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LMI를 이용한 L_1/H_∞ 준최적 제어기법

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Mixed L_1/H_∞ Suboptimal Control : A LMI Approach

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Abstract

In this paper, we consider the mixed L_1/H_∞ problems of finding internally stabilizing controllers which minimize the peak-to-peak gain of a certain closed loop transfer function with H_∞-norm constraint on other closed loop transfer function(or vise versa). This problem is a useful framework for designing a controller with the norm constraints upon time and frequency domain. We formulate the mixed L_1/H_∞ problem as LMI problems by using the reachable set. This paper offers the sufficient condition for the existence of suboptimal state feedback controller, and shows that suboptimal solution can be obtained by solving a finite-dimensional convex optimization and a line search.

1 Introduction

In many controller design problems, one is to design a controller to meet certain objectives for a given plant. In practice, plant has some uncertainties and is affected by disturbances or noise at either its input or output. Thus one tries to model such uncertainties and disturbances, and to design the closed loop system minimizing the effect of them in some senses.

It is assumed that the disturbances are persistently bounded signal and formulated L_1 optimal control problem minimizing the peak-to-peak gain of closed loop system in [1]. The complete solution for this problem was obtained from L_1 optimal control theory [2], [3]. And L_1 optimal control theory deals with time domain specifications, on the other hand H_∞ control theory mainly treats frequency domain design specification. In [4], a large class of time domain design specifications was expressed in terms of linear constraints and was incorporated into linear program.

In this paper, we consider the problem of designing stabilizing controllers that minimizing the L_1 norm of a certain closed loop transfer function with maintaining the H_∞ norm of other closed loop transfer function under a prescribed value or vice versa. This problem can arise in minimizing the peak-to-peak gain of some close loop transfer function under the known uncertainty of system. Alternatively, it can be thought as the problem of designing a controller that achieves good nominal L_1 performance as well as robust stability in the sense of H_∞-norm.

It has been shown that the discrete-time mixed l^1/H_∞ problem can be solved by finite dimensional convex constrained optimization problem and standard unconstrained H_∞ problem. Furthermore, the continuous-time mixed L_1/H_∞ controller can be obtained from discrete-time Euler approximating system[6]. Since these obtained controllers may be arbitrarily high order system, we need to reduce the order. However, instead of minimizing the L_1-norm, new technique that minimizes *-norm by LMI approach, has been introduced in [7]. These controllers has the same degree as generalize plant from the properties of LMI.

In this paper, we propose an alternative approach to obtain a mixed L_1/H_∞ controller though expressing the bounds of L_1-norm and H_∞-norm in LMI terms. The main result of this paper shows that the minimizing state feedback controller can be obtained by a two-stage procedure entailing a finite-dimensional convex optimization and a one-dimensional line search.

This paper is organized as follows: In section 2, we introduce some notations to be used. In section 3, we shows that the mixed L_1/H_∞ problem can be formulated as a finite-dimensional optimization problem and one-dimensional line search. Finally, in section 4, we discuss our main results and directions of future research.

2 Preliminaries

In this section, we summarize some basic notations and theorems which are used for deriving our main results in section 3.

2.1 Basic Notations

Let $\Re_+$ be nonnegative real number.

$L^\infty(\Re_+)$: The real normed space of measurable function $f(t)$. The norm is defined:

$$\|f\|_\infty = ess\sup_{\Re_+}|f(t)|$$

$L_1(\Re_+)$: The space of Lebesgue integrable functions on $\Re_+$. The norm is defined:

$$\|f\|_1 = \int_0^\infty f(t)dt$$

$$\|H\|_1 = \sup_{\|u\|\le 1}\|H*u\|_\infty = \max_i \sum_{j=1}^n \|h_{ij}\|_1 \prec \infty$$

2.2 Modified sup-norm and $*$-norm

We now define modified sup-norm to apply LMI technique. Given a function $f(t) \in L^\infty$, we will define the norm $\|f(t)\|_{\infty,e} = \sup_{t\ge 0}\{f'(t)f(t)\}^{1/2}$, i.e. the supremum over time the pointwise euclidian norm of the vector $f(t)$. For an operator $H: L_\infty^n \to L_\infty^m$, we will define the induced norm $\|H\|_{1,e}$, i.e.

$$\|H\|_{1,e} = \sup_{\|u\|_{\infty,e}\le 1}\|H*u\|_{\infty,e}.$$

Note that for scalar signals these norms coincide with the usual $\|\cdot\|_\infty$ and $\|\cdot\|_1$. However, in general case, we have following relation

$$\frac{1}{n^{1/2}}\|H\|_1 \le \|H\|_{1,e} \le m^{1/2}\|H\|_1.$$

In the following theorem, $\|H\|_{1,e}$ can be expressed in terms of R, i.e. the set of all reachable states from the origin with input norm-bounded by one. The reachable set R is bounded by an ellipsoid that comes from a LMI as in [7,8].

