TIME-DEPENDENT FRACTURE OF ARTICULAR CARTILAGE: PART 1 - THEORY & VALIDATION

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Abstract
A time-dependent large deformation fracture theory is developed for application to soft biological tissues. The theory uses the quasilinear viscoelastic theory of Fung, and particularizes it to constitutive assumptions on polyvinyl-chloride (PVC) (Part I) and cartilage (Part II). This constitutive theory is used in a general viscoelastic theory by Christensen and Naghdi and an energy balance to develop an expression for the fracture toughness of the materials. Experimental methods are developed for measuring the required constitutive parameters and fracture data for the materials. Elastic stress and reduced relaxation functions were determined using tensile and shear tests at high loading rates with rise times of 25 - 30 msec, and test times of 150 sec. The developed method was validated, using an engineering material, PVC to separate the error in the testing method from the inherent variation of the biological tissues. It was found that the the proposed constitutive modeling can predict the nonlinear stress-strain and the time-dependent behavior of the material. As an approximation method, a pseudo-elastic theory using the J-integral concept, assuming that the material is a time-independent large deformation elastic material, was also developed and compared with the time-dependent fracture theory. For PVC, the predicted fracture toughness is 1.2 ± 0.41 and 1.5 ± 0.23 kN/m for the time-dependent theory and the pseudo-elastic theory, respectively. The methods should be of value in quantifying fracture properties of soft biological tissues. In Part II, an application of the developed method to a biological soft tissue was made by using bovine humeral articular cartilage.

1. Introduction

Only a limited number of studies have been made to measure fracture properties of biological materials. Brittle and stiff materials such as wood, sea shells, bones (Behri, 1984), etc. were readily measured using the *stress-intensity*, K, or the *energy release rate*, G, because of their similar characteristics to engineering materials such as metals. However, for soft tissues, only a few materials have been studied for their fracture behavior (Broom, 1984, 1986; Purslow, 1980, 1983a, 1983b; Chin-Purcell, 1991). Particularly in the field of orthopaedics, the breakdown of musculo-skeletal soft tissues such as cartilage, meniscus, ligaments, and tendon appears to be often involved in the desease process or injury. Understanding the fracture behavior of these soft tissues appears to be important in understanding the pathophysiology of the tissues.

Recently Chin-Purcell (1991) introduced the methods of fracture mechanics to quantify fracture toughness of cainine patellar cartilage. Although the approach seemed appropriate, the particular application of the method she used was limited because she assumed small deformation elastic behavior for the tissue in order to calculate the fracture parameter from the experimental fracture data. In reality, cartilage is known to be highly time dependent and, because of its high compliance, likely undergoes large deformations. There are questions with the absolute magnitude of the fracture parameters for cartilage calculated by Chin-Purcell because she ignored these nonlinearities.

The specific objectives of the present research were to: (1) establish a nonlinear viscoelastic model which can describe the constitutive behavior of highly deformable time-dependent biological soft tissues; (2) develop a methodology of fracture measurement for soft tissues on the basis of this large deformation, viscoelastic constitutive theory; (3) measure the material parameters necessary to apply this theory to the tissues, (4) verify the methods, both theoretical and experimental, necessary to deduce fracture toughness for the tissue using these methods, and (5) use the theories and methods to assess the effect of large deformations and time dependency on the predicted fracture parameter for a soft tissue.

A single phase viscoelastic model was used as an appropriate constitutive model to account for the large deformations as well as the time-dependent behavior of the tissues. For the application to plane stress analysis of thin striplike specimens, the quasilinear theory of viscoelasticity (Fung, 1972, 1981) was generalized to a three-dimensional form by using the general thermodynamic consideration of a viscoelastic medium (Christensen, 1971). As a fracture measurement for viscoelastic materials, the fracture surface energy of tissue is defined based on an energy balance (Knauss, 1970, 1971; Schapery, 1975), the terms of which can be determined by solution of an appropriate boundary value problem and experimental measurements. The proposed method of fracture measurement for soft tissues consists of theoretical modeling, numerical solution methods, and experimental measurement of the material's pre-failure and fracture properties. It is important to assess the validity and the accuracy of the overall procedure as well at each stage of the test procedures. However, the direct use of biological tissues for this purpose appears to be inadequate because there are considerable variations between individual specimens. In order to validate the proposed fracture measurement method, an engineering material, PVC, which has a similar deformational and fracture behavior to soft biological tissues was chosen and tested at each stage of the experimental and numerical procedures, because most of biological tissues have significant natural variation between the specimens.

Methods

2. Theory

2.1. Constitutive Modeling of Soft Biological Tissues

Assuming soft biological tissues to be an isotropic and compressible hyper-viscoelastic material, a nonlinear viscoelastic constitutive theory was developed. Although the theory has originally been developed for the materials which can be assumed to be a single phase solid, it could be extended to a biphasic formulation by utilizing the model in describing of the flow-independent viscoelasticity of the tissue matrix alone, as proposed in the poroviscoelastic models (Mak, 1986; Huyghe, 1991). The theory is a combination of Fung's quasi-linear viscoelastic theory (1972) and the general nonlinear viscoelastic theory of Christensen and Naghdi (1967; or Christensen, 1968, 1971; Coleman, 1964a, b). The resulting expression is similar to the recent model proposed by Huyghe (1991), but differs in that i) the stored strain energy is history dependent in our theory, allowing a dissipation term to be computed, ii) the reduced relaxation function is defined to be a fourth order tensor.

