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Stochastic traffic assignment Models for Dynamic Route Guidance

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Stochastic Traffic Assignment Models for Dynamic Route Guidance
(動的 길잡이 裝置를 위한 確率的 通行 配定 模型 開發에 관한 硏究)

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國文要約

尖端 交通 體系(Intelligent Transport Systems)의 중요한 要素인 尖端 交通 管理 體系(Advanced Traffic Management Systems)의 성공 여부는 交通情報를 어떻게 提供하고 統制하는데 의존한다. 즉, 情報 提供 方式과 이에 대한 運轉者의 反應을 정확하게 把握하고 豫測하여야 ITS를 성공적으로 構築할 수 있다. 이 論文에서는 動的 車輛 길잡이 裝置의 效用性을 評價하기 위한 確率的 通行配定模型을 開發하는 것이다. 개발된 通行配定模型은 운전자의 動的行態調整(Dynamic Behavioural Adjustment)을 明白하게 確率 過程(Stochastic Process)으로 표현하여 기존의 模型에 비해 通行者들의 行態를 더욱 實際的으로 반영한다. 특히, 각 통행자들에게 K개의 最小經路時間을 提供해줌으로 인하여 통행자의 路線選擇에 대한 選擇幅을 增加시켜 준다. 通行路線의 선택폭의 증가는 爭點으로 대두되는 問題(交通統制所에서는 車輛 길잡이 保有 운전자에게 體系最適(System Optimum)와 利用者最適(User Equilibrium)중 어떠한 原則하에 交通情報를 提供하여야 하는가)에 대한 解決 方案이다. 왜냐하면 만약 交通統制所에서 운전자에게 통행정보를 體系 最適을 하기 위해 情報를 提供하고자 하면, 길잡이 裝着 運轉者는 더 이상 제공된 정보를 따르지 않고 자기 스스로의 經驗에 의해 利用者 最適을 달성하고자 할 것이다. 이 論文의 目的은 이러한 복잡한 통행자의 路線選擇行爲를 반영하는 確率的 平衡 通行 配定 模型을 여러가지 統計技法을 導入하여 開發하는 것이다.

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1 Introduction

Modelling human behaviour is undoubtedly difficult. No two individuals will react identically to every situation, and yet it is often impractical to model at a sufficiently microscopic level to imbue each individual with personal traits. We are left with two possibilities. The first is to define a model with a single deterministic solution, hoping that it will mirror the system studied "on average". The alternative is to incorporate a random element into the model, calling all those elements which we cannot hope to represent explicitly "noise". Traffic assignment models seek to represent traveller behaviour, and therefore suffer the difficulties just described. The vast majority of work on equilibrium traffic assignment has employed the deterministic approach. Wardrop equilibrium (Wardrop, 1952) is a case in point. Stochastic user equilibrium (Daganzo and Sheffi, 1977), despite the name, is another since the traditional solution is a fixed flow pattern.

Despite the past popularity of the deterministic approaches, they have serious shortcomings. For example, fixed flow patterns which are meant to represent traffic equilibria cannot account for day-to-day variations. Whilst we might hope that the degree such variation is relatively small compared to the overall flow over a substantial time period, such an argument will not be appropriate when modelling at a relatively fine level of temporal aggregation; cf. Cascetta (1989) and the report of May's (1991) findings documented in Watling and Van Vuren (1993). Furthermore, traveller information and learning is an area of increasing importance, yet one which cannot be effectively covered by the class of deterministic models. It would seem perverse, for instance, to represent errors in drivers' perceptions by anything other than a probability distribution. Watling and Van Vuren discuss similar issues.

Stochastic models offer a far greater potential than deterministic models for providing a rich representation of traveller behaviour. This has been recognized by some authors, and a number of random process models have recently been published. See, for example, Cascetta and Cantarella (1991) and Watling (1994). Nevertheless, there is a price to pay for modelling stochastically rather than deterministically, in that the former type of model tend to be far less tractable than the latter. For example, the behavioural definition of stochastic user equilibrium is that each traveller minimizes his own *perceived* traveller costs, where his perceptions are error prone. Modelling these perceptual errors by a random variables implies that the solution (i.e. resulting flow pattern) of stochastic user equilibrium should be an *equilibrium probability distribution.* However, this probability distribution cannot be explicitly written down, with the result that Daganzo and Sheffi (1977) concentrated on the quasi-solution of a (pseudo) expected traffic flow pattern. We emphasize that despite being an *equilibrium* model, stochastic user equilibrium has the potential to represent fluctuations in flow; day-to-day variability, for example. It is the determinism of the quasi-solution which stunts this potential.

