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Gradient Projection법을 이용한 철골평면구조물의 최적설계연구

Study on Optimum Design of Steel Plane Frame By Using Gradient Projection Method

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ABSTRACT

The general conceptual constitution of structural optimization is formulated. The algorithm using the gradient projection method and design sensitivity analysis is discussed. Examples of minimum-weight design for six-story steel plane frame are taken to illustrate the application of this algorithm. The advantages of this algorithm such as marginal cost and design sensitivity analysis as well as system analysis are explained.

1. INTRODUCTION

The economics regarding the structural design may not be parameterized by a single variable. Some of the most important factors seem to be the structural volume or weight, construction labor cost, maintenance and repair expenses throughout the life of the building, etc. However these factors have different effect on the total cost of construction and maintenance of building structures from region to region and from time to time. For example, in some country, the labor cost is much more important than in other countries where the material cost is generally the higher one. therefore, if designers are concerned with the real cost of construction and maintenance of building structures, it is very difficult to visualize or qunatify these economic parameters in a simple way.

Nevertheless, it is commonly accepted by structural designers that minimization of structural weight or structural material volume is of the utmost importance in economic parameters because the weight or volume of structure appears to be roughly proportional to the material and construction labor cost regardless of time and place of construction.

Therefore, in this structural optimization program, the objective of design is assumed to be only the minimization of weight or volume of structure while all the othersare included in the design constraints.

2. FORMULATION OF MATHMATICAL MODEL[1]

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2.1 Design Variable and State Variable

The behavior of most engineering system is governed by some law of physics. this behavior is described analytically by a set of variables called state variables. For structural systems, state variables may include displacements and stresses at certain points, eigenvectors, eigenvalues etc. Let $z \in R^n$ be a state variable vector representing displacements at key points of structure, and let $y \in R^n$ and ζ represent an eigenvector and eigenvalue, respectively.

There is second set of variables called design variables that describes the system. Let $b \in R^k$ represent a vector of design variables which mean the moment of inertia in the case of moment-resisting steel structures.

2.2 Relationship Between Design Variable and Associated Section Properties

The member sections are classified as three types, economy beam section, W14 series column section and W12 series column section. The different equations corresponding to differnt types of sections have been set up by least square curve fitting. The range of interest in the design variable, the moment inertia, I is assumed 100 to 4000 in^4, which corresponds to the member sizes of medium-rise structures.[2]
The equations representing the fitted curves are follows:
Economy beam series:

$$\begin{Bmatrix} A \\ S \end{Bmatrix} = \begin{Bmatrix} 0.493I^{0.498} \\ 0.589I^{0.743} \end{Bmatrix} \tag{2.1a}$$

W14 column series:

$$\begin{Bmatrix} A \\ S \end{Bmatrix} = \begin{Bmatrix} 0.0693I^{0.861} \\ 0.262I^{0.908} \end{Bmatrix} \tag{2.1b}$$

W12 column series:

$$\begin{Bmatrix} A \\ S \end{Bmatrix} = \begin{Bmatrix} 0.103I^{0.833} \\ 0.375I^{0.867} \end{Bmatrix} \tag{2.1c}$$

Where A and S mean section area and elastic section modulus, respectively.

The shape factor between plastic section modulus, Z, and elastic section modulus, S, is assumed to be 1.15 .

2.3 General Mathematical Model

Now a general mathematical model for optimum design of structural system is determined as follows:

<u>Definition 1</u>

Find a design variable vector $b \in R^k$ that minimizes objective function

$$\psi_0(\mathbf{b}, \mathbf{z}, \zeta) \tag{2.2}$$

and satisfies the equillibrium equations and design constraints which can be expressed in a simplified form as shown in eq(2.3)

$$\psi_i(\mathbf{b}, \mathbf{z}, \zeta) \le 0, \; i=1, 2, \ldots, m. \tag{2.3}$$

<u>Definition 2(Constraint set)</u>

A set of points that satisfies all the constraints of problem of Definition 1, is called a constraint set. It is defined as

$$D = \{b \in R^k : k(\mathbf{b})z=f, K(b)y=\zeta M(\mathbf{b})y, \psi_i(\mathbf{b}, \mathbf{z}, \zeta) \le 0, \; i=1, 2, \ldots, m\} \tag{2.4}$$

Since all the constraints equations are continuous with respect to **b**, the set D of eq(2.4) is closed. It is assumed that D is bounded and hence compact. It is further assumed that the objective function of (2.2) is continuous on D. Therefore, the mathematical model for optimum structural design of Definition 1 has an absolute minimum in D.