Theorem 2.1 Suppose H=[A,B,C,0] and A is stable. Then,

$$\|H\|_{1,e} = \max_{x\in R}\|Cx\| \le \inf_Q \max_{x'Q^{-1}x\le 1}\|Cx\| = \|H\|_*$$

where Q is any symmetric, positive definite matrix such that for some real number $\alpha \succ 0$,

$$AQ + QA' + \alpha Q + \frac{1}{\alpha}BB' \le 0 \tag{1}$$

As in Theorem 2.1, we define the $*$-norm as the smallest upper bound on $\|H\|_{1,e}$, that offers a upper bound for the 1-norm of the system. This upper bound comes from approximating the set of reachable states with norm-bounded input by ellipsoid[9]. The computing of the $*$-norm is equivalent to the minimization of convex function over an interval of the real line. The function and its subgradient can be efficiently evaluated by solving Lyapunov equation[7].

Proposition 1 Suppose that H=[A,B,C,0] and A is stable.

Let Q be any solution of (1) and $\|H\|_* = r$.

Then, for any $\eta > 0$ such that satisfies

$$AQ + QA' + \frac{1}{\eta^2}BB' + QC'CQ \le 0,\ \eta \ge r.$$

This proposition addresses that for any symmetric positive definite matrix Q which satisfies (1), the invariant ellipsoid $E = \{\xi : \xi' Q^{-1}\xi \le 1\}$ contains the reachable set R with unit peak input. Furthermore, the invariant ellipsoid E also contains the reachable set R_∞. Thus, for a obtained invariant ellipsoid E approximating the reachable set R with unit peak input, we impose a H_∞-norm constraint on the invariant ellipsoid E.

2.3 Bounded H_∞-norm:

Now, we consider a system $H=[A,B,C,D]$ and the condition of $\|H\|_\infty < \gamma$. We recall the well known Bounded Real Lemma for continuous-time systems. The following theorem, which will be used in our main result, can be easily obtained from bounded real lemma.

Theorem 2.2 Suppose that A is stable. The following statements are equivalent.

(a) $\|H\|_\infty < \gamma$

(b) There exists a symmetric positive definite matrix X satisfying the following LMI;

$$\begin{pmatrix} A'X + XA & XB & C' \\ B'X & -\gamma I & D' \\ C & D & -\gamma I \end{pmatrix} < 0$$

3 Problem Formulation

In this section, we formulate the mixed L_1/H_∞ problem and the cost function. Consider the system shown in Figure 1, where u and y represent the control inputs and the outputs available to the controller, respectively. The external disturbances and regulated outputs are represented by ϖ and z_1, z_∞.

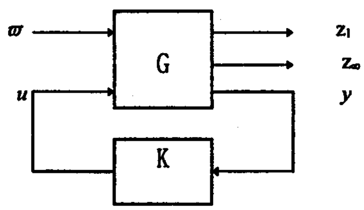

Figure 1: Problem setup

3.1 Cost function J

The mixed L_1/H_∞ problem can be stated as:

For the given generalized plant G, find an internally stabilizing controller K such that minimizes the cost function $J(T_{z_1\varpi}) = \|T_{z_1\varpi}\|_1$ under the H_∞-norm constraint $\|T_{z_\infty\varpi}\|_\infty \le \gamma$.

To solve this problem, we modify the cost function as follows:

$$J_1(T_{z_1\varpi}) = \|T_{z_1\varpi}\|_* \text{ subject to } \|T_{z_\infty\varpi}\|_\infty \le \gamma.$$

$$J_\infty(T_{z_1\varpi}) = \|T_{z_\infty\varpi}\|_\infty \text{ subject to } \|T_{z_1\varpi}\|_* \le \delta.$$

In this paper, we consider only J_1 as the cost function. The formulation using J_∞ instead of J_1 gives similar result. In section 4, we shall state this problem. As we stated in section 2, $*$-norm is an upper bound for 1-norm. If we minimize the $*$-norm, 1-norm also decreases.