In principle, arbitrarily accurate numerical approximations to any property of a stochastic model can be obtained by simulation. For example, if we wish to know the expected flow pattern then we need simply simulate our stochastic process a large number of times, and then calculate the mean of our simulations. The Weak Law of Large Numbers (see Grimmett and Stirzaker, 1982, chapter 7) ensures that by increasing the number of simulations we can attain an arbitrary degree of accuracy. Nevertheless, *direct* simulation of the appropriate probability distribution frequently impossible. The difficulty is as follows. Models of traffic assignment tend to be defined at a microscopic level, in terms of the choices made by an individual traveller under given conditions. In other words, we know the conditional probability route choice distribution of every traveller. However, in order to simulate the global flow pattern, we must be able to draw samples from the joint distribution of all travellers' route choices. If there are n travellers, then this probability distribution will be n dimensional, and (assuming that travellers interact in a plausible fashion) with a complicated covariance structure. In most cases it is not possible to analytically calculate this joint distribution when only the individual conditional distributions are known. Hence, direct simulation is not usually feasible for even moderately small values of n.

This type of problem, where macroscopic solutions are unavailable for attractive microscopic models, was encountered by physicists and chemists in the 1950's. They defined models of the atomic structure of materials in terms of local interactions between proximate atomic particles, but were unable to explicitly derive the global state of the substance, nor directly simulate it. However, Metropolis *et al.* (1953), in a landmark paper, demonstrated how such processes could be indirectly simulated by sampling from a Markov Chain whose stationary distribution is identical to the probability distribution required. This technique and related methodologies are called *Markov Chain Monte Carlo methods*, and have seen increasingly widespread use during the last ten years. See Neal (1993) for an overview.

The aim of this paper is to demonstrate how existing stochastic process models of traffic assignment can be generalized to permit greater realism, and to show how equilibria of such models can be computed using Markov Chain Monte Carlo techniques. In the next section existing stochastic models are reviewed. We note that these models have restricted representations of traveller interaction due to the perceived need for a tractable model solution. Markov Chain Monte Carlo methods are introduced in section three. Their application to calculation of traffic assignment equilibria is described in general, and illustrated using simple examples. These examples also serve to indicate how use of Markov Chain simulation techniques will allow new stochastic models to avoid some of the problems associated with the existing models, as outlined in section two. The findings of the paper are summarized in the last section, and application of the techniques developed to complex models of traveller learning, incorporating advanced traveller information systems for example, is discussed.

2 Stochastic Assignment Models

2.1 Notation

Let the triple (N, L, K) represent a finite transport network, where N is the node set, L the set of directed links, and K a set of link cost functions. Let D be the demand matrix, whose ijth element d_{ij} is the demand from node i to node j. Let $n = \mathbf{1}^T D \mathbf{1}$ be the total network demand (number of travellers) where $\mathbf{1}$ is a column vector of ones.

Define I to be a set indexing all feasible routes (acyclic paths) and let I_j be the indexing set for the feasible routes for the jth traveller. Let the vector $\mathbf{R} = (R_1, \ldots, R_n)$ the vector of traveller route choices, so that $R_j \in I_j$. For the purposes of stochastic models, $\mathbf{R}$ as a random vector. We denote its probability mass function by $p(\mathbf{r}) = \Pr(\mathbf{R} = \mathbf{r})$. Throughout this paper we will discuss *sequences* or *chains* of route choice vectors. We will write $\mathbf{R}(t)$ for the tth member of such a sequence; sometimes this will be referred to as the tth iteration of $\mathbf{R}$. On occasions t

may allow an interpretation as real time, but in general it acts simply as a counter.

For the most part during this paper we will define the state of the network in terms of the vector **R** for the sake of generality. (For example, such a representation allows models which define the traits of individuals at any level of detail, from homogeneous through to entirely heterogeneous populations.) However, from a practical point of view it will often be simply the total route (or link) flows which are of interest. The flow down path i, where $i \in I$, is given by

$$F_i = \sum_{j=1}^{n} \delta(R_j, i),$$

where δ is the Kroneker delta which is zero unless its arguments are identical, in which case it takes the value one. If **F** is the vector of path flows, and A the link-path incidence matrix, then the vector of link flows, **X**, is given by

$$\mathbf{X} = A\mathbf{F}.$$

As with the vector of route choices, $\mathbf{F}(t)$ and $\mathbf{X}(t)$ are path and link flows at the tth iteration, whenever we discuss evolution of the system.