a) Generalized quasilinear viscoelastic theory

From the general thermodynamic consideration for nonlinear viscoelastic media (Christensen, 1967), the following local energy balance and entropy inequality are assumed,

$$\rho r - \rho(\dot{A} + \dot{T}S + T\dot{S}) + \sigma_{ij}d_{ij} - Q_{i,i} = 0 \ : \ \text{energy balance} \quad (1)$$

$$\rho T\dot{S} - \rho r + Q_{i,i} - Q_i\,(T_{,i}/T) \geq 0 \ : \ \text{entropy inequality} \quad (2)$$

where ρ is the mass density at time t, r is the heat supply function per unit mass, A is the Helmholtz free energy per unit mass, T is the absolute temperature, S is the entropy per unit mass, Q_i are the Cartesian components of the heat flux vector measured per unit area per unit time, σ_{ij} is the Cauchy stress, and d_{ij} is the velocity gradient,

$$2\,d_{ij} = v_{i,j} + v_{j,i}\,, \quad \text{with } v_i(\tau) = \dot{x}_i\,(X_K,\tau),\ \ v_i = v_i(t) \quad (3)$$

A superimposed dot is used for differentiating with respect to t, holding the reference configuration coordinate X_K fixed. For an isothermal condition ($\dot{T}$, $T_{,i} = 0$), combining the two gives

$$-\rho\,\dot{A} + \sigma_{ij}\,d_{ij} \geq 0 \quad (4)$$

At this point, we diverge from Christensen and Naghdi (1967) in the assumed form of the stored strain energy. It was wished to have a strain energy function consistent with Fung's quasi linear theory.

In viscoelastic materials, input work is intantaneously stored but then relaxes as time elapses. In other words, the work input at the present time t is stored completely in the body, while the work previously input at any time τ (where τ< t) has been somewhat dissipated depending upon how far the time τ is from the present time t. For a mathematical description of such a dissipation behavior, an appropriate weight function is introduced, assuming that the amount of stored energy can be expressed as a function of the entire history of input work. If the superposition principle is valid, such a dissipation behavior of the material can be written as a Stieltjes integral for the arbitrary input work history:

$$\rho_o A(t) = \int_{-\infty}^{t} g(t-\eta)\,\dot{W}(\eta)\,d\eta, \quad \text{where } g(0) = 1 \quad (5)$$

where ρ_o is the density in the initial configuration and thus $\rho_o A$ and W are the strain energy and input work per unit volume of the material in the initial state, respectively. The function g(t) is a normalized weight function describing the energy dissipation behavior of the material, similar to the classical representation theorem of the viscoelastic stress-strain relation.

Recalling $dW = S_{KL}\,dE_{KL}$, it can be rewritten

$$\rho_o A(t) = \int_{-\infty}^{t} g(t-\eta) S_{KL}(\eta) \frac{dE_{KL}(\eta)}{d\eta}\,d\eta \quad (6)$$

The stress in this expression is assumed to be given by a generalized form of Fung's quasilinear viscoelastic constitutive equation, i.e.:

$$S_{KL}\,(\bar{E}(t);t) = \int_{-\infty}^{t} G_{KLMN}(t-\tau)\,\frac{\partial S^e_{MN}(\bar{E}(\tau))}{\partial \tau}\,d\tau \quad (7)$$

where $G_{KLMN}(t)$ is the *generalized reduced relaxation function* of the material. $\bar{S}^e$ is a non-physical variable, the *elastic stress*, i.e., the stress that is reached instantaneously when the strains are assumed to be suddenly increased from 0 to the present state of strain $\bar{E}(t)$. It is important to note in this formulation that the reduced relaxtion function $G_{KLMN}(t)$ is a function of time t only, while the elastic stress tensor $\bar{S}^e$ is a function of strain only.

Substituting this sress-strain relationship into equation (6) leads to

$$\rho_o A(t) = \int_{-\infty}^{t}\int_{-\infty}^{\eta} g(t-\eta)\, G_{KLMN}(\eta-\tau)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,\frac{\partial E_{KL}(\eta)}{\partial \eta}\,d\tau\,d\eta \quad (8)$$

For mathematical simplicity, it is now assumed that

$$g(t-\eta)\,G_{KLMN}(\eta-\tau) = G^*_{KLMN}(\eta-\tau,\ t-\eta) \quad (9)$$

The stored strain energy density expression then becomes

$$\rho_o A(t) = \int_{-\infty}^{t}\int_{-\infty}^{\eta} G^*_{KLMN}(\eta-\tau,\ t-\eta)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,\frac{\partial E_{KL}(\eta)}{\partial \eta}\,d\tau\,d\eta \quad (10)$$

This expression represents the strain energy density function associated with the generalized quasilinear viscoelastic constitutive equation in equation (7).

By taking the time derivative of this, then from Leibnitz's rule, we have

$$\rho_o \dot{A}(t) = \dot{E}_{KL}(t)\int_{-\infty}^{t} G^*_{KLMN}(t-\tau,\ 0)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,d\tau + \int_{-\infty}^{t}\int_{-\infty}^{\eta} \frac{\partial}{\partial t} G^*_{KLMN}(\eta-\tau,\ t-\eta)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,\frac{\partial E_{KL}(\eta)}{\partial \eta}\,d\tau\,d\eta \quad (11)$$

This can be thought of as the initial constitutive assumption for our theory. Substituting this into the entropy inequality in equation (3) and using $\dot{E}_{KL} = d_{ij}\,x_{j,L}x_{i,K}$, it is obtained

$$\sigma_{ij}d_{ij} - \rho\dot{A}(t) = \left[\sigma_{ij}(t) - \frac{\rho}{\rho_o}\int_{-\infty}^{t} G^*_{KLMN}(t-\tau,\ 0)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,d\tau\ x_{i,K}\,x_{j,L}\right] d_{ij} - \frac{\rho}{\rho_o}\int_{-\infty}^{t}\int_{-\infty}^{\eta} \frac{\partial}{\partial t} G^*_{KLMN}(\eta-\tau,\ t-\eta)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,\frac{\partial E_{KL}(\eta)}{\partial \eta}\,d\tau\,d\eta \geq 0 \quad (12)$$

