포화된 퍼지 Syntopogenous 공간

Saturated Fuzzy Syntopogenous Spaces

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In [4], Katsaras introduced the notion of fuzzy syntopogenous structures and it is closely connected with Lowen fuzzy uniformities, Artico-Moresco fuzzy proximities and fuzzy neighborhood spaces.

In this paper, we introduce the notion of saturated fuzzy topogenous orders and using this order, we introduce the notion of saturated fuzzy syntopogenous spaces and we prove basic properties of this space. Also we show that the category [FSyn] of saturated fuzzy syntopogenous spaces and continuous maps is coreflective in the category KFSyn of fuzzy syntopogenous spaces (in the sense of Katsaras) and continuous maps.

For definitions and results on fuzzy syntopogenous structures we will refer to [4].

Definition 1.1 1) A fuzzy syntopogenous structure on a non-empty set X is a non-empty family S of fuzzy semi-topogenous orders on X satisfying the following axioms:

FS1) S is directed in the sense that given ζ, $\eta \in S$ there exists $\xi \in S$ with ζ, $\eta \le \xi$.

FS2) Given $\zeta \in S$ and $\varepsilon > 0$ there exists $\eta \in S$ such that $\zeta \le \eta^2 + \varepsilon$.

2) If S consists of a single fuzzy topogenous order, we say that it is a fuzzy topogenous structure.

Definition 1.2 A map f of a fuzzy syntopogenous space (X, S) into a fuzzy syntopogenous space (Y, T) is said to be continuous if for any $\zeta \in T$, there exists $\eta \in S$ such that $f^{-1}(\zeta) - \varepsilon \le \eta$, where $f^{-1}(\zeta)(\mu, \rho) = \zeta(f(\mu), 1 - f(1 - \rho))$ for $\mu, \rho \in I^X$.

Notation The class of all fuzzy syntopogenous spaces and continuous maps between them forms a concrete category, which will be denoted by FSyn.

Notation For $a \in I$ we will denote a the fuzzy set which assumes the value a at each $x \in X$. For $a \in I$ and $A \subseteq X$ we will denote A_a the fuzzy set $a \wedge A$, and we will A^a denote the fuzzy set $a \vee X - A$.

Notation For $a \in I$ we will denote a the fuzzy set which assumes the value a at each $x \in X$. For $a \in I$ and $A \subseteq X$ we will denote A_a the fuzzy set $a \wedge A$, and we will A^a denote the fuzzy set $a \vee X - A$.

Let X be a set and ζ a fuzzy semi-topogenous order on X. We define a map $[\zeta] : I^X \times I^X \to I$ as follows:

$$\text{for } \mu, \rho \in I^X, \quad [\zeta](\mu, \rho) = \wedge\{\zeta(x_{\mu(x)}, y^{\rho(y)}) : x, y \in X\}.$$

Then $[\zeta]$ is a fuzzy semi-topogenous order on X, which will be called saturation of ζ.

Definition 1.3 1) Let ζ be a fuzzy semi-topogenous order on a set X. If $[\zeta] = \zeta$, then we say that ζ is a saturated fuzzy topogenous order.

2) A fuzzy syntopogenous structure S on a set X is said to be a saturated fuzzy syntopogenous structure on X if each member of S is saturated.

Proposition 1.4 Let ζ and η be fuzzy semi-topogenous orders on a set X. Then one has the following:

1) For $x, y \in X$ and $a, b \in I$, $[\zeta](x_a, y^b) = \zeta(x_a, y^b)$.

2) If $(A_\lambda)_{\lambda \in \Lambda}$ is a family of subsets of X and $\rho \in I^X$, then $\wedge\{[\zeta](\cup\{A_\lambda : \lambda \in \Lambda\}, \rho)$ $= \wedge\{[\zeta](A_\lambda, \rho) : \lambda \in \Lambda\}$.

3) $\zeta \leq [\zeta]$.

4) If $\zeta \leq \eta$, then $[\zeta] \leq [\eta]$.

5) $[[\zeta]] = [\zeta]$.

6) $[[\zeta]o[\eta]] = [\zeta]o[\eta]$.

Corollary 1.5 If S is a fuzzy syntopogenous structure on X, so is $[S] = \{[\zeta] : \zeta \in S\}$ is a fuzzy syntopogenous structure on X.