Finally, we define $C(i)$ to be the actual cost to a traveller taking the ith route, $i \in I$. It follows that $C(R_j)$ is the actual cost experienced by the jth traveller. In all but the most trivial models this cost is dependent upon the route choices of all travellers, and hence may be written more fully as $C(i|\mathbf{R})$, which is to be read as the actual cost of route i given the traveller route choice vector **R**. Often we will wish to discuss perceived costs. The perceived cost of the ith route to the jth traveller will be written as $\tilde{C}_j(i)$. The perceived cost could be a function of the present, and possibly past, states of the network. It will also usually contain a random element, which we generically term ϵ. Thus, $\tilde{C}_j(i)$ could be written "long-hand" as

$$\tilde{C}_j(i) = \tilde{C}(i|\mathbf{R}(t), \mathbf{R}(t-1), \ldots, \epsilon_j).$$

2.2 Existing Stochastic Assignment Models and Their Equilibria

The only truly stochastic models to have received much attention in the transportation literature are those in which travellers make route choices based solely upon memory, as we now describe. Let $\{\mathbf{R}(t)\} = \mathbf{R}(1), \mathbf{R}(2), \ldots$ be a sequence of route choice vectors from a stochastic assignment process, where the index t has a real temporal interpretation; counting days, for example. Cascetta (1989), Cascetta and Cantarella (1991) and Watling (1994) have all considered models in which each travellers route choice at time t is based upon memory of the finite past (say the previous t_0 epochs). Hence, the route choice probability distribution at time t is naturally defined conditional on the past, and written $p(\mathbf{r}(t)|\mathbf{r}(t-1), \mathbf{r}(t-2), \ldots, \mathbf{r}(t-t_0))$. For example, each traveller might choose the route which had minimum perceived cost during the last epoch.

A simple, logit based example of this type of model, in which memory lasts only a single epoch, would define the conditional route choice distribution for the jth traveller by

$$\Pr(R_j(t) = i|\mathbf{r}(t-1)) = \frac{\exp\{-\theta C(i|\mathbf{r}(t-1))\}}{\sum_{k \in I_j} \exp\{-\theta C(k|\mathbf{r}(t-1))\}}. \quad (1)$$

Here, $C(i|\mathbf{r}(t-1))$ is the cost of travelling on route i when the flow pattern was defined by route choices $\mathbf{r}(t-1)$. The parameter θ calibrates the extent of travellers perceptual errors. Given an initial route choice vector $\mathbf{r}(0)$, equation 1 fully defines a memory based stochastic assignment model. We shall refer to it on occasions as a *memory model.*

We note that for any general memory model, travellers' route choices for the present epoch are independent conditional on the past. Hence, the joint conditional route choice distribution is simply the product of every individual's marginal choice distribution. For instance, the joint conditional distribution of the route choice vector $\mathbf{R}(t)$ for the simple example above is given by

$$p(\mathbf{r}(t)|\mathbf{r}(t-1)) = \prod_{i=1}^{n} \Pr(R_j(t) = i|\mathbf{r}(t-1)).$$

This conditional independence allows such traffic assignment processes to be easily simulated, a matter discussed further below.

One reason for the interest in memory based assignment processes is that they can be studied as Markov Chains, about which a great deal is known.

Definition 1: Markov Chain. *The stochastic process $Y(t)$ is a Markov Chain if*

$$\Pr(Y(t) = y|Y(t-1), Y(t-2), \ldots, Y(0)) = \Pr(Y(t) = y|Y(t-1))$$

for all possible y.

Whilst $\mathbf{R}(t)$ will not generally be a Markov Chain itself (only in the case where $t_0 = 1$), the cross-product of route choices across t_0 epochs, $\mathcal{R}(t) = \mathbf{R}(t) \times \mathbf{R}(t-1) \times \ldots \times \mathbf{R}(t-t_0+1)$ will be. Because $\mathbf{R}$ can only take finitely many values, and t_0 is defined to be finite, the Markov Chain can only take finally many values (which makes its properties easier to study than those for processes taking values in an infinite set.)

An important property of finite Markov Chains is that they converge to a unique equilibrium distribution (also called invariant and stationary distribution) under very mild conditions.

Definition 2: Equilibrium (Stationary State) of a Markov Chain. *The Markov Chain $Y(t)$ is in equilibrium if*

$$\Pr(Y(t) = y) = \Pr(Y(t-1) = y)$$

for all possible y. When in this state, the distribution of $Y(t)$ is called the equilibrium distribution and denoted by $p^*(y)$.

For stochastic traffic assignment memory models, sufficient (although not necessary) conditions for the existence of a unique equilibrium state are that each traveller has a non-zero probability of selecting any feasible route. (However, this non-zero probability can be extremely small for highly costly routes without adversely effecting the practical rate of convergence in any plausible model.) In order to simulate the equilibrium state of memory models the first t_0 values of $\mathbf{R}$ can be chosen arbitrarily, and then each present state simulated conditional on its past. (Recall that such simulation is rendered simple by the conditional independence of the present given the past.) The theory of Markov Chains then ensures that $\mathbf{R}(t)$ will settle down to its equilibrium probability distribution as t becomes large. We note that whilst the selection of the first t_0 route choice vectors is arbitrary, convergence to equilibrium will be swifter the closer these route choices are to typical equilibrium choice vectors.