For an arbitrary deformation history, for equation (12) to be satisfied it is necessary that the coefficient of d_{ij} vanish, giving

$$\sigma_{ij}(t) = \frac{\rho}{\rho_o}\left(\int_{-\infty}^{t} G^*_{KLMN}(t-\tau,\ 0)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,d\tau\right) x_{i,K}\,x_{j,L} \quad (13)$$

which leaves equation (12) as

$$\rho\Lambda \geq 0\,,$$

$$\text{with } \Lambda = -\frac{1}{\rho_o}\int_{-\infty}^{t}\int_{-\infty}^{\eta} \frac{\partial}{\partial t} G^*_{KLMN}(\eta-\tau,\ t-\eta)\,\frac{\partial S^e_{MN}(\tau)}{\partial \tau}\,\frac{\partial E_{KL}(\eta)}{\partial \eta}\,d\tau\,d\eta \quad (14)$$

This last relationship states the result that the rate of dissipation of energy must be nonnegative. Using the relation between the

Kirchhoff stress and the Cauchy stress tensor,

$$S_{KL}(t) = \frac{\rho_o}{\rho} x_{L,j}\, x_{K,i}\, \sigma_{ij}(t) \tag{15}$$

The constitutive equation in (13) can be rewritten in the sense of the Kirchhoff stress as;

$$S_{KL}(t) = \int_{-\infty}^{t} G_{KLMN}(t-\tau) \frac{\partial S^e_{MN}[\bar{E}(\tau)]}{\partial \tau} d\tau$$

$$= S^e_{MN}(0^+)\, G_{KLMN}(t) + \int_0^t G_{KLMN}(t-\tau) \frac{\partial S^e_{MN}[\bar{E}(\tau)]}{\partial \tau} d\tau \tag{16}$$

with the assumption that $G_{KLMN}(t-\tau) = G^*_{KLMN}(t-\tau, 0)$.

The possible existence of the resulting generalized expression (16) was previously suggested by Fung (1984). Comparing this result with the one-dimensional constitutive expression:

$$S(t) = \int_{-\infty}^{t} G(t-\tau)\ \dot{S}^e(\tau)\, d\tau$$

$$= S^e(0^+)\, G(t) + \int_0^t G(t-\tau)\ \dot{S}^e(\tau)\, d\tau \quad , \tag{17}$$

one can realize that the proposed expression is a general tensorial form of the quasilinear viscoelastic constitutive law.

If the material is assumed to be isotropic, it will have only two independent components. Substituting the most general expression for a 4th order isotropic tensor:

$$G_{KLMN}(t) = \lambda(t)\, \delta_{KL}\delta_{MN} + \mu(t)\left(\delta_{KM}\delta_{LN} + \delta_{KN}\delta_{LM}\right) \tag{18}$$

into equation (16),

$$S_{KL}(t) = \int_{-\infty}^{t} \lambda(t-\tau)\, \delta_{KL} \frac{\partial S^e_{PP}(\tau)}{\partial \tau} d\tau + 2\int_{-\infty}^{t} \mu(t-\tau) \frac{\partial S^e_{KL}(\tau)}{\partial \tau} d\tau \tag{19}$$

where δ_{KL} is the Kronecker delta function, and $\lambda(t)$ and $\mu(t)$ are independent relaxation functions for the material. Since the expression above is written in terms of the elastic responses $S^e_{KL}(t)$ instead of strains $E_{KL}(t)$, the two relaxation functions $\lambda(t)$ and $\mu(t)$ are only mathematically analogous to the Lamé constant in the theory of elasticity. The relaxation function in shear, $\mu(t)$, of articular cartilage has been measured by several investigators (Zhu, 1986; Sprit, 1989). In a recent study by Huyghe (1991), it was assumed that the stress S_{KL} is split up into two components, one resulting from elastic volume change of the tissue (S^c_{KL}), the other from viscoelastic shape change (S^s_{KL});

$$S_{Kl} = S^c_{KL}(\bar{E}) + S^s_{KL}(\bar{E}(t), t) \tag{20}$$

and that the viscoelastic component S^s_{Kl} is described in a spectral form of quasilinear viscoelasticity as follows:

$$S^s_{KL}(\bar{E}(t), t) = \int_{-\infty}^{t} G(t-\tau) \frac{dS^e_{KL}[\bar{E}(\tau)]}{d\tau} d\tau \tag{21}$$

It is worth while to note that if the function $\lambda(t)$ in equation (19) is assumed to a constant, then the first term of the equation becomes a function of strain only. Thus one may see that the Huyghe's constitutive form in equation (20) is a special case of equation (19).

b) Form of the elastic response S^e_{ij}

Most of biological soft tissues show a gradual stiffening behavior as the strain increases and thus in general have a nonlinear stress-strain relationship. Using the most general strain energy expression for nonlinear elastic materials of Rivlin (1953),

$$W = \sum^{\infty} C_{ijk}\, (I_1 - 3)^i\ (I_2 - 3)^j\, (I_3 - 1)^k , \tag{22}$$

with the hyper-elasticity assumption;

$$S^e_{ij} = \frac{1}{2}\left(\frac{\partial W}{\partial E_{ij}} + \frac{\partial W}{\partial E_{ji}}\right) \tag{23}$$

the elastic stress, S^e_{ij} is expressed. W is the elastic strain energy density function, C_{ijk} is the coefficient for each term, and I_1, I_2, and I_3 are the first, the second, and the third strain invariants, respectively, given by;

