On Calculating Eigenvalues In Large Power Systems Using Modified Arnoldi Method

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Abstract

This paper presents a method of calculating a selective number of eigenvalues in power systems, which are rightmost, or are largest modulus. The modified Arnoldi method in conjunction with implicit shift QR-algorithm is used to calculate the rightmost eigenvalues. Algorithm requires neither a prior knowledge of the specified shifts nor the calculation of inverse matrix. The key advantage of the algorithm is its ability to converge to the wanted eigenvalues at once. The method is compared with the modified Arnoldi method combined with S-matrix transformation, where the eigenvalues having the largest modulus are to be determined. The two methods are applied to the reduced Kansai system. Convergence characteristics and performances are compared. Results show that both methods are robust and has good convergence properties. However, the implicit shift QR method is seen to be faster than the S-matrix method under the same condition.

Keywords: small-signal stability, selective eigenvalue calculation, preconditioning, implicit shift QR, S-matrix, modified Arnoldi

I. Introduction

A large eigenvalue problems arise in a variety of works in power system planning and operations. Power uitlities in Japan have used eigenvalue analysis as one of important tools in dealing with low frequency oscillation problems. In manufacturing companies, one of key steps involved in designing process of FACTS equipment is the decision of dynamic characteristics. The dynamic characteristics of FACTS for maintaining entire uitlity system stability are usually decided from eigenvalue analysis. In addition to these examples, applications of eigenvalue analysis to the power systems are widespread and need to conduct large eigenvalue analysis is being increasingly recognized .

The modified Arnoldi method is a popular algorithm for solving large eigenvalue problems, which has been first introduced to power systems in [1]. It is widely accepted that the method is the most appropriate for finding a few eigenvalues and corresponding eigenvectors of a large nonsymmetric matrix. However, there is a notable lack of general purpose to find eigenvalues of interest in power systems. Although eigenvalues we want to find are rightmost one (or close to imaginary axis), Arnoldi iterations are converged to the eigenvalues having the largest modulus. Thus, some initial transformation (preconditioning) is necessary to have the rightmost eigenvalues dominant.

Shift-invert transformation is very popular but several shifts are usually necessary to determine the rightmost eigenvalues. To be run efficiently, this job requires a more trained engineer. From this viewpoint, the best known method is S-matrix method [2]. It needs to be applied only once for the calculation of all the rightmost eigenvalues. Refs. [3,4] reported their attempts to combine S-Matrix transformation with the modified Arnoldi method. In [5], Mori et al. addressed a problem inherent to the S-matrix transformation and proposed an improved approach.

In ref. [6], authors made an attempt to calculate the rightmost eigenvalues using the modified Arnoldi method combined with the implicit shift QR method. Main characteristics of the method and its implementing procedures are described in the next section. Contrary to the S-matrix method, this algorithm does not require an initial transformation. So, a matrix factorization required in the transformation is not necessary. The main objective of this paper is to compare the performance and convergence properties of the two methods.

2. Modified Arnoldi with Implicit Shift QR Method

The key steps of the basic modified Arnoldi method are described in Figure 1. (See ref [6] for the details.) As mentioned in preceding section, the modified Arnoldi method needs preconditioning to find the rightmost eigenvalues. In addition to this, it is difficult to extract the rightmost eigenvalues if the rightmost eigenvalues are clustered together while the others are separated favorably from one another. To avoid these shortcomings, implicit shift QR method is applied in conjunction with the modified Arnoldi method. The implicit shift QR algorithm amplifies the components of the vector toward the direction of the wanted eigenvalues and at the same time damping those in the remaining eigenvectors. This mechanism is adapted to calculate a new starting vector in Arnoldi iterations. So, sorting the rightmost eigenvalues and calculating a new starting vector will provide a new set of the eigenvalues that are located further rightside compared to the previous set. Implementation of the implicit shift QR algorithm is described in Figure 2. The procedures in Figure 2 are replaced with step k of Figure 1 to calculate the rightmost eigenvalues.

