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연속 시간 선형 주기 시스템에 대한 주기 예측 구간 H_∞ 추적 제어

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Intervalwise Receding Horizon H_∞ Tracking Control for Continuous Linear Periodic Systems

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Abstract

In this paper, a fixed-horizon H_∞ tracking control (HTC) for continuous time-varying systems is proposed in state-feedback case. The solution is obtained via the dynamic game theory. From HTC, an intervalwise receding horizon H_∞ tracking control (IHTC) for continuous periodic systems is obtained using the intervalwise strategy. The conditions under which IHTC stabilizes the closed-loop system are proposed. Under proposed stability conditions, it is shown that IHTC guarantees the H_∞-norm bound.

1 Introduction

The receding horizon control strategy has been developed as a proper control strategy for tracking performance and time-varying systems. It is well known that this strategy presents more practical aspects in the applications to real systems than the infinite horizon control strategy, because it needs only informations for only a finite future time. The receding horizon strategy is obtaining a solution to optimize a finite future cost horizon. There are two receding horizon strategies, the pointwise and intervalwise one. As well shown in [2], in the pointwise strategy, the terminal point of a fixed-length finite cost horizon continuously recedes at each time instant. In the intervalwise strategy the terminal point is kept fixed for a period of finite cost horizon and, after one period, the terminal point moves by one period and fixed during the next period.

In the aspect of computational load, the intervalwise strategy is superior to the other one since the intervalwise one requires calculation of control gain per a period of every cost horizon while the other one requires it per every time instant. The pointwise strategy has been developed for general time-varying systems [5], [6], [7], and the intervalwise strategy only for periodic and time-invariant systems [1], [2], [9].

There has been a few studies on the receding horizon tracking problems and its stability property in the H_∞ problem. In order to obtain the receding horizon tracking control, the tracking control for a fixed finite cost horizon should be obtained at first. Very recently, in [3], an H_∞ tracking control for a finite cost horizon was proposed in continuous systems. In [7], the stability conditions for continuous systems were proposed using the pointwise strategy. But the intervalwise receding horizon strategy has not been investigated in the tracking problems and the H_∞ problem to author's knowledge.

In this paper, an intervalwise receding horizon H_∞-tracking control (IHTC) for continuous periodic systems is proposed. Our fixed finite horizon H_∞ tracking control (HTC) which is first obtained to derive IHTC is different from that of [5]. The solution (HTC) is obtained via the dynamic game theory as shown in [4]. The conditions under which closed loop stability and infinite horizon H_∞-norm bound are quaranteed with IHTC are proposed, respectively.

This paper is organized as follows. In Section 2, HTC for continuous time-varying systems is derived in state-feedback case. In Section 3, a stabilizing IHTC for continuous periodic systems is proposed. In Section 4, it is also shown that the stabilizing IHTC guarantees the H_∞-norm bound. Finally, our conclusions follow in Section 5.

2 H_∞-tracking control

We derive a finite horizon H_∞-tracking control (HTC) using the previous result [4] in which only the regulation problem is dealt with. Consider the following discrete time-varying system:

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B_2(t)u(t), \quad x(0) = 0 \quad (1)$$

$$z(t) = \begin{bmatrix} C(t)x(t) \\ u(t) \end{bmatrix}, \quad z_r(t) = \begin{bmatrix} y_r(t) \\ 0 \end{bmatrix}$$

where

$$x(t) \in R^n, \ \ u(t) \in R^m, \ \ w(t) \in R^l, \ \ z(t) \in R^{p+m}$$

and the finite horizon cost index with the finite terminal weighting matrix $F > 0$:

$$\begin{aligned} J(z_r, u, w) &= [z(t_f) - z_r(t_f)]^T F[z(t_f) - z_r(t_f)] + \\ &\int_{t=0}^{t_f} [\|z(t) - z_r(t)\|_2^2 - \gamma^2 \|w\|_2^2] \qquad (2) \end{aligned}$$

where the tracking commands $y_r(1)$, $y_r(2)$, $\cdots$, $y_r(N)$ are assumed to be available over the future horizon N. In the following theorem, we introduce the existing result on the finite horizon H_∞-regulation problem where the tracking commands $y_r(t) = 0$ for $\forall t$. From now on, we substitute B_1 with $B_\gamma = \gamma^{-1} B_1$. At first, we introduce the following Riccati differential equation (RDE)

$$\begin{aligned} -\dot{Z}(t) &= Z(t)A(t) + A^T(t)Z(t) + Q(t) - Z(t)[B_2(t)B_2^T(t) \\ &\quad - B_{\gamma_c}(t)B_{\gamma_c}^T(t)]Z(t); \ \ Z(t_f) = Q_f \qquad (3) \end{aligned}$$

and

$$\begin{aligned} \hat{\gamma}^{CL} &= inf\{\gamma > 0 : The \ \ RDE \ \ (3) \ \ does \ \ not \ \ have \\ &\quad a \ \ conjugate \ \ point \ \ on \ \ [0, t_f]\} \qquad (4) \end{aligned}$$

Theorem 1 *[4], For the linear-quadratic zero-sum differential game with closed-loop information structure, defined on the time interval $[0, t_f]$, let the parameter $\hat{\gamma}^{CL}$ be as defined by (4). Then if $\gamma > \hat{\gamma}^{CL}$, the differential game admits a unique feedback saddle-point solution, which is given by*

$$\begin{aligned} u^*(t) &= -B_2^T(t)Z(t)x(t) \\ w^*(t) &= \gamma^{-2}B_1^T(t)Z(t)x(t), \ \ x^*(0) = x(0) \end{aligned}$$

Now we can derive HTC for continuous systems using the above result as follows:

Theorem 2 *Then, the differential game for (1) admits a unique feedback saddle-point solution, which is given by*

$$\begin{aligned} u^*(t) &= -B_2^T(t)[Z(t)x(t) + g(t)] \qquad (5) \\ w^*(t) &= \gamma^{-2}B_1^T[Z(t)x(t) + g(t)] \qquad (6) \end{aligned}$$

where

$$\begin{aligned} -\dot{g}(t) &= A^T(t)g(t) - Z(t)[B_2(t)B_2^T(t) - B_\gamma(t)B_\gamma^T(t)]g(t) \\ &\quad - C^T(t)y_r(t) \\ g(t_f) &= -C^T(t_f)Fy_r(t_f) \end{aligned}$$

proof: The proof procedures are almost the same as [8] and Theorem 2 of [9].