3.2 State-feedback controller

Now, we consider the generalized plant G:

$$G = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & 0 & D_{22} \\ I & 0 & 0 \end{array}\right) \qquad (3)$$

For the simplicity, we assume that $C_1'D_{12} = 0$ and $D_{12}'D_{12} = I$. The following theorem shows that necessary and sufficient condition of bounded $*$-norm.

Theorem 3.1 The following statements are equivalent:

(a) There exits a finite-dimensional LTI controller K such that $\|T_{z_1\omega}\|_* \le \delta$.

(b) There exists a constant gain controller $u=Kx$ such that $\|T_{z_1\omega}\|_* \le \delta$.

(c) There exists a scalar α such that following LMI admits a solution:

$$\begin{pmatrix} AQ + B_2Y + QA' + Y'B_2' + \alpha Q & B_1 \\ B_1' & -\alpha I \end{pmatrix} \le 0$$

$$\begin{pmatrix} \delta^2 I & \begin{pmatrix} C_1Q \\ Y \end{pmatrix} \\ \begin{pmatrix} QC_1' & Y' \end{pmatrix} & Q \end{pmatrix} > 0 \qquad (4)$$

, where $Q = Q', Q > 0$.

Moreover, if any of these statements hold, then the controller $u = YQ^{-1}x$ achieves $\|T_{z_1\omega}\|_* \le \delta$

3.3 Problem Solution

In this subsection, we analyze the optimal controller for the cost function J defined as J_1. The main result shows that the suboptimal solution can be obtained by solving a finite-dimensional convex optimization problem and one-dimensional line search. From Theorem 2.2 and 3.1, we can obtain following result. Firstly, we define the following LMI problem as LMIRP(LMI formulation for robust performance problem).

$$\begin{pmatrix} AQ + B_2Y + QA' + Y'B_2' + \alpha Q & B_1 \\ B_1' & -\alpha I \end{pmatrix} \le 0$$

$$\begin{pmatrix} \delta^2 I & \begin{pmatrix} C_1Q \\ Y \end{pmatrix} \\ \begin{pmatrix} QC_1' & Y' \end{pmatrix} & Q \end{pmatrix} > 0 \qquad \text{(LMIRP1)}$$

$$\begin{pmatrix} AQ + B_2Y + QA' + Y'B_2' & B_1 & QC_2' + Y'D_{22} \\ B_1' & -\gamma I & 0 \\ C_2Q + D_{22}Y & 0 & -\gamma I \end{pmatrix} < 0 \qquad \text{(LMIRP2)}$$

, where $Q = Q', Q > 0$.

Remark: For a given $\gamma > 0$, LMIRP can be solved by finite-dimensional convex optimization problem and one-dimensional line search.

Proposition 2 Suppose that the generalized plant G is given as (3). Then, if there exists a scalar α such that following LMI admits a solutions, then (a) and (b) are satisfied

(a) There exits a finite-dimensional LTI controller K such that $\|T_{z_1\omega}\|_* \le \delta$ and $\|T_{z_\infty\omega}\|_\infty \le \gamma$.

(b) There exists a constant gain controller $u=Kx$ such that $\|T_{z_1\omega}\|_* \le \delta$ and $\|T_{z_\infty\omega}\|_\infty \le \gamma$.

Furthermore, the controller $u = YQ^{-1}x$ achieves

$\|T_{z_1\omega}\|_* \le \delta$ and $\|T_{z_\infty\omega}\|_\infty \le \gamma$.

Proposition 2 states that controllers can be obtained by solving a finite-dimensional convex optimization problem and a one-dimensional line-search

4 Conclusions

In this paper, we considered the mixed L_1/H_∞ problems of finding internally stabilizing controllers which minimize the peak-to-peak gain(or H_∞-norm) of a certain closed loop transfer function with H_∞-norm(or peak-to-peak gain) constraint on other closed loop transfer function. This problem is a useful framework for designing a controller with the norm constraints upon time and frequency domain.

Recently, many solutions for this problem are introduced. However, to obtain a suboptimal solutions, the degrees of controllers can be arbitrarily large. To avoid this high order property, LMI approach was introduced in [7]. We formulated the mixed L_1/H_∞ problem as LMI problems by using the reachable set. The main result of this paper offers the sufficient condition for existence of suboptimal state feedback controller, and shows that suboptimal solution can be obtained by solving a finite-dimensional convex optimization and a line search. Research is being carried out toward extending these results to the output feedback case.

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