a. set up v_1 $(V = v_1)$

b. $u_i^1 = Jv_i \quad (k = 1)$

c. $u_i^{k+1} = (I - V_i V_i^H) u_i^k$

d. $\dfrac{\|u_i^k\|_2}{\|u_i^{k+1}\|_2} \le \theta \quad (\theta > 1)$

e. $h_{i+1,i} = \|u_i^{kr+1}\|_2$ where $k_r = k$

f. $v_{i+1} = \dfrac{u_i^{k+1}}{h_{i+1,i}}$

g. $h_i = V_i^H J v_i$

h. $h_i = [h_i \quad h_{i+1,i} \quad 0 \quad \dots \quad 0]^T$

i. if $i \neq m$, $V_{i+1} = [V_i \; v_{i+1}]$, $i = i+1$, go to b

j. Calculate all eigenvalues λ_i of H

k. $v_1^{new} = V_m P \bar{p}$

l. Repeat steps b - k until finding all eigenvalues

Figure 1. Algorithm of the modified Arnoldi method [6]

3. Modified Arnoldi with S-matrix

S-matrix algorithm is combined with the modified Arnoldi method by replacing the matrix J in step b of Figure 1 with the following matrix

$$J_s = I + (h_1 - h_2)[J - h_1 I]^{-1} \qquad (1)$$

where I is an identity matrix. Eq. 1 is a general expression of the S-Matrix transformation of the matrix J, which is so-called Cayley transformation. Symmetry axis will be located in the center point between h1 and h2. Obvioiusly, the wanted eigenvalues in new domain appears as eigenvalues having largest modulus. In step k.1 of Figure 2, largest modulus eigenvalues are sorted instead of rightmost eigenvalues. It should be noted that the calculation of inverse matrix is required in Eq. 1 whereas it is not required in implicit shift QR algorithm.

k.1 Sort eigenvalues

- m rightmost eigenvalues
- p rest eigenvlaues as shifts

k.2 $Q = I_{m+p}$

k.3 *for* $j = 1,2,.....,p$

$$H = Q_j^T H Q_j$$

$$Q = QQ_j$$

k.4 $V = (VQ)e_{m+1}$

k.5 $V_+ = VQ$

k.6 $r = ve_{m+1}^T He_m + re_{m+1}^T Qe_m$

Figure 2. Calculation of the starting vector for the rightmost eigenvalues [6]

4. Numerical Results

Reduced Kansai system is considered for comparing two algorithms. The composition and size of the system is summarized in Table 1.

Table 1. Description of test system

buses	generators	AVR	PSS	Total No. of states
961	26	26	13	291

Complete eigenanalysis has been performed using LAPACK routine DGEEV to verify the results obtained in this section. Figure 3 shows distribution of the eigenvalues, in which the eigenvalues having real parts greater than -2 are included. The DGEEV run required 102.46 [sec] to compute all eigenvalues and the associated eigenvectors. Calculations reported in this paper were performed with double precision on an HP 735/125 workstation.

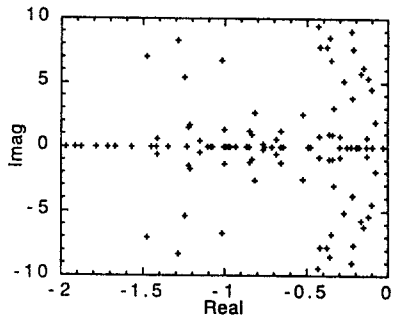

Figure 3. Eigenvalue distribution of the reduced Kansai system

4.1 Modified Arnoldi with implicit shift QR

The system is analyzed first using the modified Arnoldi with the implicit shift QR to obtain the rightmost eigenvalues. In Arnoldi step, iteration was halted when residue became less than 1.0E-6. This ad hoc stopping rule allowed the iteration to halt quite early in cases where it was difficult to make a clear separation between the wanted eigenvalues and unwanted eigenvalues. Computational efficiency varies for different numbers of wanted eigenvalues and additional steps. Detailed observation on this matter appears in [6]. After trying different number of wanted eigenvalues, the best results are selected for the number of additional steps, 22. Table 2 shows the results, where 4 pais of rightmost complex eigenvalues are determined.

