DEVELOPMENT OF NONLINEAR FEEDBACK LINEARIZATION CONTROLLER FOR AN EMS SYSTEM WITH FLEXIBLE RAIL

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Abstract

In this paper, we consider a nonlinear control problem for an Electro-Magnetic Suspension(EMS) system with flexible rail. In controller design based on feedback linearization, we apply the feedback linearization technique to the part of the system which provides nonlinearities to the plant. The experimental results demonstrate that the feedback linearization controller shows good performance.

1. Introduction

Feedback linearization is an approach to the nonlinear control problem, which has developed considerably in recent years. This approach differs entirely from the standard method based on the Jacobian approximation, in that linearization is achieved by exact coordinate transformation and feedback, rather than by linear approximation[1],[2].

In this paper, we consider a nonlinear control problem for an attraction-type EMS (Electro-Magnetic Suspension) system which consists of flexible rail beam and a single magnet.

In the previous research dealing with the EMS system[3], the rail beam is assumed to be a rigid body, so that the dynamics of the rail beam was neglected. However, if the length of rail beam in the real EMS system is considerably long, the effect of rail dynamics on the levitation of the magnet can not be neglected[4]. In this paper, we propose a 5th-order nonlinear model for the EMS system considering the rail beam dynamics and design a feedback linearization controller keeping the vertical levitation gap constant. We also demonstrate the effectiveness of the proposed feedback linerization controller by experiments.

2. Mathematical Model

A schematic diagram of the EMS system with flexible rail beam is depicted in Fig. 1.

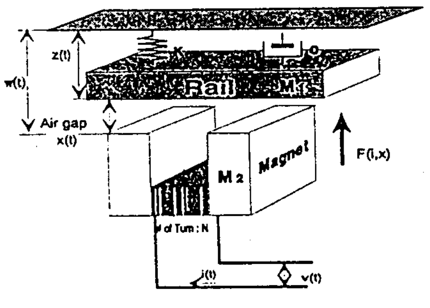

Fig. 1 EMS system with flexible rail beam

Flexible rail beam is a distributed parameter system and its model can be expressed as a partial differential equation[5]. In this paper, we model the rail beam as a 2nd-order lumped parameter system(a mass-spring-damper system) by introducing some assumptions[6].

The state equation describing the EMS system with flexible rail can be expressed as the following equations :

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ h(z,x) \end{bmatrix} \tag{2.1a}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ [-\frac{\mu_0 N^2 A}{4M_2}\frac{x_3^2}{x_1^2} + G] - h(z,x) \\ -\frac{2R}{\mu_0 N^2 A} x_1 x_3 + \frac{x_2}{x_1} x_3 + \frac{2x_1}{\mu_0 N^2 A} u \end{bmatrix} \tag{2.1b}$$

where

$$h(z,x) = -\omega_n^2 z_1 - 2\zeta\omega_n z_2 + \frac{\mu_0 N^2 A}{4M_1}\frac{x_3^2}{x_1^2} + G$$

and the input u is the applied voltage to the electro-magnet circuit.
The control purpose is that the system output y maintains the desired reference gap length r.

$$y = x_1 = x \tag{2.2}$$

3. Controller Design

In this paper, we linearize the whole EMS system (2.1a) and (2.1b), by constructing a linearization loop for the part of the system (2.2b) which brings nonlinearities and select the appropriate linear feedback gains to compensate the effect due to the the rail beam, dynamics. By using this approach, the controller does not contain the uncertain rail beam dynamics. As a result, the proposed controller is insensitive to the effect of the rail beam dynamics and its structure becomes simple.

3.1 Feedback Linearization

We regard the effect due to the rail beam dynamics as the external disturbance to the system, and apply input-state linearization technique to the electro-magnet system which brings nonlinearities to the EMS system. The system equation (2.1b) representing the electro-magnet dynamics can be written in the input-affine form:

$$\dot{x}=\begin{bmatrix} x_2 \\ -\frac{\mu_0 N^2 A}{4M_2}\frac{x_3^2}{x_1^2}+G \\ -\frac{2R}{\mu_0 N^2 A}x_1x_3+\frac{x_2}{x_1}x_3 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \frac{2x_1}{\mu_0 N^2 A} \end{bmatrix}u+\begin{bmatrix} 0 \\ \frac{f_d}{M_2} \\ 0 \end{bmatrix} \qquad (3.1)$$

where f_d is the term describing the external disturbance applied to the system and the effect of rail vibration dynamics is assumed to be included in this term.