3 Stability of IHTC for continuous periodic systems

Consider continuous T-periodic systems. T-periodic systems mean that $A(t+T) = A(t)$, $B_1(t+T) = B_1(t)$, $B_2(t+T) = B_2(t)$, and $C(t+T) = C(t)$ for $\forall t$. Consider a generally time-varying matrix function $L(\cdot)$. The symbol $L_\tau(\cdot)$ will denote a T-periodic matrix function such that $L_\tau(t) = L(t)$ for $\tau \le t \le \tau + T - 1$ and $L_\tau(t+T) = L_\tau(t)$ for $\forall \ t \ge \tau$.

Here we consider the problem of applying the control (6) to continuous T-periodic systems with the intervalwise strategy. We propose an intervalwise receding horizon H_∞-tracking control (IHTC) which stabilizes continuous T-periodic systems. To study the stability of IHTC, we have only to consider the state feedback gain. Assume that $t_f \ge T + 1$. Here t_f is both the cost horizon and the horizon that the tracking signal is given. Let the initial point be τ and Q_f be the fixed value. Among the solutions obtained over $[\tau, \tau + t_f]$, we use the solutions over $[\tau, \tau + T]$. Next the initial point moves to $\tau + T$ and the terminal point of the cost horizon moves to $\tau + T + t_f$. This procedure repeats. Therefore $Z(\cdot)$ is T-periodical.

The periodic Riccati equation is as follows:

$$\begin{aligned} -\dot{Z}_{\tau+kT}(t) &= Z_{\tau+kT}(t)F_k(t) + F_k^T(t)Z_{\tau+kT}(t) + K_2^T(t)K_2(t) \\ &\quad + K_1^T(t)K_1(t) + Q(t) \qquad (7) \end{aligned}$$

where $F_k(t) = A(t) + B_2(t)K_2(t)$, $K_2(t) = -B_2^T(t)Z_{\tau+kT}(t)$, $K_1(t) = -B_\gamma^T(t)Z_{\tau+kT}(t)$. We define $\Theta(t)$ used in Theorem 3:

$$\Theta(t) = Q(t) + K_2^T(t)K_2(t) + K_1^T(t)K_1(t)$$

Theorem 3 *Let $Z(\cdot)$ be the solution of (3) for continuous T-periodic systems, and assume that*

1) $Z(t) > 0$ on $t \in [\tau, \tau + T]$

2) $\int_{t=\tau}^{\tau+T} \Theta(t) > 0 \ dt$

Then, $A(\cdot) - B_2(\cdot)B_2^T(\cdot)Z_{\tau+kT}(\cdot)$ is asymptotically stable for each $k \ge 0$.

proof:

$$\begin{aligned} Z_{\tau+kT}(\tau) &= \Phi_{F_k}^T(\tau)Z_{\tau+kT}(\tau)\Phi_{F_k}(\tau) + \\ &\quad \int_{t=\tau}^{\tau+T} \Psi_{F_k}^T(t, \tau - T)\Theta(t)\Psi_{F_k}(t, \tau - T) \ dt \end{aligned}$$

where

$$\Psi_F(\tau+T,\tau) = F(\tau+T-1)F(\tau+T-2)\cdots F(\tau+1)F(\tau) \quad (8)$$

$$\Phi_F(\tau) = \Psi_F(\tau+T,\tau) \quad (9)$$

Let v be an eigenvector of $\Phi_{F_k}(\tau)$ associated with the eigenvalue λ. Then, $Z_{\tau+kT}(\tau)$ yields:

$$(1-|\lambda|^2)v^* Z_{\tau+kT}(\tau+1)v = \int_{t=\tau}^{\tau+T} v^*\Psi_{F_k}^T(t,\tau-T) \Theta(t)\Psi_{F_k}(t,\tau-T)v \ dt$$

All characteristic multipliers of $\Phi_{F_k}(\tau)$ belong to the open-unit disk by assumptions. ■

4 The H_∞-norm bound of the stabilizing IHTC

Now consider the H_∞-norm bound for continuous systems.

Corollary 1 *With the stabilizing IHTC $u^*(t)$ for continuous systems when $z_r(\cdot)=0$, the H_∞-norm bound of the closed-loop system is guaranteed.*

proof: Proof procedures are almost identical to them of 3.4 in [7] with $P(t,t+\tau)$ replaced by $Z_{\tau+kT}(t)$. ■

5 Conclusion

In this paper, a fixed finite horizon H_∞ tracking control (HTC) for continuous time-varying systems is first derived. And then, an intervalwise receding horizon H_∞ tracking control (IHTC) is proposed for continuous periodic systems. It is shown that the proposed IHTC guarantees closed loop stability, infinite horizon H_∞ norm bound under the proposed conditions. One of the advantages of the proposed IHTC is that it extends the previous results on the intervalwise receding horizon control to the tracking controller and the H_∞ control. Another advantage that computation burdens are lessened makes IHTC easily applied to real-time tracking systems.

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