4.2 Modified Arnoldi with S-Matrix

In Eq. 1, there is no criterion for choosing optimal shifts h1 and h2. To have an insight of the transforming mechanism, we perform the S-matrix transformation of the original eigenspace (not eigenvalues) defined in Figure 3 using various shifts. Results of the transformations

Table 2. Converged eigenvalues for implicit shift QR

Eigenvalue (original)	Eigenvalue (converged)	Residue	CPU time [seconds]
-0.031			
-0.078+j1.910	x	5.02E-10	
-0.095			
-0.102+j4.481	x	2.80E-11	789.38
-0.121+j5.373	x	9.86E-09	
-0.130+j0.692	x	1.61E-06	
-0.132			
-0.152+j6.196			
-0.167+j5.731			

are shown in Figure 4. In transformed space, density changes dramatically depending on the value of the shifts. For small h (0.5) shown in Figure 4.a, density is extremely high in right-hand part and fairly low in opposite part of the figure. Results of increasing h are shown in Figures 4.b-4.f. As clearly observed in those figures, incresing h changes both geometrical shape and density in the transformed space. The mapped areas in those figure are gradually shrunken to left side and dense part also gradually moves to left side. Contrary to Figure 4.a, a transformed space in Figure 4.f is severely shrunken and very dense.

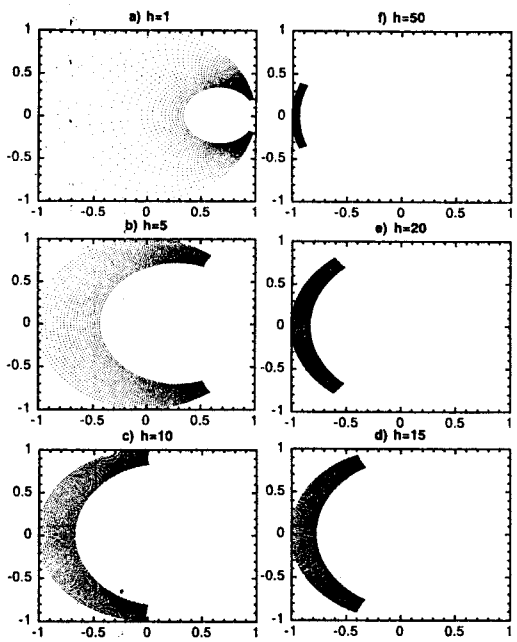

Figure 4. S-matrix transformation of eigenspace for different shifts h (h1=h2) varying from 1 to 50

S-matrix transformations of original eigenvalues are shown in Figure 5, where an ellipse represents a unit circle. Each figure corresponds to different value of the shifts. We experienced very poor convergence when imaginary axis become a symmetry axis. So, non-imaginary axis is recommended to become a symmetry axis. Here, a vertical line crossing the horizontal axis at -0.2 is selected as a symmetry axis. Eigenvalues having real part greater than -0.2 will appear outside a unit circle. As observed in preceding pagraph, density is very high at either right-hand part (Figure 5.a) or left-hand part (Figure 5.d) in the two extreme values. Distribution of eigenvalues are more or less spreaded in Figures 5.b and 5.c compared to other two extremes.

Determining rightmost eigenvalues becomes a problem determining the largest modulus eigenvalues in the S-matrix transformation. However, the rank of the rightmost eigenvalues we want to find is not always identical to the rank of eigenvalues having the largest modulus. For example, in Figure 5.a, eigevalues having the largest modulus are several real eigenvalues located outside circle. Obviously, the rightmost eigenvalues are irrespective of the eigenvalues having the largest modulus in the figure. Table 3 shows the rank of the modulus.

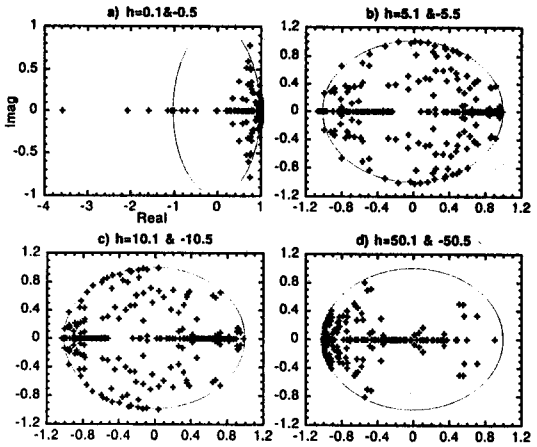

Figure 5. Transformed eigenvalue distribution using S-Matrix

First column in Table 3 contains the original rightmost eigenvalues in descending order. The other four columns show the rank of the corresponding eigenvalues in the transformed space with respect to the modulus. Discrepancies between the two ranks are very large in column a). It is because the wanted eigenvalues are clustered in dense area, whereas many unwanted real eigenvalues are sepatated favorably. However, when the shift h increases, the discrepancies become small. In column d), we can see the two ranks are identical, which is for large shift.

