ON TL–FINITE STATE MACHINES

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ABSTRACT. In this paper we introduce the notions of a TL-finite state machine, TL-retrievability, TL-separability, TL-connectivity and discuss their basic properties.

1. Introduction. Automata theory is one of basic and important theories in computer science. Following Zadeh [4] who introduced the concept of a fuzzy set, Wee [3] introduced the idea of fuzzy automata. There has been considerable growth in the area of fuzzy automata [1]. The use of algebraic techniques in determining the structure of autom- ata has been significant. However, in fuzzy automata, the algebraic approach is lacking. Recently Malik, Mordeson and Sen [2] applied algebraic techniques to study fuzzy automata. In this paper we introduce the notion of a TL-finite state machine that is a generalization of the notion of a fuzzy finite state machine and apply algebraic techniques to study TL-automata.

This is a part of a forthcoming paper with the same title .

2. TL-finite state machines

We let L denote a complete lattice that contains at least two distinct elements. The meet, join, and partial ordering will be written as $\wedge$, $\vee$, and $\leq$, respectively. We also write 1 and 0 for the greatest element and least element of L, respectively.

From this definition one gets immediately $T(0,a) = 0$ and
$T(a,b) \leq a \wedge b$ for all $a, b \in L$. A t-norm T on L is said to be $\vee$-distributive if $T(a, b \vee c) = T(a,b) \vee T(a,c)$ for all $a, b, c \in L$. And T is said to be positive-definite if $T(a,b) > 0$ for all $a, b \in L \setminus \{0\}$.

Throughout this paper, T shall mean a positive-definite and $\vee$-distributive t-norm on L unless otherwise specified.

DEFINITION 2.1. *An L-finite state machine is a triple $M = (Q, X, \tau)$ where Q and X are finite nonempty sets and τ is an L-subset of $Q \times X \times Q$, i.e., τ is a function from $Q \times X \times Q$ to L.*

Let $M = (Q, X, \tau)$ be an L-finite state machine. Then Q is called the set of states and X is called the set of input symbols. Let X^* denote the set of all words of elements of X of finite length. Let λ denote the empty word in X^* and $\mid x \mid$ denote the length of x, where $x \in X^*$.

DEFINITION 2.2. *Let $M = (Q, X, \tau)$ be an L-finite state machine. Define $\tau^* : Q \times X^* \times Q \longrightarrow L$ by $\tau^*(q, \lambda, p) = 1$ if $q = p$ and $\tau^*(q, \lambda, p) = 0$ if $q \neq p$ and $\tau^*(q, xa, p) = \vee\{T(\tau^*(q, x, r),$*

$\tau(r,a,p)) \mid r \in Q\}$ where $p,q \in Q$, $x \in X^*$, $a \in X$. When T is applied to M as above, M is called a TL-finite state machine (briefly, a TL-fsm).

The notion of a TL-fsm is a generalization of the notion of a fuzzy finite state machine. In fact, every fuzzy finite state machine is a TL-fsm with $T = \wedge$ and $L = [0,1]$ obviously. However, not all TL-fsms are fuzzy finite state machines.

PROPOSITION 2.3. *Let $M = (Q,X,\tau)$ be a TL-fsm. Then $\tau^*(q,xy,p) = \vee\{T(\tau^*(q,x,r),\tau^*(r,y,p)) \mid r \in Q\}$ where $p,q \in Q$ and $x,y \in X^*$.*

DEFINITION 2.4. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $p,q \in Q$. p is called an immediate successor of q if there exists $a \in X$ such that $\tau(q,a,p) > 0$. p is called a successor of q if there exists $x \in X^*$ such that $\tau^*(q,x,p) > 0$.*

PROPOSITION 2.5. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $p,q,r \in Q$. Then*

(1) q is a successor of q,

(2) if p is a successor of q and r is a successor of p, then r is a successor of q.

When $M = (Q,X,\tau)$ is a TL-fsm, we denote $S_M(q)$ the set of all successors of q, where $q \in Q$.