2.3 Contemporaneous Traveller Interaction

Whilst memory models are attractive in their explicitly modelling of travellers' reaction to past experience, they allow no real sense of contemporaneous traveller interaction, as we now explain. If epochs $t, t+1, t+2, \ldots$ in a memory model are interpreted as representing the same time period on consecutive days, then there is clearly no modelling of contemporaneous traveller interaction; travellers react only to day-old events. On the other hand, if we view each epoch as a time period of arbitrarily short length within a day, then memory models explicitly require *every* traveller to reselect his or her route from moment to moment. This can lead to a massive shift in flow pattern over short time periods which is intuitively unattractive. Watling (1994) described the results of simulations on memory models where large groups of travellers were constantly flitting back and forth between a number of relatively attractive routes, giving rise to a multimodal route choice equilibrium distribution which seems somewhat unlikely to occur in practice.

Such difficulties can be countered, to an extent, by introducing "damping" effects on travellers' rerouting behaviour. An example of such an effect would be to allow travellers to change route only if they perceive a drop in travel cost greater than some threshold value. By doing so, more travellers will choose not to reroute from epoch to epoch, and hence the flow pattern will change more gently through time. Nevertheless, the philosophy of memory models is not aimed at representing contemporaneous traveller interaction. Since it is clear that travellers do react contemporaneously (diverting to a side road so as to avoid growing congestion downstream, for example), models defined explicitly in terms of such behaviour are worthy of consideration.

We develop such a model below based upon the following behavioural foundation (which is that chosen by Sheffi and Daganzo, 1977).

Definition 3: Stochastic User Behaviour. *A traveller displaying stochastic user (SU) behaviour will always choose the route with minimum perceived cost at present.*

In other words, an SU behaviour traveller will select from a route choice probability distribution based upon the instantaneous route costs. This definition is microscopic, in that it is in terms of individual behaviour. Nevertheless, it implies the following definition of macroscopic stochastic user equilibrium. (We note that our definition of this equilibrium is at odds to the commonly used deterministic solution of pseudo mean flow, discussed in the introduction.)

Definition 4: Stochastic User Equilibrium (SUE). *The route flow vector* $\mathbf{R}$ *is in stochastic user equilibrium if the probability distribution of* $\mathbf{R}$ *is invariant to any traveller displaying SU behaviour.*

We note that if the travellers have no perceptual errors, then SU behaviour is to choose the cheapest available route (in terms of actual measured cost), and SUE becomes the state in which no traveller has a cheaper alternative route; that is, Wardrop equilibrium.

Mathematical analysis of the SUE distribution $p^*(\mathbf{r})$ seems impossible for all but the most trivial cases. Simulation from this distribution also appears difficult, since it is clear that in the equilibrium state, travellers' route choices will be highly interdependent. Furthermore, because the definitions of SU is atemporal (unlike the behavioural foundations of memory models), it is not immediately obvious that SUE can be viewed as the limiting state of some evolving stochastic process. Nevertheless, we demonstrate in the next section that we can find an artificially constructed Markov Chain whose stationary distribution is identically that of SUE, and this leads to a methodology for simulating SUE.

3 Monte Carlo Markov Chain Methods

3.1 Simulation

The aim of Monte Carlo Markov Chain Methods is to simulate from complicated equilibrium distributions by sampling a Markov Chain whose stationary state is precisely the equilibrium distribution required. In the case of memory models we do not need to search for such Markov Chains, since their equilibria are *defined* as the stationary states of particular Markov processes. All that needs to be done in order to simulate a memory model is to select an initial state $\mathcal{R}(0)$, and then generate each ensuing flow pattern conditional on the previous t_0 patterns. The independence of present route choices given the past flow patterns (as discussed earlier) makes this a simple procedure.

Despite the basically atemporal definition of SUE, this state can be viewed as stationary for a carefully constructed Markov Chain. The definition of SUE implies that any particular traveller reselecting a route according to SU behaviour will leave the equilibrium distribution $p^*(\mathbf{r})$ invariant. It follows that *any sequence* of SU behaviour traveller reroutes leave this equilibrium invariant. But since this holds for any general sequence of traveller actions, it must hold for specific sequences. In particular, $p^*(\mathbf{r})$ must be the stationary distribution for the Markov Chain defined by each traveller considers rerouting in turn. However, we know that under mild conditions any Markov Chain settles down its equilibrium distribution. We conclude that SUE can be simulated by looking at the long term behaviour of sequentially rerouting travellers one-by-one.