Table 3. Rank of largest modulus in transformed space

Eigenvalue	shifts h1 & h2			
(original)	a)0.1&-0.5	b)5.1&-5.5	c)10.1&-10.5	d)50.1&-50.5
-0.031	1	1	1	1
-0.078+j1.910	9,10	2,3	2,3	2,3
-0.095	2	4	4	4
-0.102+j4.481	11,12	8,9	5,6	5,6
-0.121+j5.373	13,14	10,11	10,11	7,8
-0.130+j0.692	5,6	5,6	7,8	9,10
-0.132	3	7	9	11
-0.152+j6.196	16,17	13,14	12,13	12,13
-0.167+j5.731	18,19	15,16	14,15	14,15
-0.178	4	12	16	16
-0.190	7	17	17	17
-0.195	8	18	18	18
-0.200	15	19	19	19

Results of applying the S-matrix method are summarized in Table 4. First column is the rightmost eigenvalues in descedning order as shown in Table 3. The other four columns indicate converged eigenvalues for the different shifts. The converged eigenvalues are marked by x. In all the cases, the converged eigenvalues are the ones having the largest moduli. A number of converged eigenvalues in the table is not same from one another. For example, in column a), relatively many eigenvalues are obtained compared to the other cases. Among the eigenvalues obtained, however, some of the eigenvalues are unwanted real eigenvalues while some wanted eigenvalues are missing. This result was well expected from the observations of the eigenspace and eigenvalue distributions shown in Figures 4 and 5. The reason we have many unwanted eigenvalues are because geometrical distances betwen the eigenvalues close to real axis are expanded large in the transformed space whereas others not close to the real axis are scontracted. When the shift increases, a number of the converged eigenvalues is reduced but they are more or less all wanted eigenvalues.

The CPU times required for the above calculations are given in Table 5. Compared to the implicit shift QR method, the S-matrix method required almost double CPU time under the same condition. Choosing the different shifts also affects CPU time. Interestingly, case b) took the least CPU time among the casese. This result is somewhat expected from the observations made in the preceding paragraphs.

Table 4. Converged eigenvalues for 1000 Arnoldi iterations

Eigenvalue	shifts h1 & h2			
(original)	a)0.1&-0.5	b)5.1&-5.5	c)10.1&-10.5	d)50.1&-50.
-0.031	x	x	x	x
-0.078+j1.910	x	x	x	x
-0.095	x	x	x	x
-0.102+j4.481		x	x	x
-0.121+j5.373				x
-0.130+j0.692	x	x	x	
-0.132	x	x		
-0.152+j6.196				
-0.167+j5.731				
-0.178	x			
-0.190	x			
-0.195	x			
-0.200				

Table 5. CPU time comparison

	shifts h1 & h2			
	a)0.1&-0.5	b)5.1&-5.5	c)10.1&-10.5	d)50.1&-50.
CPU time [s]	1486.55	1350.29	1562.57	1575.67

5. Conclusions

The paper has described two methods based on the modified Arnoldi method for the calculation of rightmost eigenvalues in large power systems. From the numerical results, it is clearly observed that these two methods give practically identical results. The methods are robust and have good convergence properties. Performances of the two methods are heavily affected by the number of additional steps. From the CPU time comparisons, the modified Arnoldi with implicit shift QR is seen to be faster than the modified Arnoldi with the S-matrix under the same condition.

Sensitivity of the S-matrix transformation with respect to the shift h was visualized. Observation of the geometrical properties involved in the S-matrix transformation was useful for the analyses of the numerical resutls. The selection of the shift requires a particular attention.

Although computational results for the modified Arnoldi method are quite promising, the mechanism of determining the rightmost eigenvalues is yet preliminary to deal with the problems existing in the practical system. Our limited experience indicates that a better understanding of the convergence estimates would be helpful to better deal with the close eigenvalues. Work is under way to improve the efficiency and speed of the method.

6. References

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