DEFINITION 2.6. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $R \subset Q$. The set of all successors of R, denoted by $S_M(R)$, in Q is defined to be the set $S_M(R) = \bigcup\{S_M(q) \mid q \in R\}$.*

We will write $S(q)$ and $S(R)$ for $S_M(q)$ and $S_M(R)$, respectively.

THEOREM 2.7. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $A,B \subset Q$. Then*

(1) if $A \subset B$, then $S(A) \subset S(B)$,

(2) $A \subset S(A)$,

(3) $S(S(A)) = S(A)$,

(4) $S(A \bigcup B) = S(A) \bigcup S(B)$,

(5) $S(A \bigcap B) \subset S(A) \bigcap S(B)$.

DEFINITION 2.8. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $R \subset Q$. Let ν be an L-subset of $R \times X \times R$ and let $N = (R,X,\nu)$. The TL-fsm N is called a submachine of M if $\tau \mid_{R\times X\times R} = \nu$ and (2) $S_M(R) \subset R$.*

For the convenience sake, we assume that $\emptyset = (\emptyset, X, \nu)$ is a submachine of a TL-fsm M. A submachine $N = (R,X,\nu)$ of $M = (Q,X,\tau)$ is called proper if $R \neq Q$ and $R \neq \emptyset$. Clearly, if K is a submachine of N and N is a submachine of M, then K is a submachine of M. Note that the number of all submachines of M is finite because Q is finite.

DEFINITION 2.9. *Let $M = (Q,X,\tau)$ be a TL-fsm. Let $R \subset Q$ and $\{N_i = (Q_i,X,\tau_i) \mid i \in \Lambda\}$ be the collection of all submachines of M whose state set contains R. Define $< R >= \bigcap_{i\in\Lambda}\{N_i \mid i \in \Lambda\} = (\bigcap_{i\in\Lambda} Q_i, X, \wedge_{i\in\Lambda}\tau_i)$. Then $< R >$ is called the submachine generated by R.*

In Definition 2.9, $< R >$ is clearly the smallest submachine of M whose state set contains R. The union $\bigcup_{i\in\Lambda} N_i$ of a collection $\{N_i = (Q_i,X,\tau_i) \mid i \in \Lambda\}$ of submachines of M is $(\bigcup_{i\in\Lambda} Q_i, X, \nu)$ where $\nu = \tau \mid_{(\bigcup_{i\in\Lambda} Q_i)\times X\times(\bigcup_{i\in\Lambda} Q_i)}$. The union of submachines of M is clearly a submachine of M.

DEFINITION 2.10. *Let $M = (Q, X, \tau)$ be a TL-fsm. Let $R \subset Q$. R is called free if $r \notin S(R \setminus \{r\})$ for all $r \in R$. And R is called a basis of M if R is free and $< R >= M$.*

3. TL-retrievability

DEFINITION 3.1. *A TL-fsm $M = (Q, X, \tau)$ is said to be TL-retrievable if it satisfies the following; for $p, q \in Q$ if there exists $y \in X^*$ such that $\tau^*(q, y, p) > 0$, then there exists $x \in X^*$ such that $\tau^*(p, x, q) > 0$; or equivalently, $q \in S(p)$ if and only if $p \in S(q)$ where $p, q \in Q$.*

THEOREM 3.2. *Let $M = (Q, X, \tau)$ be a TL-fsm. If M is TL-retrievable, then M has a basis $\{q_1, \cdots, q_n\}$ and $M = \bigcup_{i=1}^{n} < q_i >$ in which the state sets of $< q_i >$ are disjoint.*

DEFINITION 3.3. *Let $M = (Q, X, \tau)$ be a TL-fsm. Let $p, q \in Q$ and let $R \subset Q$. If $q \in S(R \bigcup \{p\})$, whenever $p \in S(R \bigcup \{q\})$ and $p \notin S(R)$, then M is said to satisfy the TL-exchange property.*

PROPOSITION 3.4. *Let $M = (Q, X, \tau)$ be a TL-fsm. Then the following are equivalent:*

(1) M satisfies the TL-exchange property.