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 2\,(E_{11} + E_{22} + E_{33}) + 3 ,$$

$$I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} = \frac{4\,(E_{11} + E_{22} + E_{33} + E_{11}E_{22} + E_{22}E_{33} + E_{33}E_{11}) + 3}{(2E_{11}+1)\,(2E_{22}+1)\,(2E_{33}+1)},$$

$$I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 = (2E_{11} + 1)\ (2E_{22} + 1)\,(2E_{33} + 1). \tag{24}$$

In the expression, λ's are extension ratios, the deformed length of a line element over its initial length. E's are the principal strains in the sense of Green and St. Venant (or Lagrangian) whereas

$$E_{ii} = \frac{1}{2}\,(\lambda_i^2 - 1), \quad \text{where } i = 1, 2, 3, \text{ but no summation index.} \tag{25}$$

In the expression above, material incompressibility requires that $I_3 = 1$, i.e.,

$$\lambda_1\, \lambda_2\, \lambda_3 = 1 \tag{26}$$

It is important to note that the strain-energy density function, W, in the equations above is the "elastic" strain energy density representing the amount of the stored energy when the body is subjected to elastic (very high strain rate) deformation. The general expression of the strain energy density function, $\rho_o A$, for large deformation viscoelastic media is given by equation (8) in the previous section.

c) Form of reduced relaxation functions

Direct measurement of $\lambda(t)$ for soft biological tissues is very difficult, because this parametric function implicitly represents the dilatation contribution to the material's viscoelasticity and since most soft biological tissues are mostly water, they are nearly incompressible. The function $\lambda(t)$ can however be approximated by using two other measurable relaxation behaviors (the uniaxial tensile reduced relaxation function G(t) and the shear relaxation function $\mu(t)$), under the assumption that the material is isotropic. Since in uniaxial tension tests, the specimen can be assumed to satisfy the condition that the lateral stress components $S_{22}(t) = S_{33}(t) = 0$ and the strains $E_{22}(t) = E_{33}(t)$ (thus $S^e_{22} = S^e_{33}$ as well), $\lambda(t)$ can be expressed in terms of the tensile relaxation function G(t) and the shear relaxation function $\mu(t)$ from equation (19). Applying the Laplace transform to the result gives

$$\bar{G}(s) = \frac{3\,\bar{\lambda}(s)\bar{\mu}(s) + 2\bar{\mu}(s)^2}{\bar{\lambda}(s) + \bar{\mu}(s)} \tag{27}$$

and in turn,

$$\bar{\lambda}(s) = \frac{\bar{G}(s)\bar{\mu}(s) - 2\bar{\mu}(s)^2}{3\bar{\mu}(s) - \bar{G}(s)} \tag{28}$$

In order to measure the two independent relaxation behaviors G(t) and $\mu(t)$ of a specimen, we perform sequentially a tensile and a shear test on the same specimen. The acquired data has been numerically processed using the generalized reduced relaxation function in the following form:

$$G(t) = B + (1 - B)\exp(-At^C) \tag{29}$$

The constant parameters A, B, and C for the two relaxation functions G(t) and $\mu(t)$ are determined, respectively, by using a nonlinear least-square curve fitting. In the equation, the constant B characterizes the total relaxation of the material, and therefore the smaller the B, the more pronounced the viscoelastic properties will be, while the constants A and C determine the shape of the relaxation curve. The two resulting curves for G(t) and $\mu(t)$ will be used for determining $\lambda(t)$.

2.2. Crack Propagation in Viscoelastic Media

The goal of the present study is to provide a more accurate value for fracture toughness for soft tissues by taking into account both large strain as well as time and history-dependent deformation behavior of the material. Since the J integral is not a widely accepted fracture parameter for viscoelastic materials, an alternative concept, the fracture energy, is selected as a fracture measure and is computed based on an energy balance.

In dealing with the problem proposed, similar to other physical or mechanical cases, continuum models are utilized. Aside from the regular field equations such as equations of motion and kinematic relations, the system must at all times obey the fundamental principle of the global conservation of energy during the crack propagation. The rate of work done by external forces and all energies that enter or leave the material body containing cracks per unit time must equal the time rate of change of the internal and kinetic energies plus the energies associated with the formation of cracks, (assuming all other energy exchanges to be negligible). On this basis, the global conservation of energy per unit time for a cracking viscoelastic medium containing 'm' cracks at any time t becomes

$$\dot{W}(t) = \dot{E}(t) + \dot{D}(t) + \dot{K}(t) + \sum_{n=1}^{m} \dot{\xi}_n(t) \tag{30}$$

where $W(t)$ is the mechanical work done by external loads, $E(t)$ is the stored elastic strain energy of the uncracked portion of the medium, $D(t)$ is the energy dissipated by the uncracked portion of the medium, $K(t)$ is the kinetic energy of the uncracked medium, and $\xi_n(t)$ is the energy absorbed by the n-th crack developed in the medium.

For any volume V of a viscoelastic medium bounded by closed surfaces S (figure 1), all the parameters in (30) can be expressed in elementary parameters as:

$$\dot{W}(t) = \int_{S-S_f} n_i \sigma_{ij} \frac{\partial u_j}{\partial t} dS \tag{31}$$

$$\dot{E}(t) = \int_V \rho_o \dot{A}\ dV \tag{32}$$

and $$\dot{D}(t) = \int_V \Lambda\ dV \tag{33}$$

where S_f is the surface over the failure zone at the crack tip region, and ρ_o is the uniform initial density of the material, A is specific stored strain energy, Λ is rate of energy dissipation per unit volume, n_i are components of the unit outward normals to S.

