미지 입력을 가진 쌍선형 시스템의 관측기 구성

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Observer Design for Bilinear Systems with Unknown Inputs

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Abstract: In this paper, we considers the problem of designing an observer for bilinear systems with unknown input. A sufficient condition for the asymptotic stability of the proposed observer is derived by means of detectability, invariant zeros, and stable subspace. In sufficient condition, the bound which guarantees the asymptotic stability was derived, which based on the Lyapunov stability. And Observer existing conditions are suggested in various cases. Through a simple example, we derived the observer structure and the bound which guarantees the asymptotic stability.

Keywords: Bilinear system, unknown input, Lyapunov stability

1. Introduction

The problem of designing an observer for systems with unknown input has attracted some attention in the literature. Meditch and Hostetter(1973) proposed a method which assumes some a priori knowledge of the disturbance. Kudva *et. al.*(1980) gave necessary conditions for this kind of observer to exist. In Bhattacharyya(1978), a geometric approach has been proposed. Also, Hara and Furuta(1976) and Funahashi(1979) proposed methods which construct stable minimal-order observers for bilinear systems.

We suggest various sufficient conditions for existence and asymptotical stability of bilinear observer by means of detectability, invariant zeros, and stable subspace, according to the relations of measurement and unknown input. And we propose a perturbation bound to guarantee the asymptotic stability of the observer.

2. Observer for Bilinear Systems with Unknown Inputs

Consider a particular kind of bilinear systems with unknown inputs:

$$\dot{x}(t) = [A_0 + \sum_{i\in I} p_i(t) A_i]x(t) + [B_0 + \sum_{j\in J} q_j(t) B_j]u(t) + Dv(t) \tag{1.1}$$

$$w(t) = Cx(t), \tag{1.2}$$

where $x(t)\in R^n$, $u(t)\in R^p$, $w(t)\in R^m$ and $v(t)\in R^q$, are state, input, output and unknown input, respectively; $I = \{1, \cdots, i_{max}\}$, $J = \{1, \cdots, j_{max}\}$; A_0, $A_i \in R^{n\times n}$, $B_0, B_j \in R^{n\times p}$, $D\in R^{n\times q}$, $C\in R^{m\times n}$, $p_i(t)$ and $q_j(t)$ are input or time-varying components lying within certain bounds,

$$p_i^- \le p_i(t) \le p_i^+, \ i\in I, \tag{2.1}$$

$$q_j^- \le q_j(t) \le q_j^+, \ j\in J. \tag{2.2}$$

Without loss of generality, it can be assumed that D has full column rank and C has full row rank.

For the system (1.1), (1.2), an observer which reconstructs the state $x(t)$ without the knowledge of the unknown input $v(t)$ is constructed as follows:

$$\dot{z}(t) = [F_0 + \sum_{i\in I} p_i(t) F_i]z(t) + [G_0 + \sum_{j\in J} q_j(t) G_j]u(t) + [L_0 + \sum_{i\in I} p_i(t) L_i]w(t) \tag{3.1}$$

$$x_e(t) = z(t) - Ew(t), \tag{3.2}$$

where $z(t)\in R^n$, $x_e(t)\in R^n$; F_0, $F_i\in R^{n\times n}$; G_0, $G_j\in R^{n\times p}$; L_0, $L_i\in R^{n\times m}$; $E\in R^{n\times m}$.

Define the estimation error $e(t) = x_e(t) - x(t)$. Then,

$$\begin{aligned}\dot{e}(t) &= \dot{x}_e(t) - \dot{x}(t) \\ &= [F_0 + \sum_{i\in I} p_i(t) F_i]e(t) \\ &+ [F_0 - (EC+I)A_0 + \sum_{i\in I} p_i(t)(F_i - (EC+I)A_i)]x(t) \\ &+ [G_0 - (EC+I)B_0 + \sum_{j\in J} q_j(t)(G_j - (EC+I)B_j)]u(t) \\ &+ [L_0 + F_0 E + \sum_{i\in I} p_i(t)(L_i + F_i E)]w(t) - [(EC+I)D]v(t).\end{aligned}$$

Since $e(t)$ is required to converge to zero irrespective of $v(t)$, E is chosen to satisfy the relation:

$$(EC+I)D = 0. \tag{4}$$

Let $P := EC + I$, $\overline{L}_0 := L_0 + F_0 E$, and $\overline{L}_i := L_i + F_i E$.