2.4 Design Sensitivity Analysis

A major step in any direct method of nonlinear programming is to calculate gradients of the objective and constraint function with respect to design variables at the current design point.

A first variation of the function $\psi_i(\mathbf{b},\mathbf{z},\zeta)$ gives

$$\delta\psi_i = \frac{\partial \psi_i}{\partial b}\delta b + \frac{\partial \psi_i}{\partial z}\delta z + \frac{\partial \psi_i}{\partial \zeta}\delta\zeta \tag{2.5}$$

where all the derivatives are calculated at the given values of **b** and the computed values for **z** and ζ. The object of design sensitivity analysis is to express the terms with δz and $\delta\zeta$ as functions of δb;Thus the first variation of eq(2.5) is written as

$$\delta\psi_i = (\mathbf{l}^i)^T\delta\mathbf{b} \quad i=0,1,\ldots,m \tag{2.6}$$

where $\mathbf{l}^i$ is then gradient for the function ψ_i with respect to design variables at current design point.

In order to eliminate $\delta\mathbf{z}$ from eq(2.5), one writes a first variation of the state equation and obtains $\delta\mathbf{z}$ as

$$\delta z = -K^{-1}\frac{\partial}{\partial b}[K(b)z-f\,]\delta b \tag{2.7}$$

Also to eliminate $\delta\zeta$ from eq(2.5), one writes a first variation of the eigenvalue problem and after some algebraic manipulation obtains a well-established formula for $\delta\zeta$:

$$\delta\zeta = 1^{\zeta T}\delta\mathbf{b} \tag{2.8}$$

where l^{ζ} is the design dervative vector for the eigenvalue ζ given in eq(2.9) with $\mathbf{y}^T\mathbf{M}(\mathbf{b})\mathbf{y}=1$.

$$l^{\zeta} = \frac{\partial}{\partial b}[y^TK(b)y - \zeta y^TM(b)y\,]^T \tag{2.9}$$

Substituting for $\delta\mathbf{z}$ and $\delta\zeta$ from eqs(2.7) and (2.9) into eq(2.5) and comparing the result with eq(2.6), one obtains

$$l^i = [\frac{\partial \psi_i}{\partial b} - \frac{\partial \psi_i}{\partial z}K^{-1}\frac{\partial}{\partial b}(K(b)z - f\,) + \frac{\partial \psi_i}{\partial \zeta}l^{\zeta T}\,]^T \tag{2.10}$$

Calculation of $\mathbf{K}^{-1}$ in eq(2.10) can be avoided by an algebraic manipulation. Define a vector $\mathbf{q}^i$ as follows.

$$[q^i]^T \equiv \frac{\partial \psi_i}{\partial z}K^{-1} \qquad i=0,1,2,\ldots,m \tag{2.11}$$

Then, the gradient vector $\mathbf{l}^i$ in eq(2.10) becomes

$$l^i = [(\frac{\partial\psi_i}{\partial b})^T - \frac{\partial}{\partial b}(K(b)z-f)q^i + l^\zeta \frac{\partial\psi_i}{\partial\zeta}] \qquad (2.12)$$

Now postmultiplying eq(2.11) by **K** and taking the transpose of the resulting equation, one obtains $\mathbf{q}^i$ as a solution of the equation

$$Kq^i = [\frac{\partial\psi_i}{\partial z}]^T \qquad (2.13)$$

Eq(2.13) is called an adjoint equation, and $\mathbf{q}^i$ is called an adjoint vector. Note that the adjoint equation (2.13) has the same cofficient matrix as the original state equation.
Thus the previous decomposition of **K** can be used to solve for $\mathbf{q}^i$.