If the inertia contribution to this energy balance equation is assumed to be neglected, the failure zone is small relative to the crack and body, and a single crack is assumed, equation (30) can be rewritten as;

$$\int_V \rho_o \dot{A}\ dV + \int_V \Lambda\ dV - \int_{S-S_f} n_i \sigma_{ij} \frac{\partial u_j}{\partial t} dS = -\dot{\xi}(t) \tag{34}$$

where $\dot{\xi}(t)$ is the rate of energy absorption in the failure zone. Equation

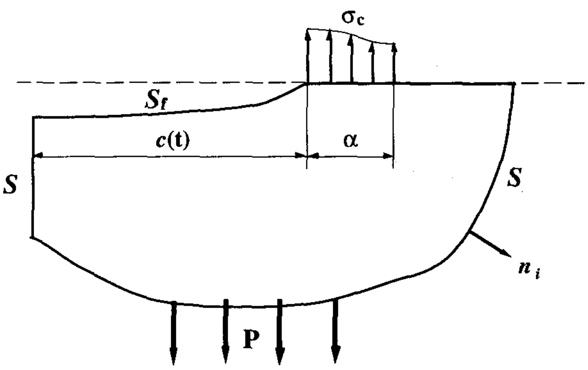

Figure 3-1 Two dimensional SEN crack showing the vicinity of a crack

(34) can also be written in the following form by using the cohesive stress σ_c and the opening displacement at the crack tip cohesive zone Δu_j ;

$$\int_V \rho_o \dot{A}\ dV + \int_V \Lambda\ dV - \int_{S-S_f} n_i \sigma_{ij} \frac{\partial u_j}{\partial t} dS = -\int_0^{\alpha} \sigma_c \frac{\partial \Delta u_j}{\partial t} dx \tag{35}$$

Alternatively,

$$\dot{\xi}(t) = 2\gamma b(t)\dot{c}(t) \tag{36}$$

where $b(t)$ and $\dot{c}(t)$ are the speciemen's thickness and the crack speed, respectively. γ is the fracture energy which is absorbed by the failure zone. A fracture criterion of the form (36) has been proposed by Knauss and coworkers (1970a, 1970b, 1971). Similar local energy balance criterion have also been considered by Schapery (1975a, 1975b). From equations (33)-(36), the fracture energy γ can be determined if the crack propagation speed $\dot{c}(t)$, $\dot{W}$, $\dot{A}$, and Λ are known. The rate functions for stored and dissipated energy are given by equations (8) and (14), respectively. These equations constitute the proposed fracture theory.

2.3. Numerical solution methods

The stress-strain field and the energy terms in the theory for actual specimen configurations are determined numerically. Based on the minimization of potential energy principle (Christensen, 1971), a nonlinear finite element code has been written for the plane stress analysis of a large deformation visco-elastic medium. The numerical procedures previously developed for large strain problems (Lindley, 1971) was largely employed in the present finite element program with some modifications in order to consider the time- dependence of the problem. The program was written in the manner that each nodal point of a specimen is considered in turn and moved to a position which minimizes the total energy of the specimen. The numerical solutions are obtained using the super computer Cray-XMP.

3. Application to PVC

The stress-strain curves and the relaxation behavior at high strain rates of uncross-linked PVC were measured through uniaxial tensile and simple shear tests. Based on the quasilinear viscoelastic formulation discussed earlier, the constitutive parameters, that is, the elastic stress and the reduced relaxation functions, of PVC were determined. Comparing the experimental stress and strain at various loading rates with the theoretically predicted values, the proposed constitutive theory was validated. The accuracy and sensitivity of the experimental methods were also assessed by performing the tests repeatedly with different sizes of testpieces. The constitutive material properties found were used for calculation of the material's fracture property, the fracture energy here, in conjunction with the proposed time-dependent fracture theory.

3.1. Specimens

The PVC specimens used in the present validation procedures were prepared from medical gloves (Brand name: B-D Tru-Touch, Bector-Dickinson Co.). The material was selected because it showed a very similar overall deformational and fracture behavior with our object material, articular cartilage. It is highly viscoelastic as well as very compliant with the maximum recoverable strain of ~100% and thus modeled as a quasilinear viscoelastic medium. Cracks in the PVC grow in a relatively stable manner from a moderate low strain level (about 30 - 40 percent). To measure the viscoelasticity of the material, instead of the conventional dumbbell configuration, a rectangular strip-like shape of the secimens ranging in section dimensions of 15 - 20 mm x 5 - 15 mm x 0.1 - 0.15 mm was used. The dimensions of the testpieces were intentionally selected in a comparable size of cartilage specimens for the consistency of testing (i.e., Parts I & II). For the fracture tests, single edged notched (SEN) specimens (10mm x 10 mm x 0.15 - 0.25 mm) with the notch depth of 1 mm were used. The molecular structure of the PVC was confirmed by a simple experiment checking the material solubility in NMP(methyl 2-pyrrolidinone) and Acetone. However, neither the specific recipe nor the treatment for processing of this particular PVC is available. These information are currently under protection by the manufacturer.

3.2. Experimental Methods

In order to measure the elastic stress-strain curve as well as the relaxation behavior of the material in each loading mode, the specimens were tested under uniaxial tensile and simple shear loading on an MTS test machine with various strain rates. Both tests are basically displacement-control relaxation tests consisting of two different phases, a constant-rate ramp displacement phase and a fixed displacement phase. A typical force response for a transient displacement in relaxation tests is shown in the figure. In reducing data, the relaxation curve was extrapolated as shown in the figure. From the force-displacement and force-time data from each phase, the material's elastic stress-strain relation (Phase I) and reduced relaxation functions (Phase II) were obtained, respectively.