Since the nonlinear system (3.1) satisfies two conditions for input-state linearization, it is input-state linearizable[1],[2].
Now the coordinate transformation for the electro-magnet system (3.1) can be obtained as follows :

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= L_f y_1 = x_2 \\ y_3 &= L_f^2 y_1 = -\frac{\mu_0 N^2 A}{4M_2}\frac{x_3^2}{x_1^2}+G \end{aligned} \qquad (3.2)$$

Also the feedback linearizing input u for the electro-magnet system (3.1) is given by :

$$\begin{aligned} u &= \frac{(v-L_f^3 y_1)}{L_g L_f^2 y_1} \\ &= -M_2\frac{x_1}{x_3}\left(v-\frac{R}{M_2}\frac{x_3^2}{x_1}\right) \end{aligned} \qquad (3.3)$$

where v is the linear feedback input which will be designed in terms of new state variables.

Applying the coordinate transform (3.2) and the linearizing input (3.3) for the system (3.1) to the EMS system (2.1a) and (2.2b), the linearized EMS system can be written as :

$$\dot{z}=\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}z+\begin{bmatrix} 0 \\ -\frac{M_2}{M_1}y_3 \end{bmatrix} \qquad (3.4a)$$

$$\dot{y}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}y+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}v+\begin{bmatrix} 0 \\ \omega_n^2\hat{z}_1+2\zeta\omega_n z_2+\frac{M_2}{M_1}y_3 \\ 0 \end{bmatrix} \qquad (3.4b)$$

where $z=[\,\hat{z}_1=z_1-\frac{(M_1+M_2)}{K}G \quad z_2\,]^T$ and

$y=[\,y_1-r \quad y_2 \quad y_3\,]^T$.

Now we design the linear feedback input v constructing the closed-loop system as follows :

$$v=-k_1(y_1-r)-k_2(y_2+z_2)-k_3y_3 \qquad (3.5)$$

where k_1, k_2, k_3 are the linear feedback gains which will be selected later.

3.2 Linear feedback gain selection

If the rail beam is a rigid body, the EMS system (3.4), with the linear feedback input (3.5) being applied, can be expressed as the following 3rd- order linear state equation [3] :

$$\dot{y}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 \end{bmatrix}y+\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\frac{f_d}{M_2} \qquad (3.8)$$

We can obtain a condition for the feedback gains k_1, k_2, k_3 which forces this linear system be stable as follows :

$$k_1>0,\ k_3>0,\ k_2k_3>k_1 \qquad (3.7)$$

If the external disturbance f_d is bounded, the steady-state response of output y_1 due to the constant disturbance is obtained by the Final value theorem[7] :

$$\lim_{t\to\infty}y_1(t)=\lim_{s\to 0}sY_1(s)=\frac{1}{M_2}\frac{k_3}{k_1}f_d \qquad (3.8)$$

where $Y_1(s)$ a represent the Laplace transforms of y_1.
Therefore, in order to reject the effect of the external disturbance(including the effect due to the rail dynamics) on the output y_1, the gain k_1 needs to be maximized and the gain k_3 minimized as much as possible.

Now, we would like to select the feedback gains k_1, k_2, k_3 of the linear input v which make the EMS system (3.4a) and (3.4b) be stable.
The EMS system (3.4a) and (3.4b), with the linear feedback input (3.5) having been applied, can be viewed as a feedback-connected system which consists of the following two linear subsystems, as depicted in Fig. 2.