The required sequentially rerouting Markov Chain can be constructed as follows. Let $\mathbf{R}'(t)$ be an assignment process such that $\mathbf{R}'(t)$ and $\mathbf{R}'(t-1)$ differ only in their (t mod n)th component. For example, $\mathbf{R}'(1)$ is obtained from the initial state $\mathbf{R}'(0)$ by letting the first traveller sample from his route choice distribution, $\mathbf{R}'(2)$ is obtained from $\mathbf{R}'(1)$ by rerouting the second traveller and so on. Recycling is done in the obvious way, so that $\mathbf{R}'(n+1)$ is obtained from $\mathbf{R}'(n)$ by again rerouting the first traveller for instance. In each case, let the costs defining the route choice distribution for $\mathbf{R}'(t)$ be defined in terms of the flow at $\mathbf{R}'(t-1)$. Thus, the *transition* probability q_t (that is, the conditional probability of $\mathbf{R}'(t)$ given $\mathbf{R}'(t-1)$) is given by

$$q(\mathbf{r}'(t)|\mathbf{r}'(t-1)) = \begin{cases} \Pr(\mathbf{R}'(t) = \mathbf{r}'(t)|\mathbf{r}'(t-1)) & \text{if } R'_k(t) = r'_k(t-1) \text{ for all } k \neq t \bmod n \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

If a logit route choice model were used, for example, then

$$q(\mathbf{r}'(t)|\mathbf{r}'(t-1)) = \Pr(R'_{t\bmod n}(t) = i|\mathbf{r}'(t-1)) = \frac{\exp\{-\theta C(i|\mathbf{r}'(t-1))\}}{\sum_{k\in I_j}\exp\{-\theta C(k|\mathbf{r}'(t-1))\}}. \quad (3)$$

Now let $\mathbf{R}(t)$ be the subsequence of $\mathbf{R}'(t)$ defined by $\mathbf{R}(t) = \mathbf{R}'(nt)$. Consecutive realizations of the process $\mathbf{R}(t)$ are obtained by sequentially allowing every traveller to sample from his route choice distribution once. Then $\mathbf{R}(t)$ is a Markov Chain. Furthermore, its equilibrium distribution, attained as $t \to \infty$, is invariant to sequences of SU traveller route selections, and is therefore identical to SUE as required. This particular method of simulation using Markov Chains is called *Gibbs sampling*; further details may be found in Neal (1993, section 4.1).

It should be noted that despite the fact that $\mathbf{R}(t)$ involves rerouting all travellers at each iteration, this process does not constitute a memory model. The important difference is that when sampling SUE, the underlying process $\mathbf{R}'(t)$ is generated by updating route costs after every single traveller route selection. (See the definition of $q(\mathbf{r}'(t)|\mathbf{r}'(t-1))$ above.) However, for memory models the costs are only updated after every traveller has reselected his route. This distinction is by no means trivial – as we shall see in the numerical examples of section 3.3, SUE and memory models can have dramatically dissimilar equilibrium distributions. Another fundamental difference between the Markov Chains used in simulating memory models and SUE concerns the interpretation of t. For memory models, t can be viewed as time, with consecutive iterations of the process being seen as route flows on consecutive epochs (possibly days). When simulating SUE, on the other hand, the Markov Chain is no more than a mathematical tool. We are only interested in sampling from its stationary distribution. Values of the process on the way to convergence do not usually admit any sensible physical interpretation, and t should be viewed simply as a counter.

3.2 Testing for Convergence

Our aim is to sample from the equilibrium distribution of the model under consideration. We have seen that this may be achieved by running an associated Markov Chain until it reaches its stationary state which coincides with the required equilibrium. To implement this methodology in practice it is necessary to know when the Markov Chain has converged. A simple method of diagnosing convergence is to define some summary statistic, $S(\mathbf{R}(t))$ (such as some function of mean flow pattern, or of the variance of the simulated flows) and to plot a graph of this statistic against iteration number. Figure 1 displays a typical example, taken from a SUE model which is described further in section 3.3 below. The pictured graph displays the average number (taken over consecutive groups of twenty iterations) of travellers selecting a particular route plotted against iteration number. Note that the *sampling distribution* of this average seems to have settled down after about 400 iterations. (This is not to say that this flow statistic is converging to a particular value; rather, its probability distribution has tended to its stationary functional form.) This is some indication that the whole system may have (approximately) converged to its equilibrium distribution by iteration 400.

This technique can be refined by running several parallel simulations and comparing the behaviour of $S(\mathbf{R}(t))$ between runs. More sophisticated methods for detecting convergence, such as the "Gibbs stopper" (Ritter and Tanner, 1992) have been proposed. However, visual checks will probably be sufficient for most intents and purposes in stochastic traffic assignment modelling.