Then, with a choice of E satisfying equation (4),

$$\dot{e}(t) = [F_0 + \sum_{i\in I} p_i(t) F_i]e(t)$$

$$+[F_0-PA_0+\overline{L}_0C+\sum_{i\in I}p_i(t)(F_i-PA_i+\overline{L}_iC)]x(t)$$

$$+[G_0-PB_0+\sum_{j\in J}q_j(t)(G_j-PB_j)]u(t).$$

If F_0, F_i, G_0, G_j, L_0 and L_i are constructed such that

$$F_0 - PA_0 + (L_0+F_0E)C = 0, \tag{5}$$

$$F_i - PA_i + (L_i+F_iE)C = 0, i\in I, \tag{6}$$

$$G_0 - PB_0 = 0, \tag{7}$$

$$G_j - PB_j = 0, j\in J, \tag{8}$$

then, for all $p_i(t)$ and $q_j(t)$,

$$\dot{e}(t)=[F_0+\sum_{i\in I}p_i(t)F_i]e(t). \tag{9}$$

3. Construction of Bilinear Observers

First, E is chosen to satisfy equation (4) and F_0 and L_0 are constructed to satisfy (5) while F_0 remains stable. Next, F_i, L_i, $i\in I$, G_0 and G_j, $j\in J$ are constructed according to (6), (7), and (8). And to guarantee the stability of $F_0+\sum_{i\in I}p_i(t)F_i$ for chosen F_0 and F_i, $i\in I$, we regard $\sum_{i\in I}p_i(t)F_i$ as a perturbation and propose a condition on $p_i(t)$ to ensure the stability of $F_0+\sum_{i\in I}p_i(t)F_i$.

In the following, it is assumed that

$$rank CD = q, \quad q\le m, \tag{10}$$

which is necessary and sufficient condition for the existence of E satisfying equation (4). And let $R=I-D(CD)^+C$.

Theorem 1 : Assume that $rank CD = q\le m$. If (C, RA_0) is detectable, then there exist E, F_0 and L_0 such that equations (4) and (5) are satisfied and F_0 is stable.

Proof: If

$$E = -D(CD)^+, \tag{11}$$

then equation (4) is satisfied and P becomes R. Therefore, (C, PA_0) is detectable, which implies the existence of an $\overline{L}_0$ such that $PA_0-\overline{L}_0C$ is stable. Now F_0 and L_0 are constructed with this $\overline{L}_0$ as follows:

$$F_0 = PA_0-\overline{L}_0C, \tag{12}$$

$$L_0 = \overline{L}_0-F_0E. \tag{13}$$

If P is chosen, then equations (6), (7) and (8) can be satisfied by F_i, L_i, G_0 and G_j constructed as follows :

$$F_i = PA_i, i\in I, \tag{14}$$

$$L_i = -F_iE, i\in I, \tag{15}$$

$$G_0 = PB_0, \tag{16}$$

$$G_j = PB_j, j\in J. \tag{17}$$

Theorem 2 : Assume that $rank CD = q\le m$. If all the invariant zeros of the system (C, A_0, D) have negative real parts, there exist E, F_0 and L_0 such that equations (4) and (5) are satisfied and F_0 is stable.

Proof: Suppose that a complex number λ corresponds to an unobservable mode of (C, RA_0). Then, there exists a nonzero vector $x\in R^n$ such that

$$\begin{bmatrix}\lambda I_n-RA_0\\ C\end{bmatrix}x=0,$$

or equivalently,

$$\begin{bmatrix}\lambda I_n-A_0 & D\\ C & 0\end{bmatrix}\begin{bmatrix}x\\ (CD)^+CA_0x\end{bmatrix}=0,$$

which implies that

$$rank\begin{bmatrix}\lambda I_n-A_0 & D\\ C & 0\end{bmatrix} < n+q.$$

Since $q\le m$, λ is an invariant zero of the system (C, A_0, D). Hence, if all the invariant zeroes of the system (C, A_0, D) have negative real parts, then (C, RA_0) is detectable and the assertion follows from Theorem 1.