Theorem 1.6 Let ζ be a saturated fuzzy syntopogenous order on a set X. Then one has the following:

1) $\zeta(\mu \vee \rho, \sigma) = \zeta(\mu, \sigma) \wedge \zeta(\rho, \sigma)$ and $\zeta(\mu, \rho \wedge \sigma) = \zeta(\mu, \rho) \wedge \zeta(\mu, \sigma)$ iff for $x, y \in X$ and a, b $c, d \in I$ with $a \leq c$, $d \leq b$,

$$\zeta(x_c, y^b) \leq \zeta(x_a, y^b) \text{ and } \zeta(x_a, y^d) \leq \zeta(x_a, y^b).$$

2) ζ is perfect iff for $x, y \in X$ and $(a_\lambda)_{\lambda \in \Lambda} \subseteq I$ and $b \in I$,

$$\zeta(x_{\vee a_\lambda}, y^d) = \wedge\{\zeta(x_{a_\lambda}, y^b) : \lambda \in \Lambda\}.$$

3) ζ is biperfect iff for x, y $\in$ X and $(a_\lambda)_{\lambda\in\Lambda} \subseteq$ I and $(b_\gamma)_{\gamma\in\Gamma} \subseteq$ I,

$$\zeta(x_{\vee a_\lambda}, y^{\wedge b_\gamma}) = \wedge\{\zeta(x_{a_\lambda}, y^{b_\gamma}): \lambda \in \Lambda \text{ and } \gamma \in \Gamma\}.$$

4) ζ is symmetrical iff for x, y $\in$ X and a, b $\in$ I

$$\zeta(x_a, y^b) \leq \zeta(y_{1-b}, y^{1-a}).$$

Definition 1.7 Let f be a map of a set X into a set Y, S a family of fuzzy semi-topogenous orders on Y and ζ a fuzzy semi-topogenous order on Y. Then

1) f is said to be compatible with ζ if for any p, q $\in$ Y and any $\mu, \rho \in I^X$,

$\zeta(\vee\{p_{\mu(x)} : x \in f^{-1}(p)\}, \wedge\{q^{\rho(y)} : y \in f^{-1}(q)\}) = \wedge\{\zeta(p_{\mu(x)}, q^{\rho(y)}) : (x, y) \in f^{-1}(p) \times f^{-1}(q)\}$.

2) If f is compatible with every member of S, then we say that f is compatible with S.

Proposition 1.8 Let f be a map of a set X into a set Y and ζ be a fuzzy semi-topogenous order on Y. Then one has the following:

1) If f is compatible with ζ, then $f^{-1}([\zeta]) = [f^{-1}(\zeta)]$.

2) If ζ is perfect (resp. biperfect, resp, symmetrical), then $[f^{-1}(\zeta)]$ is perfect (resp, biperfect, resp. symmetrical).

Theorem 1.9 1) [FSyn] is coreflective in FSyn.

2) [FSyn] is coreflective in KFSyn.

Theorem 1.10 1) The full subcategory [bFSyn] of [FSyn] determined by all biperfect saturated fuzzy syntopogenous spaces and the category QFUnif of fuzzy quasi-uniform spaces and uniformly continuous maps are isomorphic.

2) The full subcategory [sbFSyn] of [FSyn] determined by all symmetrical biperfect saturated fuzzy syntopogenous spaces and the category FUnif of Lowen fuzzy uniform spaces and uniformly continuous maps are isomorphic.

Theorem 1.11 1) The full subcategory [ptFSyn] of [FSyn] determined by all perfect saturated fuzzy topogenous spaces is coreflective in the full subcategory ptKFSyn of KFSyn determined by all perfect fuzzy topogenous spaces.

2) [ptFSyn] and the category FTop of all fuzzy neighborhood spaces and continuous maps are isomorphic.

Definition 1.12 An Artico-Morseo fuzzy proximity δ on a set X is called saturated if for $\mu, \rho \in I^X$, $\delta(\mu, \rho) = \vee\{\delta(x_{\mu(x)}, y_{\rho(y)}): x, y \in X\}$.

Theorem 1.13 1) The full subcategory [tFSyn]of [FSyn] determined by all symmetrical saturated fuzzy topogenous spaces and the category [FProx] of all saturated fuzzy proximity spaces and all proximity maps are isomorphic.

2) [tFSyn] is coreflective in the full subcategory tKFSyn of KFSyn determined by all symmetrical fuzzy topogenous spaces.

3) [FProx] is coreflective in the category FProx of all fuzzy Artico-Moresco proximity spaces and all proximity maps.

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