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단일 부궤환 제어시스템의 근궤적에 관한 특성

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Some Properties About the Root Loci for Unity Negative Feedback Control Systems

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Abstract

We consider the interval of a gain within which it is guaranteed that a feedback control system is stable. This paper presents the condition under which either a unity feedback control system is stable for a connected gain interval with a proportional compensator cascaded with an open loop forward transfer function. By the connected interval we mean that all the numbers between any two numbers in the connected interval belongs to the connected interval. The condition may be described by a frequency inequality in terms of the denominator and/or numerator of the closed loop transfer function. We also consider the conditions for the discrete-time control systems and the time delay continuous-time control systems. We show that this condition cannot be extended for the transfer function having complex coefficients via a counterexample.

요 약

단일부궤환 제어시스템이 안정화되는 비례보상기의 이득의 구간을 연구한다. 이 논문에서 제어시스템이 연결된 구간에 대하여 안정화되는 조건을 구한다. 연결된 구간은 그 구간에 있는 어떤 임의의 두 실수를 취할때 두 실수 사이의 모든 실수가 그 구간에 속하는 경우를 말한다. 조건은 폐루프 전달함수의 분자나 분모의 주파수를 매개변수로한 부등식으로 표현된다. 또한 이산 시간 제어시스템과 시간지연 연속시스템에 관한 조건도 구한다.

I. INTRODUCTION

The basic properties of the root loci of the unity negative feedback control system are first due to Evans (1948)[1]. An important study in the linear control system is the investigation of the trajectories of the roots of the characteristic equation, so--called the root loci, when a certain system parameter varies. In this paper, we consider the linear unity negative feedback control system with a compensator having a varying proportional gain. When we vary the gain from the negative infinity to positive infinity, the unity negative feedback control system may be stable for some intervals. The main question is that under what conditions is the control system stable for a connected interval? We derive the sufficient condition under which the system is stable for a connected interval. The definitions are in Section II. In Section III, we present the main results. In Section IV we extend the condition to the case of the discrete-time system and the time delay continuous-time control system. In Section V, four examples are included and the conclusion lies in Section VI. All proofs of the given theorems are collected in Appendix.

II. DEFINITIONS

Definition: By the connected interval we mean that all the numbers between any two number in the connected interval belong to the connected interval. The connected interval may not be a null set.

Throughout the paper we assume that the unity negative feedback control system has the strictly open loop forward transfer function. If the open loop forward transfer function is $\frac{g(s)}{f(s)}$, then the denominator of the closed loop transfer function of the control system is expressible as $1+k\frac{g(s)}{f(s)}$. Similarly, the denominator of the closed loop transfer function of the discrete-time control system and the time delay continuous-time control system may be described by

$$1+K\frac{g(z)}{f(z)} \text{ and } 1+K\frac{e^{t_s}g(s)}{f(s)},$$

respectively.

Definition (Angle Growth Condition):
If $g(s)$ satisfies the condition

$$\arctan\frac{Im(g(j\omega))}{Re(g(j\omega))} \le \left|\frac{\sin\left(2\frac{Im(g(j\omega))}{Re(g(j\omega))}\right)}{2\omega}\right|$$

for any positive ω such that $g(j\omega)\neq 0$,
then $g(s)$ is said to satisfy the *angle growth condition*.

III MAIN RESULTS

The following three theorems address the conditions on $g(s)$ and/or $f(s)$ under which the closed-loop continuous control system is stable for a connected interval. Throughout this paper, all the coefficients are assumed to be real. If this is not the case, an interesting phenomenon happens. See Example 4 in Section V for details.

Theorem 1: *Consider a unity negative feedback continuous-time control system with an open loop transfer function* $\frac{g(s)}{f(s)}$ *with a cascaded proportional compensator K. Assume that for a specific K*, the closed loop system is assumed to be stable. Under these conditions, if* $g(s)$ *satisfies the angle growth condition, then the closed loop system is stable for the connected interval including K*.*

Theorem 2: *Consider a unity negative feedback continuous-time control system with an open loop transfer function* $\frac{g(s)}{f(s)}$ *with a cascaded proportional compensator K. Assume that for a specific K*, the closed loop system is assumed to be stable,* $g_1(s)$ *satisfies the angle growth condition, and Anti(s) is an antistable polynomial. Under these conditions, if* $g(s)$ *satisfies the condition*

$$g(s) = s^{1\,or\,0}p(s^2)g_1(s)Anti(s),$$

then the closed loop system is stable for the connected interval including K.*

Theorem 3: *If* $\frac{g(j\omega)}{f(j\omega)}$ *touches the negative real axis once or nontouches the negative real axis when the angular frequency* ω *sweeps from a zero to the infinity excluding* ω *such that* $f(j\omega) \neq 0$*, then the closed loop system is stable for the connected interval* $K>0$ *provided that for a positive K*, the control system is stable.*

IV. APPLICATIONS OF MAIN RESULTS

The next two theorems address the conditions on $g(s)$ under which the cloed loop discrete time control system is stable for a connected interval.