Next, $W := D(CD)^+C$ is defined.
Then,

$$I = R+W,$$

$$R^2 = R(I-W) = R, \quad W^2 = (I-R)W = W.$$

Therefore, R and W are projection operators on R^n.
Hence, a direct sum decomposition of R^n results:

$$R^n = \mathrm{Im}\,R \oplus \mathrm{Im}\,W.$$

From the definition of W, $\mathrm{Im}\,W \subset \mathrm{Im}\,D$.
Since $WD = D(CD)^+CD = D$, $\mathrm{Im}\,W \supset \mathrm{Im}\,D$.
Hence, $\mathrm{Im}\,W = \mathrm{Im}\,D$,
and another direct sum decomposition of R^n is obtained:

$$R^n = \mathrm{Im}\,R \oplus \mathrm{Im}\,D.$$

Theorem 3 : Assume that $rank CD=q=m$. (C, RA_0) is detectable if and only if the stable subspace of RA_0 contains $\mathrm{Im}\,R$.

Proof: Since $rank CD=q=m$, $(CD)^+$ becomes $(CD)^{-1}$,

$$CR = C(I-D(CD)^{-1}C) = 0. \tag{18}$$

Hence, the observability matrix of (C, RA_0) becomes

$$W_o = \begin{bmatrix}C\\ CRA_0\\ \vdots\\ C(RA_0)^{n-1}\end{bmatrix} = \begin{bmatrix}C\\ 0\\ \vdots\\ 0\end{bmatrix},$$

which implies that $\mathrm{Ker}\,W_o = \mathrm{Ker}\,C$. On the other hand, from (18), $\mathrm{Ker}\,C \supset \mathrm{Im}\,R$. Since $q=m$, $\dim(\mathrm{Ker}\,C)=n-m$ and $\dim(\mathrm{Im}\,D)=m$. Thus it follows from the direct sum decomposition of R^n that $\dim(\mathrm{Im}\,R)=n-m$, which implies that $\mathrm{Ker}\,C = \mathrm{Im}\,R$. Therefore $\mathrm{Ker}\,W_o = \mathrm{Im}\,R$, that is, the unobservable subspace of (C, RA_0) is equal to $\mathrm{Im}\,R$. Hence (C, RA_0) is detectable if and only if the stable subspace of RA_0 contains $\mathrm{Im}\,R$.

In the following, $\sigma_{\max}(M)$ and $\sigma_{\min}(M)$ denote the maximum and minimum singular values of the matrix M, respectively.

Theorem 4 : Let F_0 be stable, Q be positive definite and symmetric, and $H=H^T$ be a solution of the Lyapunov equation,

$$F_0^T H + H F_0 + Q = 0. \quad (19)$$

Then, $F_0 + \sum_{i \in I} p_i(t) F_i$ is stable for all $p_i(t)$ satisfying

$$\sum_{i \in I} | p_i(t) |^2 < \frac{\sigma_{\min}^2(Q)}{\sum_{i \in I} \sigma_{\max}^2(F_i^T H + HF_i)}, \quad \forall t. \quad (20)$$

Proof: Since F_0 is stable, H is positive definite. A Lyapunov function $V(e) = e^T H e$ is introduced. By the Lyapunov stability theorem, $e(t)$ is asymptotically stable, if there exists $\varepsilon > 0$ so that

$$\dot{V}(e) \le -\varepsilon \| e \|_2^2, \quad \forall t. \quad (21)$$

It will be demonstrated that equation (20) implies equation (21). Let $Q_i := F_i^T H + HF_i$.

$$\dot{V}(e) = e^T[(F_0^T H + HF_0) + \sum_{i \in I} p_i(t)(F_i^T H + HF_i)]e$$
$$= -e^T Q e + e^T[\sum_{i \in I} p_i(t) Q_i] e. \quad (22)$$

It follows from equation (23) that there exists a positive number ε, $\varepsilon < \sigma_{\min}(Q)$, such that

$$(\sum_{i \in I} | p_i(t) |^2)^{1/2} (\sum_{i \in I} \sigma_{\max}^2(Q_i))^{1/2} \le \sigma_{\min}(Q) - \varepsilon, \ \forall t. \quad (23)$$