The displacement in the tests was applied by moving the test machine cross-head, while the force was measured by a separate force transducer (±500 gms) due to the smallness of its magnitude. Several different strain rates were tested to assess the error of the experimental data. For the data acquisition of the tests, two different time scales were employed. For the first Phase ($t \leq t_r$), the data were collected at every 0.002 seconds to acquire accurate elastic stress-strain data; thereafter ($t \geq t_r$) the data were collected at every 0.3 seconds until 150 seconds to investigate the specimen's stress-relaxation. Considering the definition of the elastic response in the quasilinear viscoelastic theory, measurement of the material's elastic stress-strain relationship must be conducted in a high strain rate test. However, the cross-head speeds which are too fast appears to often result in an undesirable dynamic effect on the test data. For the present tests, 22/sec (the equivalent MTS cross-head speed = 33 cm/sec) was used as an optimized strain rate. Due to the limitation of specimen dimensions, a special device was designed and installed on a microscope stage to see either specimen deformation or crack movement in each test. The tests were conducted after a few cycles of preconditioning stretches with 10 % of the strain prior to being tested, each lasting a duration of 10 - 15 sec, and an equal period of rest between stretches. During all the tests, the constant temperature condition was maintained within limits of $\pm 0.5^{\circ}C$.

Unlike the shear tests used in previous cartilage studies (Roth, 1982: Zhu, 1988) the shear relaxation tests in the present study was performed in the sense of simple shear because the specimens were prepared from PVC gloves. The shear testing was conducted in the manner that the MTS actuator was displaced parallel to the fixed head while maintaining constant clearence between the two clamps. For comparative purposes, in the shear tests, the same strain rate was used as that in the corresponding tensile tests. However, because in simple shear conditions, the specimen is actually subjected to normal tension as well as shear deformation, the testing had to be conducted in a relatively lower strain level (e.g. the maximum cross-head displacement used was 2 mm for the specimen 2mm long) to reduce the effect of tension on the specimen's relaxation behavior.

Because the proposed constitutive equation has a three dimensional form, it is necessary to define strain components triaxially as well. In the present study, using a uniaxal tension testing, three principal strains were approximated. Assuming that the two lateral strain components are equal and they are uniquely defined for a given axial strain, the lateral deflection of a strip-like testpiece was measured. Using a high-speed video camera (Kodak EktaPro Motion Analyzer, maximum speed for the split frame = 1000 frames/sec, for the full frame = 6000 frames/sec), the specimen's deformations during the rapid stretching were continually recorded. For the convenience of measurement, a number of dots were marked on the specimen surface and the displacement of each mark was measured. To create a reference gage length on each testpiece, twenty five dots were marked in 2 mm intervals with a permanent marker. By consideration of the strain rate used in the tests, a recording speed of 250 frames per second was arbitrarily selected. The obtained visual outputs were carefully analyzed in order to determine the relation between the lateral deflection and the axial elongation of the specimen. At each level of extension, the ratio of the two strain components of the central rectangular element surrounded by nine dots were measured on the pictures. It was assumed that this small portion of the specimen is subjected to simple tension, and possesses the same principal directions of strains with those of the whole specimen.

Using single edge notched (SEN) specimens with notch size 1 mm, the fracture tests were also carried out on the MTS at a constant rate with crosshead displacement 20 mm, through the onset point of crack initiation, until the crack propagated through the width of the specimen. During the tests, the growth of the crack was continuously monitored as a function of time (or strain) by using the video-camera. For the convenience of the crack growth measurement, five dots in 1 mm intervals were marked on the expected crack path with a permanent pen.

In the test, two separate output devices, the video recorder and the MTS-load cell system, were employed for measuring the crack size-time, the load-time, and the load-deflection data. Therefore, the tests were conducted in the following manner: 1) By programming the MTS displacement, the MTS was activated first, (putting in a few seconds of a zero displacement session before the actual ramp phase starts), 2) During this standby period yet before the actual MTS cross-head movement begins, the start button of the video recorder was pressed. Since the two devices have different lag times, it was necessary to find the real time. Coordinating the specimen configuration pictures with the MTS force-displacement-time data at every time point, the true zero time, when the specimen actually began to be deformed, was found.

In order to avoid a possible disturbance of the data by the rapid movement of the MTS actuator, a force transducer was fixed on the table. For the material testing, the maximun speed of the displacement was 33 cm/sec followed by a constant displacement peroid, but for the fracture test, a relatively slower speed was used (0.4 mm/sec) for easy detection of the onset point of crack propagation and crack speed. The load cell was fed into an A/D converter from a Daytronix signal recording unit which gives a digital readout of the load in grams and a clock time in seconds. The computer used in this system was an IBM PS II with data collection program ASYST. The computer was programmed to turn on the MTS and record the load cell data with time. The data from the computer and the images from the video screen were coordinated to determine the critical point at which the crack began to propagate, and the propagation speed at each time point.

4. Results and Data Reduction

4.1. Measurement of Constitutive Properties

Elastic response of PVC : A typical elastic stress-strain relationship (from the ramp strain increase phase) and the corresponding strain energy function of the PVC in tension are shown in figure 2.