2.5 Determination of Search Direction.

With the assumption of initial design point $\mathbf{b}^0$ being within constraint set, the reduced optimum design problem for $\delta\mathbf{b}$ is now defined as follows:

Find $\delta\mathbf{b}$ that minimizes a first order change in the objective function

$$\delta\psi_0 = \mathbf{l}^{0T}\delta\mathbf{b} \qquad (2.14)$$

and satisfies the linearized design constraints

$$\tilde{\psi}_i + \delta\tilde{\psi}_i \leq 0 \quad i=1,2,\ldots,p \qquad (2.15)$$

and a step size constraint

$$\delta\mathbf{b}^T\mathbf{W}\delta\mathbf{b} \leq \xi^2 \qquad (2.16)$$

Here $\mathbf{W}$ is a positive definite weighting matrix(usually diagonal) and is assumed to be identify matrix I. $\xi>0$ is a small number, a tilde () over a function indicates an ϵ -active constraint(that is $\psi_i+\epsilon\geq 0$, where $\epsilon>0$ is small number.), and p is the number of ϵ-active constraits. This ϵ is introduced in checking various constraints so that if the current design point is arbitraily close to a constraint surface, then that constraint is treated as an active constraint.

After Using Kuhn-Tucker necessary condition and doing some mathematical manipulations, the design change is expressed as

$$\delta b = -(\frac{1}{2\gamma})\delta b^1 + \delta b^2 \qquad (2.17)$$

$$\delta b^1 = W^{-1}[l^0 + \tilde{L}\tilde{\mu}^1], \quad \delta b^2 = -W^{-1}\tilde{L}\tilde{\mu}^2,$$
$$M\tilde{\mu}^1 = -\tilde{L}^T W^{-1} l^0, \quad M\tilde{\mu}^2 = \tilde{\psi}.$$
$$M \equiv \tilde{L}^T W^{-1} \tilde{L}. \quad \tilde{L} \equiv [\tilde{l}^i],$$

The step size $\frac{1}{2\gamma}$ in eq(2.17)is always ≥ 0. And it is reasonable to select the step size $\frac{1}{2\gamma}$ directly.

Let the convergence parameter ξ_p be defined as follows:

$$\xi_p \equiv \frac{\|\delta b^1\|_2}{\|W^{-1}l^0\|_2} \qquad (2.18)$$

It can be easily shown that for all optimum design problem, ξ_p lies between 0 and 1. If ξ_p =0, the relative optimum point is attained.

3. SENSITIVITY ANALYSIS AND SHADOW VALUE OF DESIGN CONSTRAINT

The geometry, load conditions and design variables are shown in Figure 1 and the imposed constraints are as follows:

(1) Moment: $f_s \leq 0.66F_y$ (compact section)

(2) Deflection: Interstory drift index ≤ 0.0025

(3) Soft story failure load factor $\lambda_{s.s.} \geq 1.1\lambda_{p.c.}$

At every step of the optimization procedure, the program calculates the amount of violations for all the imposed design constraints. The optimization algorithm needs the design sensitivity analysis of each active constraint to implement the gradient projection method. Sometimes it is desirable to have a quantitative intuition about how much influence on a certain active constraint the increase of each design variable by one unit can have. This can be done by calculating the design sensitivity vector, each component of which represents the contribution of the corresponding design variable. Therefore whenever designers inspect the components of the design sensitivity vector corresponding to a certain violated constriant, they can find the relative contribution of each design variable to remedy that violated constriant. Two examples of minimum-weight design with different upperbounds on the fundamental period of structure are shown in Figure 2. The active constraints and corresponding shadow values are listed in Table 1 while the definition of design variables and related plastic mechanisms are shown in Figure 1 and 3, respectively. For example, two active constriants in Table 1 are taken for illustration. One is $M_{12} \leq M_{allow}$ and the other is $1.1\lambda_{p.c.} \leq \lambda_5$. The corresponding normalized constraint functions are defined as follows:

$$\psi_A \equiv \frac{M_{12}}{M_{allow}} - 1 \leq 0$$

$$\psi_B \equiv \frac{1.1\lambda_{p.c.}}{\lambda_5} - 1 \leq 0$$

Considering that the design sensitivity vector is defined as

$$[\frac{\partial \psi}{\partial b}]^T \equiv [\frac{\partial \psi}{\partial b_1}, \frac{\partial \psi}{\partial b_2}, \dots, \frac{\partial \psi}{\partial b_k}]^T$$

where k denotes the number of design variables, the data as shown in Table 2 reveals that the increase of design variable 12 by one unit causes the decrease of ψ_A by the amount of 126×10^{-6} while the other design variables have almost no influence on the change of that constriant function, to compensate the violation of this constrint, the increase of design variable, 12 is the most effective and economical. Also, for the second constraint function ψ_B, the increase of design variables, 5 and 6, and the decrease of design variables, 11 and 12, will give the most predominant compensation of the violation of constraint function ψ_B.