Applying the following inequality

$$\sigma_{\max}(\sum_{i \in I} p_i(t) Q_i) \le \sum_{i \in I} (| p_i(t) | \sigma_{\max}(Q_i))$$
$$\le (\sum_{i \in I} | p_i(t) |^2)^{1/2} (\sum_{i \in I} \sigma_{\max}^2(Q_i))^{1/2}, \quad (24)$$

it follows that

$$\sigma_{\max}(\sum_{i \in I} p_i(t) Q_i) \le \sigma_{\min}(Q) - \varepsilon, \ \forall t. \quad (25)$$

Since

$$\sigma_{\min}(Q) \| e \|_2^2 \le e^T Q e \le \sigma_{\max} \| e \|_2^2, \quad (26)$$
$$| e^T (\sum_{i \in I} p_i(t) Q_i) e | \le \sigma_{\max}(\sum_{i \in I} p_i(t) Q_i) \| e \|_2^2 \quad (27)$$

equation (25) implies equation (21), which completes the proof.

4. Example

Consider given system,

$$A_0 = \begin{bmatrix} -2 & -2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & -4 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, B_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \ C = [1 \ 0 \ 2], \ D = [2 \ -2 \ 1]^T.$$

In this case, $rank CD = q = m$. Thus this case satisfies the necessary and sufficient condition for the existence of E satisfying equation (4). Under this condition, the solution to equation (4) is given by $E = [-0.5 \ 0.5 \ -0.25]^T$, and P is obtained by definition, so $P = \begin{bmatrix} 0.5 & 0 & -1 \\ 0.5 & 1 & 1 \\ -0.25 & 0 & 0.5 \end{bmatrix}$.

(C, PA_0) is detectable, which implies that there exists an $\overline{L}_0$ such that $PA_0 - \overline{L}_0 C$ is stable. With this choice of $\overline{L}_0 = [3 \ -3 \ 3]^T$, F_0 and L_0 satisfying equation (12), (13) are

$$F_0 = \begin{bmatrix} -4 & 2 & -2 \\ 2 & -3 & 3 \\ -2.5 & -1 & -8 \end{bmatrix}, \ eig(F_0) = -9, -2, -4,$$

$$L_0 = [-0.5 \ 0.25 \ 0.25]^T.$$

With this construction of F_0 and L_0, equation (5) is satisfied and F_0 is stable. By constructing F_1, L_1, G_0, G_1, equations (6), (7) and (8) can be satisfied.

$$F_1 = \begin{bmatrix} -0.5 & -1 & 1 \\ 0.5 & 0 & -1 \\ 0.25 & 0.5 & -0.5 \end{bmatrix}, L_1 = [0.5 \ 0 \ -0.25]^T,$$

$$G_0 = [0.5 \ 0.5 \ -0.25]^T, \quad G_1 = [1 \ 1 \ -0.5]^T.$$

By the above result, observer for the bilinear system with unknown inputs is constructed as equations (3.1), (3.2).

And by the Theorem 4, we evaluate the bound on $p_1(t)$ which guarantees the stability of $F_0 + p_1(t) F_1$.

In case of $Q = I$, H satisfying equation

$$F_0^T H + H F_0 + Q = 0 \ \text{ is}$$

$$H = \begin{bmatrix} 0.1894 & 0.1101 & -0.015 \\ 0.1101 & 0.2292 & 0.0326 \\ -0.015 & 0.0326 & 0.0785 \end{bmatrix}.$$

Perturbation $|p_1(t)|^2$ satisfying equation (23) is

$$\frac{\sigma_{\min}^2(Q)}{\sigma_{\max}^2(F_1^T H + HF_1)} = 12.9556$$

5. Conclusion

An observer for the bilinear systems with unknown input was proposed. Design of bilinear observer was also derived. And observer existence condition was checked by detectability, invariant zeros, and stable subspace And a sufficient condition guaranteeing asymptotic stability of proposed observer was also derived, which is based on the Lyapunov stability theorem. And a simple example is illustrated. Through this example, we derived the observer structure and the bound which guarantees asymptotic stability of proposed observer.

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