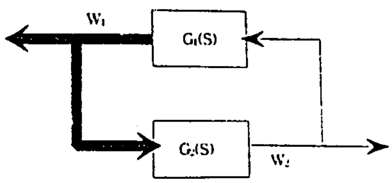

Fig. 2 Feedback connection of two linear systems

The transfer functions $G_1(s)$ and $G_2(s)$ in Fig. 2 are given by :

$$G_1(s)=\frac{1}{\Delta_1(s)}\begin{bmatrix} 2\zeta\omega_n s+\omega_n^2 \\ -k_2 s \end{bmatrix} \qquad (3.9a)$$

$$G_2(s)=\frac{\frac{M_2}{M_1}}{\Delta_2(s)}[\,-(k_2s+k_1)\quad s^2\,] \qquad (3.9b)$$

where $\Delta_1(s)=s^2+2\zeta\omega_n s+\omega_n^2$,

$\Delta_2(s)=s^3+k_3s^2+\overline{k_2}s+\overline{k_1}$,

$\overline{k_1}=(1+\frac{M_2}{M_1})k_1$, and $\overline{k_2}=(1+\frac{M_2}{M_1})k_2$

It can be easily verified that the above two transfer functions $G_1(s)$ and $G_2(s)$ are stable. To find a condition which forces the feedback-connected system to be stable, we compute the characteristic polynomial of the closed-loop system in Fig. 2.

$$\Delta_1(s)\Delta_2(s)\,det[\,I+G_1(s)G_2(s)\,]$$
$$= \frac{M_2}{M_1}(\,s^5 + a_5 s^4 + a_4 s^3 + a_3 s^2 + a_2 s + a_1\,) \qquad (3.10)$$

$$a_5 = (2\zeta\omega_n + k_3),\quad a_4 = (\omega_n^2 + k_2 + 2\zeta\omega_n k_3),$$
$$\text{where}\quad a_3 = [\,(1+\frac{M_2}{M_1})k_1 + 2\zeta\omega_n k_2 + \omega_n^2 k_3\,],$$
$$a_2 = (2\zeta\omega_n k_1 + \omega_n^2 k_2),\quad a_1 = \omega_n^2 k_1$$

Thus, if we select feedback gains which satisfy the condition (3.7) and render the characteristic polynomial (3.10) Hurwitz, the feedback system in Fig. 2 becomes stable[8].

4. Experiments

4.1 Experimental Configuration.

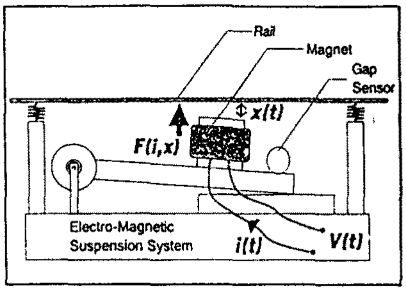

Fig. 3 Experimental setup for the EMS system

Fig. 3 shows the experimental implementation for the EMS system with rail dynamics. To provide effective vibration for the rail beam, we used a coiled spring between the rail beamand its support.

The power source is a voltage source PWM chopper with a DC link voltage of 80[V]. IBM 80486 with OROS - AU32 (32-bit DSP TMS320C31) board performs the control algorithm as well as data acquisition and signal conditioning. Using this computer, both the proposed feedback linearizing controller is implemented at sampling rate of 3 [Khz]. The following physical parameters are chosen in this experiment: r_0=7.5[mm], m= 15[kg], R= 8 [Ω], N=2000, A= 12[Cm^2].

4.2 Experimental Results

See Figure 4

5. Conclusions

In this paper, we consider a nonlinear control problem for a EMS system with flexible rail beam dynamics. We derive a 5th-order nonlinear model for the plant and propose a feedback linearization controller keeping the constant levitation gap.
In the controller design, we linearize the part of the EMS system which provides nonlinearities to the plant and select the appropriate feedback gains which forces the EMS system to be stable.

In order to verify the performance of the proposed controller in the presence of rail beam dynamics, we also carry out experiments. We implement the proposed controller employing DSP(TMS320C31). The experimental results demonstrate that the feedback linearization controller shows fairly good performance.

6. References

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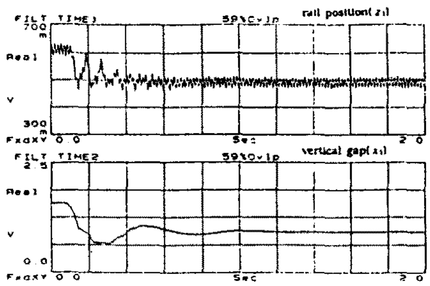

Fig. 4 Transient response in set-point regulation