Theorem 4: *Consider a unity negative feedback discrete-time control system with an open loop transfer function* $\frac{g(z)}{f(z)}$ *with a cascaded proportional compensator K. Assume that for a specific K*, the closed loop sssystem is assumed to be Schur stable. Then if the following condition holds*

$$g(z) = z^k(az+b)$$

for $k = 0, 1, 2, \cdots, \lfloor \frac{n}{2} \rfloor$,

then the closed loop system is Schur stable for the connected interval including K.*

Theorem 5: *Consider a unity negative feedback discrete-time control system with an open loop transfer function* $\frac{g(z)}{f(z)}$ *with a cascaded proportional compensator K. Assume that for a specific K*, the closed loop sssystem is assumed to be Schur stable. Let* $b(z)$ *be a monic quadratic polynomial and two roots of* $b(z)$ *has one of the following three conditions:*

i) $b(z)$ *is antischur stable;*

ii) two roots of $b(z)$ *are nonnegative real, one root* σ_1 *is Schur stable and the other* σ_2 *is not and the condition* $|\frac{\sigma_1+1}{\sigma_1-1}| \le \frac{\sigma_1+1}{\sigma_1-1}$ *is satisfied;*

iii) two roots of $b(z)$ *are nonpositive real, one root* σ_1 *is Schur stable and the other* σ_2 *is not and the condition* $|\frac{\sigma_1+1}{\sigma_1-1}| \ge \frac{\sigma_1+1}{\sigma_1-1}$ *is satisfied.*

Under these conditions, for any $i \in (0, 1, 2, \ldots, \lfloor \frac{n}{2} \rfloor)$, *let* $g(z) = b(z)z^i$. *Then the closed loop system is stable for the connected interval including K*.*

The next theorem addresses the conditions on $g(s)$ under which the closed loop time delay continuous time control system is stable for a connected interval.

Theorem 6: *Assume that the denominator of the closed loop transfer function of the time delay continuous time control system is described by*

$$1 + K\frac{g_1(s)e^{\tau_q s}}{\sum_{k=0}^{n}\sum_{j=0}^{m} a_{kj}s^k e^{\tau_j s}}.$$

Assume that for a specific K, the closed loop system is assumed to be stable. Under these conditions, if the following two conditions*

1) $2t_q \le \tau_0 + \tau_m$

2) $g(s)$ *which satisfies the angle growth condition*

holds then the closed loop system is stable for the connected interval including K.*

V. EXAMPLES

To demomstrate the usefulness of theorems presented in this paper, four examples are illustrated.

Example 1: (Continuous time control system)
Suppose that the denominator of the closed loop transfer function of the control system is

$$1+K\frac{s+1}{s(s-1)(s^2+4s+16)}.$$

Since the numerator of the open loop forward transfer function without a compensator is $s+1$ which satisfies the assumption of Theorem 1, the interval for the stable control system is connected; that is $23.3<K<35.7$. Beyond this interval, the system is unstable.

Example 2: (Discrete time control system)
Suppose that the denominator of the closed loop transfer function of the control system is

$$1+K\frac{0.000154(z+0.24587)(z+3.4667)}{z(z^3-2.7236z^2+2.4644z-0.7408)}.$$

If we choose two zeros of the numerator of the open loop forward transfer function, two roots are
$z_1=-0.2487$ $z_2=-3.4667$.
Since the inequality

$$\left|\frac{\sigma_1+1}{\sigma_1-1}\right|\geq\frac{\sigma_1+1}{\sigma_1-1}\quad(|-0.6017|\geq 0.5522)|$$

holds, by Theorem 5 we conclude that the interval for the stable control system is connected; that is $0<K<17.6$. Beyond this interval, the system is unstable.

Example 3: (Time delay control system)
Suppose that the denominator of the closed loop transfer function of the time delay control system is

$$1+K\frac{e^{-\tau s}}{s(s-p_1)}\quad(p_1<0,\tau>0).$$

Since the numerator of the closed loop transfer function satisfies the assumption of Theorem 6, we conclude that interval within which the closed loop transfer function is stable is connected; that is $0<K<\omega_1\sqrt{p_1^2+\omega_1^2}$ where ω_1 is the least value which is the distance from the origin to the root loci of the time delay control system crossing the positive imaginary axis.

Example 4 (Counterexample):
Suppose that the denominator of the closed loop transfer function of the continuous time control system is

$$1+K\frac{(s+1)}{(0.5403+j0.845)(s^2+0.18s+0.082)}=0$$

Even through the numerator s+1 of the closed loop transfer function satisfies the assumption of Theorem 1, some of the coefficients of the denominator of the closed loop transfer function are complex number. So we cannot use Theorem 1. The interval for stablity is not connected; when K=1 or K=0, the system is stable but when K=0.5 it is unstable.