Once we have decided that equilibrium has been achieved, at iteration t_{eq} say, then all those flow patterns obtained before t_{eq} should be discarded. Simulated flow patterns after t_{eq} will be from the equilibrium distribution (approximately), although consecutive interactions will be highly

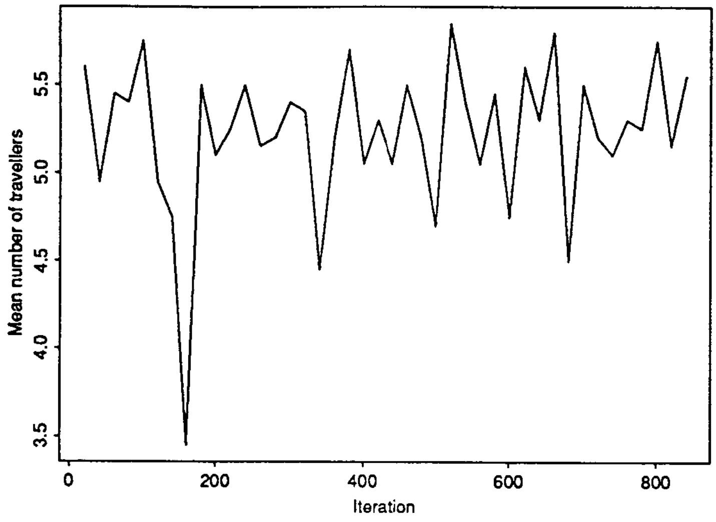

Figure 1: Variation of a route choice statistic with iteration number.

dependent realizations of this distribution. For memory models such *serial dependence* is exactly what is required, so that $\mathbf{R}(t_{eq}), \mathbf{R}(t_{eq}+1), \ldots$ represent a sequence of flows on consecutive epochs. However, for SUE models which aim to model contemporaneous traveller behaviour, such serial correlation is a nuisance. Should we be largely interested in linear functions of the flow pattern, then the existence of such serial dependence is relatively unimportant. This is because any linear function of the simulated flow, for example the mean flow on a particular link, is an unbiased approximation to its expected value (under the equilibrium distribution) whether or not serial correlation exists. However, for more complicated properties of the equilibrium distribution, it may be necessary to generate pseudo independent realizations of the stationary state. For example, using consecutive (and therefore highly dependent) simulated flow patterns in order to calculate an estimate of the variance for a particular link flow would produce highly biased results. An unbiased estimate of this variance is (generally) only available given a set of uncorrelated simulations.

A sample of such uncorrelated flow patterns can be obtained by simply running a number of independent Markov Chains in parallel and selecting only a single realization from each. It is often more efficient to run a single Markov Chain for a long period and simply take realizations which are separated by a large number of iterations (over which the serial correlation becomes suppressed). An in depth analysis of the dependence between consecutive flow patterns can be carried out using sample *autocorrelation functions* (see Grimmett and Stirzaker, 1982, chapter 9). The autocorrelation function displays the correlation between $\mathbf{R}(t)$ and $\mathbf{R}(t+\tau)$ for different *lags* τ. By viewing a plot of the autocorrelation, we can see how great a lag is required for the dependency between $\mathbf{R}(t)$ and $\mathbf{R}(t+\tau)$ to have died down, and hence how many iterations are required between pseudo independent realizations of the route choice vector. Figure 2 displays an estimate of the autocorrelation function (ACF) for the number of travellers selecting a particular route in the example introduced above. The dotted lines about zero are 95% confidence intervals for 0 correlation. It seems that by taking every 20th generated route choice vector, an (approximately) independent sample will be obtained. Neal (1993, section 6.3) discusses the matter further.

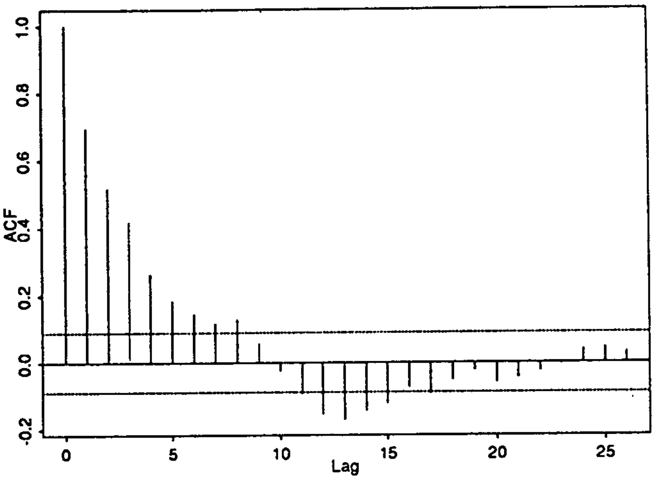

Figure 2: Autocorrelation function for number of travellers selecting a particular route.

3.3 Examples

We consider two models of traffic assignment for a total demand from O to D of 10 (identical) travellers over the simple five link network displayed in figure 3.

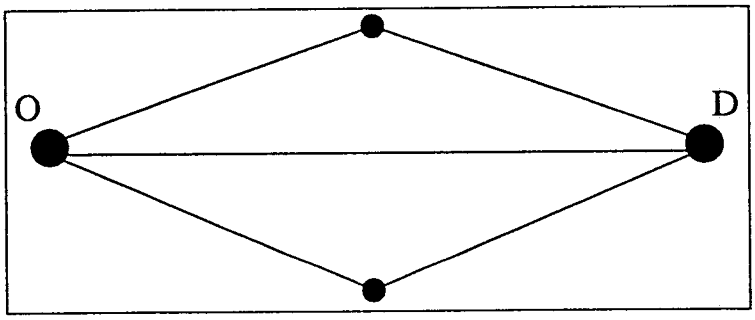

Figure 3: Example network topology.

