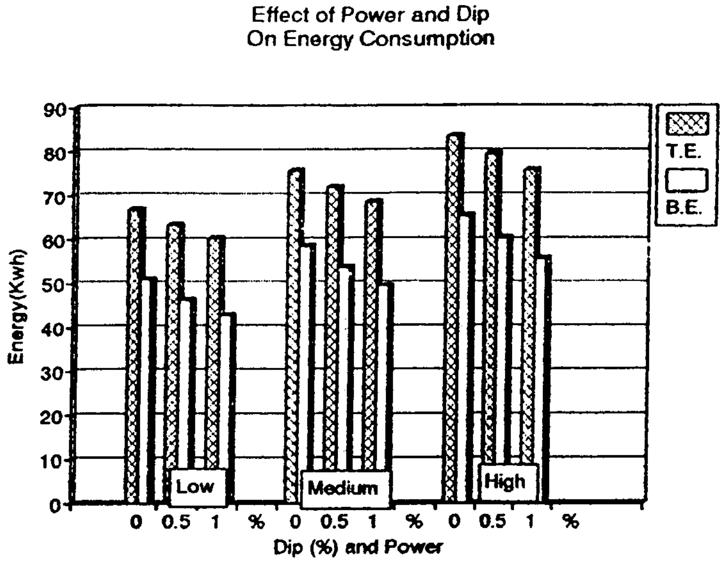


Fig. 11 **Effect of Power and Dip on Energy Consumption**

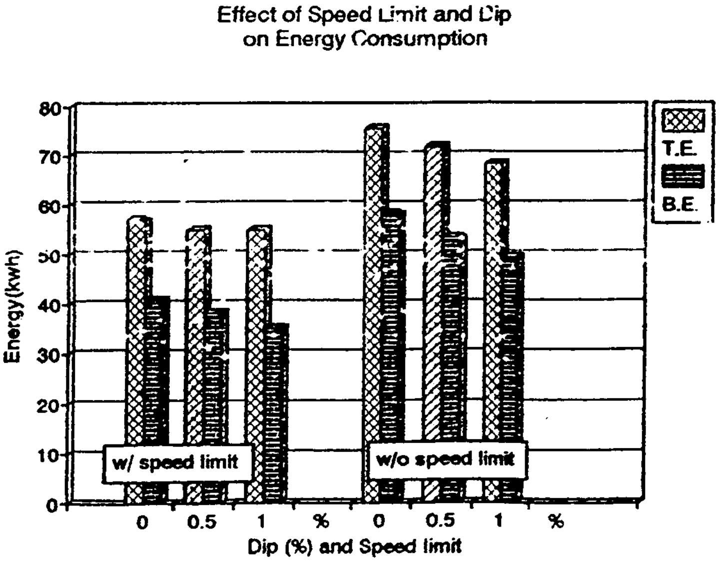


Fig. 12 **Effect of Speed Limit and Dip on Energy Consumption**

Table 8 shows that travel time decreases as dip increases by a maximum of 5.3 % for a 1.0 % dip and 2,400 m (8,000 ft) station spacing. Travel time savings are not proportional to station spacing. It is notable that for a very long spacing (e.g. 4,800 m or 16,000 ft) and 1.0 % dip the travel time can actually be higher than for a level profile because a long upgrade must be climbed after high resistance losses at high speed on the down grade. This problem might be postponed by using dips even greater than 1.0 % for long station spacings.

Table 8. **Relation between Travel Time and Station Spacing**

Station Spacing	Level(%)	Dip(0.5%)	Dip(1.0%)
2,000 ft	(0.0)*	(-1.8)	(-3.5)
(600 m)	44.1	43.3	42.5
4,000	(0.0)	(-2.9)	(-4.8)
(1,200)	67.2	65.3	64.0
6,000	(0.0)	(-3.0)	(-5.0)
(1,800)	86.1	83.5	81.8
8,000	(0.0)	(-3.2)	(-5.3)
(2,400)	103.3	100.1	97.8
10,000	(0.0)	(-3.6)	(-4.7)
(3,000)	119.0	114.7	113.5
12,000	(0.0)	(-3.8)	(-3.4)
(3,600)	134.3	129.2	129.7
14,000	(0.0)	(-3.7)	(-0.5)
(4,200)	148.0	142.6	147.2
16,000	(0.0)	(-3.9)	(1.5)
(4,800)	161.9	155.6	164.3

Unit:Seconds between stations

* :Travel time difference compared with level profile in %

8. Conclusions and Recommendations

The benefits of dipped rail alignments which can reduce propulsive energy on the downgrades after starting from a station and save braking energy on the upgrades were analyzed in this study. A model is formulated and simulated for standard urban subways. Sensitivity analysis is conducted for dips, station spacings, speed constraints, maximum acceleration, power availability.

The main findings are as follows:

1) For a given power train speed and acceleration increase significantly as the dip increases.

2) For the baseline cases savings of about 10 % of tractive energy, 15 % of braking energy and 5 % of time can be obtained from dipped profiles.

3) An appropriate dip size seems to be approximately 1.0 % of station spacing considering the maximum grade trains can climb. Benefits of a 1.0 % dip are almost double those of a 0.5 % dip.

4) Tractive energy savings percentages do not change significantly with station spacing but braking energy savings vary from 11 % to 18 % as station spacing varies from 600 m (2,000 ft) to 4,800 m (16,000 ft).

5) Both tractive and braking energy savings increase when the comfortable acceleration limit decreases.

6) Low powered trains benefit more than high powered trains from dipped profiles, although the differences are small.

7) Effects of dips are considerably more favorable if speeds are not limited.

Consequently, a dipped alignment provides considerable savings of energy and time although it may cost more to construct. Construction cost increases may be negligible if deep tunnels are used. In any case the construction costs of dipped alignment deserve further study.

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