VI. CONCLUSIONS

We have presented the condition under which either a feedback control system is stable for a connected open gain interval. The condition may be described by a frequency inequality in terms of the denominator and/or numerator of the closed loop transfer function. We also consider the conditions for the discrete-time control systems and the time delay control system. We have showed that this condition cannot be extended for the complex transfer function via a counterexample. The results in this paper may be the starting point of searching conditions under which the control system is stable for a union of disjoint connected intervals.

APPENDIX

Lemma 1 (Rantzer 1992)[3]: If $g(s)$ satisfies the angle growth condition, then stability of $f(s)$ and $f(s)+g(s)$ implies stability of $f(s)+\lambda g(s)$ for $\lambda\in(0,1)$.

We present the proof of Theorem 1:

Step 1: By contradiction, there exists an interval such that for two extremes (where K^- and K^+) the closed-loop system is stable but for an intermediate value the system is unstable.

Step 2: We construct f^* and g^* such that

$$f^*(s)=f(s)+K^-g(s)$$
$$g^*(s)=(K^+-K^-)g(s)$$

Step 3: Applying $f^*(s)$ and $g^*(s)$ to Lemma 1 and note that $g^*(s)$ satisfies the angle growth condition, we conclude the hypothesis that there exist an interval such that for two extremes (where K^- and K^+ the closed loop system is stable which contradict the hypothesis that there exist an interval such that for two extermes (where K^- and K^+) the closed loop system is stable but for an intermediate value the system is unstable. Now we complete the proof. []

Proof of Theorem 2 : The proof is similar to the proof of Theorem 1 if we use following lemma. So we skip the proof.

Lemma 2 (Kang (1994))[2]: If $g(s)$ satisfies the angle growth condition, then $g(s)A(s)P(s)$ satisfies the angle growth condition where $A(s)$ is antistable and monic and $P(s)$ is a monic polynomial having either all even or all odd powers of s. Thus stability of $f(s)$ and $f(s)+g(s)A(s)P(s)$ implies stability of $f(s)+\lambda g(s)A(s)P(s)$ for $\lambda\in(0,1)$.

Proof of Theorem 3: If the assumption in the theorem is stable, the root loci of the closed loop transfer function passes the positive imaginary axis once or nontouch the positive imaginary axis. So the control system is stable for a connected interval. Now we complete the proof.

Proof of Theorems 4, 5 and 6 : The proofs are similar to the proof of Theorem 1 if we use the lemmas presented in Rantzer (1992))[3], Kang (1994)) [2] and (Kharitonov & Zhabko (1994))[4] respectively. So we skip their proofs.

Lemma 3 (Rantzer (1992))[3]: If $g(s)$ satisfies the discrete time angle growrh condition:

$$\arctan\frac{Im(g(e^{j\omega}))}{Re(g(e^{j\omega}))}\le\frac{n}{2}+\left|\frac{\sin\left(2\frac{Im(g(e^{j\omega}))}{Re(g(e^{j\omega}))}-n\omega\right)}{2\sin(\omega)}\right|$$

for $\omega\in(0,\pi)$

where the derivative is well defined.
then Schur stability of $f(s)$ and $f(s)+g(s)$ implies Schur stability of $f(s)+\lambda g(s)$ for $\lambda\in(0,1)$.

Lemma 4 (Kang (1994))[2]: Let $b(z)$ be a monic quadratic polynomil and two roots of $b(z)$ has one of the following three conditions:

i) $b(z)$ is antischur stable;

ii) two roots of $b(z)$ are nonnegative real, one root σ_1 is Schur stable and the other σ_2 is not and the condition $\left|\frac{\sigma_1+1}{\sigma_1-1}\right|\le\frac{\sigma_1+1}{\sigma_1-1}$ is satisfied;

iii) two roots of $b(z)$ are nonpositive real, one root σ_1 is Schur stable and the other σ_2 is not and the condition $\left|\frac{\sigma_1+1}{\sigma_1-1}\right|\ge\frac{\sigma_1+1}{\sigma_1-1}$ is satisfied.

Under these conditions, for any $i\in(0,1,2,\dots,\lfloor\frac{n}{2}\rfloor)$, let $g(z)=b(z)z^i$. Then then stability of $f(s)$ and $f(s)+g(s)$ implies stability of $f(s)+\lambda g(s)$ for $\lambda\in(0,1)$.

Lemma 5 (Kharitonov & Zhabko (1994))[4]:

Suppose $g(s)=g_1(s)e^{ts}$ where $g_1(s)$ is a real polynomial.

Assume that

(1) $2r_q\le r_0+r_m$

(2) $g_1(s)$ satisfied the angle growth condition.

then stability of $f(s)$ and $f(s)+g(s)$ implies stability of $f(s)+\lambda g(s)$ for $\lambda\in(0,1)$.

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