(2) M is TL-retrievable.

DEFINITION 3.5. *A TL-fsm $M = (Q, X, \tau)$ is said to be TL-quasi-retrievable if it satisfies the following; for $q \in Q$ and $y \in X^*$ if there exists $t \in Q$ such that $\tau^*(q, y, p) > 0$, then there exists $x \in X^*$ such that $\tau^*(q, yx, q) > 0$.*

DEFINITION 3.6. *Let $M = (Q, X, \tau)$ be a TL-fsm. Let $q, r, s \in Q$. If there exists $y \in X^*$ such that $\tau^*(q, y, r) > 0$ and $\tau^*(q, y, s) > 0$, then r and s are said to be q-TL-related. If r and s are q-TL-related and if $S(s) = S(r)$, then r and s are said to be q-TL-twins.*

LEMMA 3.7. *Let $M = (Q, X, \tau)$ be a TL-fsm. Then the following are equivalent:*

(1) For $p, q, r \in Q$ and $x, y \in X^*$, if $\tau^*(q, y, r) > 0$ and $\tau^*(q, yx, p) > 0$, then $p \in S(r)$.

(2) For $q, r, s \in Q$, if r and s are q-TL-related, then r and s are q-TL-twins.

PROPOSITION 3.8. *Let $M = (Q, X, \tau)$ be a TL-fsm. Then the following are equivalent:*

(1) M is TL-retrievable.

(2) M is TL-quasi-retrievable. And for $q, r, s \in Q$ if r and s are q-TL-related, then r and s are q-TL-twins.

4. TL-separability and TL-connectivity

DEFINITION 4.1. *A nonempty submachine $N = (R, X, \nu)$ of a TL-fsm $M = (Q, X, \tau)$ is said to be TL-separated if $S(Q \setminus R) \bigcap R = \emptyset$.*

PROPOSITION 4.2. *Let $N = (R, X, \nu)$ be a nonempty submachine of a TL-fsm $M = (Q, X, \tau)$. Then N is TL-separated if and only if $S(Q \setminus R) = Q \setminus R$.*

PROPOSITION 4.3. *Let $N = (R, X, \nu)$ be a proper submachine of a TL-fsm $M = (Q, X, \tau)$. If N is TL-separated, then $C = (Q \setminus R, X, \sigma)$ is also TL-separated where $\sigma = \tau \mid_{(Q \setminus R) \times X \times (Q \setminus R)}$*

DEFINITION 4.4. *A TL-fsm $M = (Q, X, \tau)$ is said to be TL-connected if M has no TL-separated proper submachines.*

DEFINITION 4.5. *A TL-fsm $M = (Q, X, \tau)$ is called strongly TL-connected if $p \in S(q)$ for all $p, q \in Q$.*

PROPOSITION 4.6. *Let $M = (Q, X, \tau)$ be a TL-fsm. Then M is strongly TL-connected if and only if M has no proper submachines.*

THEOREM 4.7. *Let $M = (Q, X, \tau)$ be a TL-fsm. Then the following are equivalent:*

(1) M is strongly TL-connected.

(2) M is TL-connected and TL-retrievable.

(3) Every submachine of M is strongly TL-connected.

THEOREM 4.8. *Let $M = (Q, X, \tau)$ be TL-fsm. Then the following are equivalent:*

(1) M is TL-retrievable.

(2) M is the union of strongly TL-connected submachines of M.

References

1. A. Kandel and S.C. Lee, *Fuzzy switching and automata: Theory and applications,* (Crane Russak, New York, 1980).
2. D.S. Malik, J.N. Mordeson and M.K. Sen, Submachines of fuzzy finite state machines, *J. Fuzzy Math.* 2(1994) 781-792.
3. W.G. Wee, *On generalizations of adaptive algorithm and application of the fuzzy sets concept to pattern classification*, Ph.D. Thesis, Purdue Univ., June, 1967.
4. L.A. Zadeh, Fuzzy sets, *Inform. Control* 8(1965) 338-365.