Model 1 is the logit based memory model described in section 2.1. Model 2 is our version of stochastic user equilibrium, as described in section 2.1, again using logit route choices. For both models the same set of BPR (Bureau of Public Roads, 1964) link cost functions where employed. The results are summarized in figures 4 and 5. The histograms in these figures display the observed frequency distribution of the number of travellers using the "top" route from O to D. For each model, the results after 30 and 1000 iterations are shown.

The most striking aspect of the results is the complete discrepancy between the model 1 and model 2 distributions. For model 1, there is little difference between the iteration 30 and iteration 1000 cases, suggesting that convergence has been attained swiftly. The equilibrium distribution is bimodal, a result of the kind of "switching back and forth" behaviour that was described in section 2.2. It seems somewhat implausible that this type of equilibrium would be observed in reality. Model 2 converges somewhat more slowly, which is why the 30 iteration and 1000 iteration histograms are noticeably different. The stationary distribution (which we believe has been reached by the 1000th iteration) is unimodal, with a spread of flows which is not negligeable in comparison to the average.

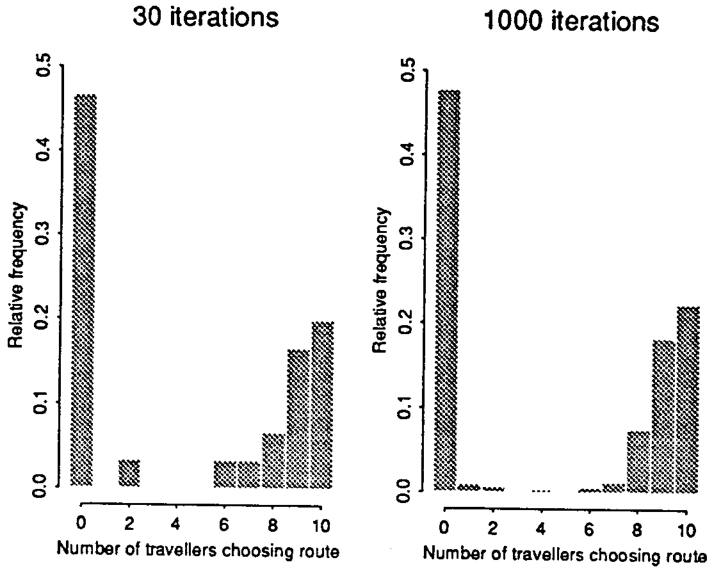

Figure 4: Route choice histograms for model 1.

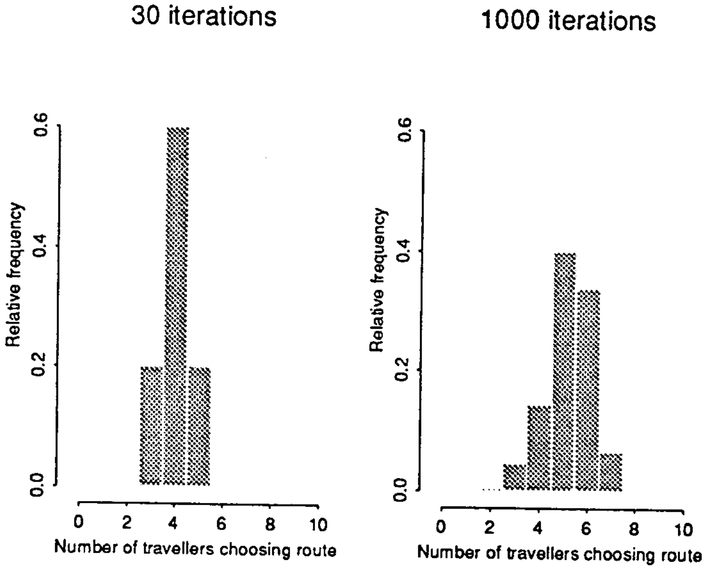

Figure 5: Route choice histograms for model 2.

Daganzo and Sheffi's (1977) deterministic quasi solution to stochastic user equilibrium was also computed for the above example. This gave a flow of 5.9 travellers on the route in question. The (approximate) mean equilibrium flow for model 2 was calculated from the last 500 iterations (for which the system was believed to have almost converged), and the corresponding mean route flow found to be 5.3. Whilst these route flow figures do not differ by a huge amount, the difference is statistically significant. This emphasizes the fact the usual deterministic solution to stochastic user equilibrium is not, precisely, the expected flow pattern that is implied by the microscopic behavioural definition of the model. To understand why this is so, note that Daganzo and Sheffi's solution calculates a *conditional* mean flow. That is, a mean flow given that the costs take fixed values; the values implied by the mean flow pattern itself, in fact.