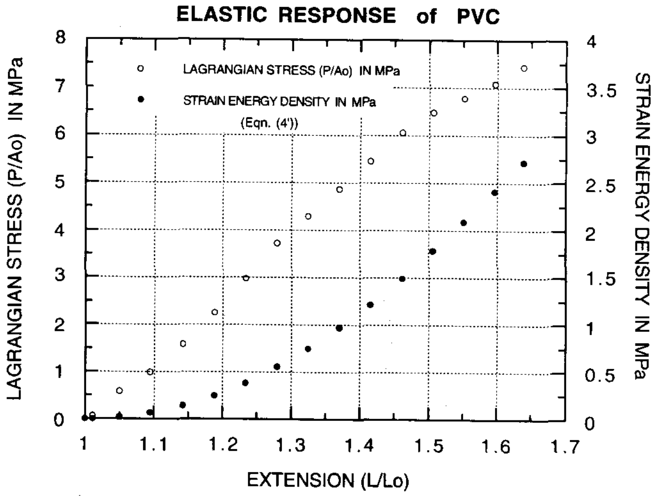

Figure 2 Elastic response of uncrosslinked PVC (for strain rate = 22 /second)

From the experimental results, it was found that the elastic stress-strain curves of PVC can be expressed by using Gent's (1958) constitutive form for natural rubbers:

$$W = C_1 (I_1 - 3) + C_2 \ln\left(\frac{I_2}{3}\right) \qquad (37)$$

where C_1 and C_2 are material constants to be determined, and I_1 and I_2 are the first and second strain invariants, respectively.

During the uniaxial tensile tests, the lateral deflection cefficient for PVC was also measured. Assuming that the lateral deflection of a specimen in uniaxial tension is a function of the axial strain alone with a functional form $\lambda_{lateral} = (\lambda_{axial})^{-\alpha}$, a relationship between two strain components was established experimentally. It was found that the ratio of the lateral strain to the axial strain slightly decreases (in its

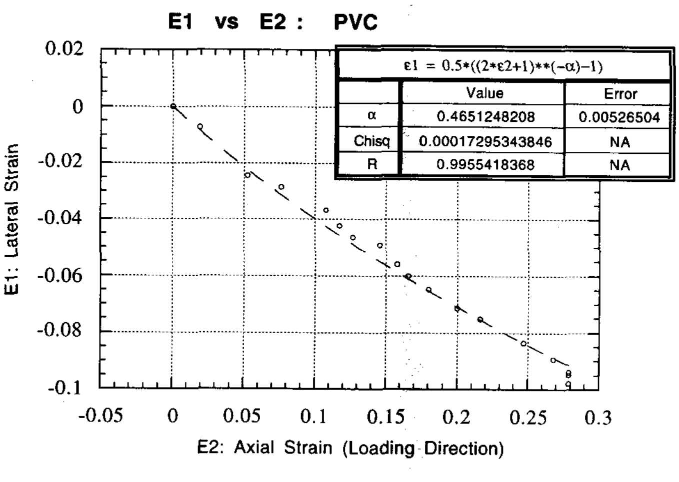

Figure 3 The axial strain vs. the lateral strain in uniaxial tension tests

ıbsolute value), as the axial extension increases. The coefficient χ was found to be 0.465, which suggests that the volume of the ;pecimen increases slightly, as the specimen was elongated, ɔecause $\alpha < 0.5$. But we assume here that $\alpha = 0.5$ for mathematical simplicity. Then, from that $I_3 = \lambda_1\lambda_2\lambda_3 = 1$ and $\lambda_2 = \lambda$, $\lambda_1 = \lambda_3 = \lambda^{-\alpha}$, the strain energy expression (35) for the case of uniaxial tension is rewritten as;

$$W = C_1 (\lambda^2 + 2\lambda^{-1} - 3) + C_2 \ln\left(\frac{\lambda^{-2} + 2\lambda}{3}\right) \qquad (38)$$

Thus, from equation (23), the corresponding elastic stress-strain equation becomes

$$S^e = S^e_{22} = \frac{\partial W}{\lambda \partial \lambda} = 2 (1 - \lambda^{-3})\left(C_1 + \frac{C_2}{\lambda^{-1} + 2\lambda^2}\right) \qquad (39)$$

By using this expression in a least squares fit of the experimental data of the uniaxial tensile tests, the material constants were found as $C_1 = 2.93 \pm 0.26$ MN/m2 and $C_2 = -3.44 \pm 0.36$ MN/m^2 for PVC (the test size, n = 6). It is important to distinguish the use of equation (38) here from the Gent's original use. Gent et al. (1958) proposed this functional form as a complete constitutive equation for a time-independent large deformation material (vulcanized rubber), while in the present study, the same expression was utilized for describing the elastic part of the stress-strain relationship of a time-dependent large deformation material, PVC.

The strain rate sensitivity of the material property was investigated by testing at several different levels of MTS cross-head speed, ranging from 1 - 400 mm/sec. Significant reductions in both peak stress and total relaxation were observed when the lower strain rate was used. It is also observed that all curves tend asymptotically to a certain value (called the equilibrium stress) at long times. Due to this material property-like nature of the long-term stress response, the equilibrium stress has often been used as a material parameter in many earlier non-time dependent investigations of cartilage (Kempson, 1972, 1980; Weightman, 1976; Woo, 1976). A significant positive correlation between the strain rates and the slope of the stress-strain curves was found.

Reduced relaxation function: Because the true reduced relaxation function $G(t) = S(t)/S(t=0)$ cannot be obtained experimentally, the present research experimentally approximates the relaxation functions of the PVC by using a strain rate of 22 /sec. The measured reduced relaxation data actually represents the ratio of the present stress S(t) against the stress at t = 0.03 second, not against the stress at t = 0. It is meaningful to compare the rise time t_r in this study with that in a previous study. The earlier work of Woo et al. (1979) defined their experimental reduced relaxation function as $G(t) = S(t)/S(t=0.25$ sec.) for bovine articular cartilage. It is reasonable to expect that the smaller the t_r used, the closer values to the true relaxation behavior of a material can be achieved.

The two independent measured reduced relaxation functions, G(t), for simple tension and $\mu(t)$, for shear, were obtained by normalizing the stress data at t against the stress at t = 0.03 sec. By using the least square procedures with the functional form (equation 29), the material constants, A, B, and C, found to be 1.59 ± 0.21, 0.22 ± 0.03, and 0.25 ± 0.03, respectively. Using the measured reduced relaxation functions, G(t) and $\mu(t)$, and equation (28), $\lambda(t)$ can be found, which implicitly represents the contribution of the specimen volume change to the material's overall stress-relaxation behavior. As shown in figure 9, a narrow difference between the tensile relaxation function G(t) and its shear relaxation function, $2\mu(t)$,was found. This implies that the viscoelasticity of the PVC is mainly due to the change of the specimen shape. Because in the uniaxial loading condition, two lateral stress components, S_{11} and S_{33}, in equation (19) can be assumed to be zero and the strain component, $E_{11} = E_{33}$. From equation (28), if $G(t) \approx 2\mu(t)$, $\lambda(t) \approx 0$, that is, the contribution of the volumetric contribution to material's viscoelasticity will be negligible. Based on this observation, the relation that $\lambda(t) = 0$ was used in the numerical process in the present study, to save the computation time.

4.2. Calculation of Fracture toughness

A typical load-deflection curve and the crack length-time curve acquired from the SEN tests for PVC is seen in figure 4 and 5, respectively. The cross mark on the load-deflection curve in figure 4 denotes the critical point for crack propagation of the specimen considered.

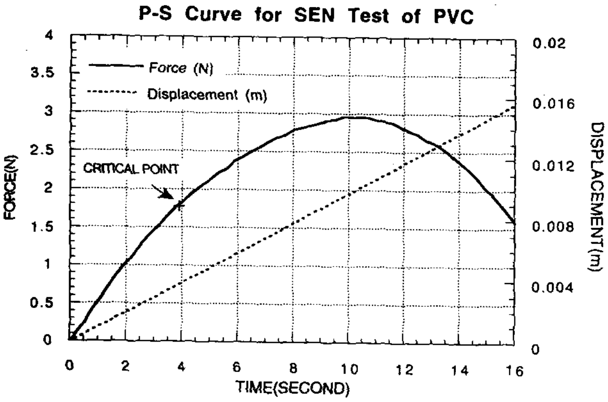

Figure 4: Typical load-deflection curve for SEN test of PVC

By use of the known material properties and the moving boundary information related to the change of crack length, the state of stresses and/or strains over the specimen were calculated as functions of time, using the finite element program. The rate of energy stored or dissipated during the fracturing process was calculated as well, by using the energy equations derived earlier (equation 29 and 35).

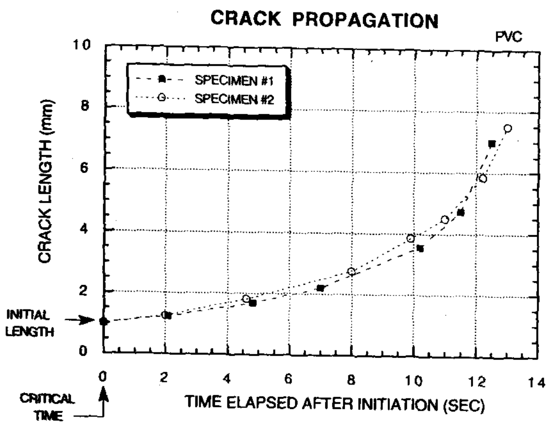

Figure 5: Crack propagation of SEN PVC test-pieces

As seen in the figure 5, the growth of cracks in the PVC specimens were stable until the crack length became approximately 60 - 70 percent of the initial width of the specimen. However, since the amount of energy for pure crack propagation is usually small as compared to the total energy of the system, it was hard to assess fracture energy by using the values for the initial few second's period showing slow crack propagation, because of computational errors. The fracture energy of the PVC specimens was therefore obtained using the energy rate data at the time points when the crack showed a relatively rapid propagation, because in these cases the difference in the energy level of the specimen would be considerable. By averaging the data for six specimens, the fracture energy, 2γ, of the PVC used was found to be 1.2 ± 0.41 kN/m.

Discussions and Conclusions

A fracture theory for time-dependent materials undergoing large deformation has been developed. A three-dimensional expression of the quasilinear viscoelastic theory of Fung (1972, 1981) has been used, based on the thermodynamic consideration of Naghdi and Christensen (1967) and Fung's original one-dimensional formulation. A new form of the elastic component of the constitutive equation for soft biological tissues has been proposed. Experimental methods for measuring the material properties at high loading rates have also been successfully developed. The validity of the theory and methods is supported by the experiments with PVC. This material is more homogeneous and isotropic than biological tissues, and, thus, its use eliminates some of the questions that might arise with application of the theory to a soft biological tissue. Validation of the constitutive equation, fracture theory, finite element solution for the deformation field, and experimental measures of load and motion was supported by the close agreement between predicted fracture toughness using these techniques and theory, and the value calculated by a pseudo-elastic assumption and use of a J-integral calculation. The accuracy of the approximate method, the pseudo-elastic J method, is unknown, but because the two methods gave fracture toughness values within 25% of each other, this supports the validity of the methods. Validity of the methods are also supported by the close agreement between the predicted and measured stress and strain versus time data at the slow loading rate, using constitutive data obtained at high loading rates. The complete method, and the approximate pseudo-elastic method, should be of value in calculating fracture parameters for soft biological materials.

Table 1: Material properties and fracture toughness : Uncross-linked PVC (Bector-Dickinson Co.).

Elastic Stress (MPa) (in the quasi-linear viscoelastic theory)	C_1 2.93 ± 0.26		C_2 -3.44 ± 0.36
Reduced relaxation fct.	A 1.59 ± 0.21	B 0.22 ± 0.03	C 0.25 ± 0.03
Fracture Toughness (kN/m)	2γ 1.2 ± 0.41		J_c 1.5 ± 0.23

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