However, the exact mean flow pattern (the expected value of the probability distribution p^*) is calculated with respected to a distribution which implies that the link costs take different values corresponding to the various possible flow patterns. Our simulated average route flow is an unbiased estimator of precisely this exact expected SUE flow.

3.4 Extensions to More Complex Stochastic Assignment Models

Neither SUE nor memory models are capable of capturing a sense of both contemporaneous traveller interaction and travellers' reaction to past experience. It seems that a hybrid "SUE with memory" model is required in order to explicitly represent both these facets of traveller behaviour. We describe one possible approach to producing such a "combined model" below.

A combined model may be defined by embedding SUE within a memory model, so that each travellers' perceived cost is a function of the present state of the network and of flow patterns from finitely many previous epochs. The result is the following two tier process. Let $\mathbf{R}(t)$ be outer iterations of this combined traffic assignment model, representing the evolution of the traffic system from day-to-day in the manner of memory models. Let $\tilde{\mathbf{R}}(\tau)$ be an inner process which attempts to capture contemporaneous traveller interaction using SUE type modelling. Suppose that the measured costs for the inner process are not simply the instantaneous travel costs (as in the SUE model of section 2.3) but are an amalgamation of both instantaneous costs and travel costs from (finitely many) past epochs of the outer process, We could then define $\mathbf{R}(t+1)$, conditional on $\mathbf{R}(t), \mathbf{R}(t-1), \ldots, \mathbf{R}(t-t_0+1)$, as the SUE route choice vector obtained by running inner process $\tilde{\mathbf{R}}(\tau)$ until convergence to equilibrium. The outer process (or rather the cross product of sets of consecutive t_0 terms) forms a Markov Chain, and will therefore settle down to a stationary state itself. In this equilibrium, each traveller tries to minimize his perceived costs which are now dependent upon both present and past states of the network.

We believe that such combined models have the potential to provide a richer and more realistic representation of traveller route choice. Our research in this area is ongoing.

4 Discussion

In this paper we have attempted to demonstrate the potential benefits of modelling traffic assignment in a truly stochastic fashion. Whilst attractive random process models tend to defy mathematical analysis, properties of their equilibria can be estimated (to an arbitrary degree of accuracy) by obtaining simulated realizations of the assignment process. Such simulation is made possible by the use of Monte Carlo Markov Chain techniques; in particular, the Gibbs sampler. By employing such simulation methodologies, transport researchers should be freed from the bounds of mathematical tractability when modelling traffic assignment, and hence allowed a wider scope when defining models at a behavioural level.

We have only begun to explore the potential of stochastic traffic assignment models, restricting attention to models based solely on memory, or solely on contemporaneous traveller interaction. Research on models combining both these facets is ongoing, and should lead to far more realistic representation of traveller behaviour. Furthermore, because mathematical tractability is of limited importance (given sufficiently powerful computer simulation techniques) we may expand existing models to incorporate new factors with little difficulty. For example, imbuing different sections of the population with varying characteristics, or defining a more complicated link cost structure does not alter the applicability of the simulation techniques developed in sections 2 and 3.

Traveller learning and the impact of advanced traveller information systems on route choice are both areas that are readily integrated into the general framework of stochastic traffic assignment modelling. Our logit based implementation of the memory model of section 2.2 represents traveller learning in a relatively crude manner, but this may easily be refined. The effect of advanced traveller information systems may be investigated by altering the behavioural model parameters for those travellers with such devices. For instance, in the logit route choice model of equation 1, those travellers with information systems would be assigned a rather larger value of θ (representing less uncertainty about travel costs) than those without.

Whatever type of stochastic assignment model that is developed, it must be calibrated for practical use. Often the estimation of model parameters will be of great interest; when studying the marginal benefits of having an in-vehicle information system, for example. In principle given sufficiently informative data, we can calculate maximum likelihood estimates of all model parameters using the equilibrium probability distributions. However, since only an simulation based approximation to this distribution is available, we can only hope to obtain so called *Monte Carlo maximum likelihood estimates*; see Diggle and Gratton (1984) and Hazelton (1995). Such estimates can be difficult to obtain when the number of parameters is large, so that it may be preferable to use a *method of moments* approach. The idea behind this method is to select those parameters which minimize some measure of discrepancy between modelled and observed values of a combination of traffic flow statistics.

Calibration is just one area of stochastic traffic assignment modelling that warrants further research. A particularly important direction for future work is empirical investigation of the properties of our simulation methodologies for large scale, real networks. Such numerical studies are in progress at the moment, including an example based upon the road network of Seoul, and will be incorporated into the final version of this paper. This research should provide guidelines as to the most efficient computational methods for implementing Monte Carlo Markov Chain simulation of stochastic traffic assignment models. Furthermore, it will help to illustrate the potential of such models by allowing results such as distribution of route choices, and correlations between different links' flows, to be displayed.

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