Whereas the design sensitivity vector shows the relative influences of design variables for the remedy of violated constriants, the shadow value of a certain active constriant function is released by the unit value. Of cource, this shadow value is an approximate estimate because the nonlinear optimization model is linearized in the local region and then this linearized model is solved using the gradient projection method. However, when designers investigate all the shadow values of active constraints, they can immediately acknowledge the relative influence of each active constriant on the reduction of the total weight of the structure. This implies that designers can have a clear idea on the cost and benefit resulting from the imposition of certain specific design constraint.

A most detailed explanation is given by using the examples of Table 3 regarding the significance of shadow values. It is intererting to note that the shadow value or price corresponding to the constraint $1.1\lambda_{p.c.} \leq \lambda_5$ is about three times as large as the next largest shadow value. When the constriants, $1.1\lambda_{p.c.} \leq \lambda_1$ and $1.1\lambda_{p.c.} \leq \lambda_5$ are released by 5% or 10% in the normalized constriant functions, the actual reductions and those expected by shadow values in structural volume are compared in Table 3. From Table 1 and 3, designers can clearly note that the contribution of design input $1.1\lambda_{p.c.} \leq \lambda_5$ to the increase or decrease of total structural volume is much larger than those of design input $1.1\lambda_{p.c.} \leq \lambda_1$.

4. SYSTEM ANALYSIS

Structures whose span lengths are varied as shown in Figure 4 have been designed by using the program OPTIMUM. The applied design constraints are as follows:

(i) Moment should be less than or equal to yield moment for all members.

(ii) Interstory drift should be less than or equal to $\frac{0.015h_x}{C_d}$ where h_x means the corresponding story height and C_d is 5.5 in case of steel frame.

(iii) Axial force should be less than or equal to 40% or 60% of yield strength.

The result of optimization are given in Figure 5. From Figure 5, the most economic system is shown to be system 2. The reduction of structural volume is about 10 to 20% of its own weight when compared with other systems.

From the minimum-weight design of system 3, it can be found that the girders with longer span are almost of the same size throughout the stories. But all the girders and columns which constitute the smaller bay are very stiff at the lower stories and the stiffness decreases at the higher stories goes up. This offers the hindsight that in order to control the lateral drift efficiently, it is more economical to strengthen the bay of the smaller span than that of longer span while the girders with the longer span mainly resist the gravity loads. This result clearly proves the economic efficiency for adding shear wall or bracing(dual system) from the optimization point of view.

However, if the span length becomes large, the design constraints on the axial force can be the controlling design input as in the case of system 3. To reduce the effect of axial force on the plastic moment and the risk of instability, ATC 3 recommends that axial force, P, should be less than 60% of yield strength P_y. But the design shown in Figure 4(c) is based on constraint $P \leq 0.4P_y$. When the structural volume for $P \leq 0.4P_y$ and that for $P \leq 0.6P_y$ are compared (135590 cubic inch versus 126410 cubic inch), the influence of the design constraint on axial force can be clearly recognized in the case of a long-span structure.

5. CONCLUSION

The design approach using this structural optimization program is basically different from that of a trial-and-error method. Even if the program OPTIMUM starts with arbitrary initial design variables, it strictly follows the direct search method which will eventually get the minimum-weight design. The real advantage of optimization is that it demonstrates what the best design is and why. Meanwhile, the trial-and-error desing procedure does not show designers clearly why a design is the best, instead giving a procedure which leads to a satisfactory decision of member sizes. When calling a design optimum, one can at least explain why it is the best for all possible choices, and how much cost it saves compared with other systems and what design input has the largest influence, etc. The design reached by using the proposed procedure clearly reveals this advantage of optimization.

6. REFERENCES

[1] Arora, J.S., "Analysis of optimality Criteria and Gradient Projection Method for Optimal Structural Design," in Computer Method in Applied Mechanics and Engeneering 23, North-Holland Publishing Co:, 1980, pp185-213

[2] Han-Seon Lee, "Use of Shakedown Analysis Technique in Optimum Seismic Design of Mement-Resisting Steel Structure", Ph.D. Dissertation at Department of Civil Eng., University of California at Berkeley, May 1989.

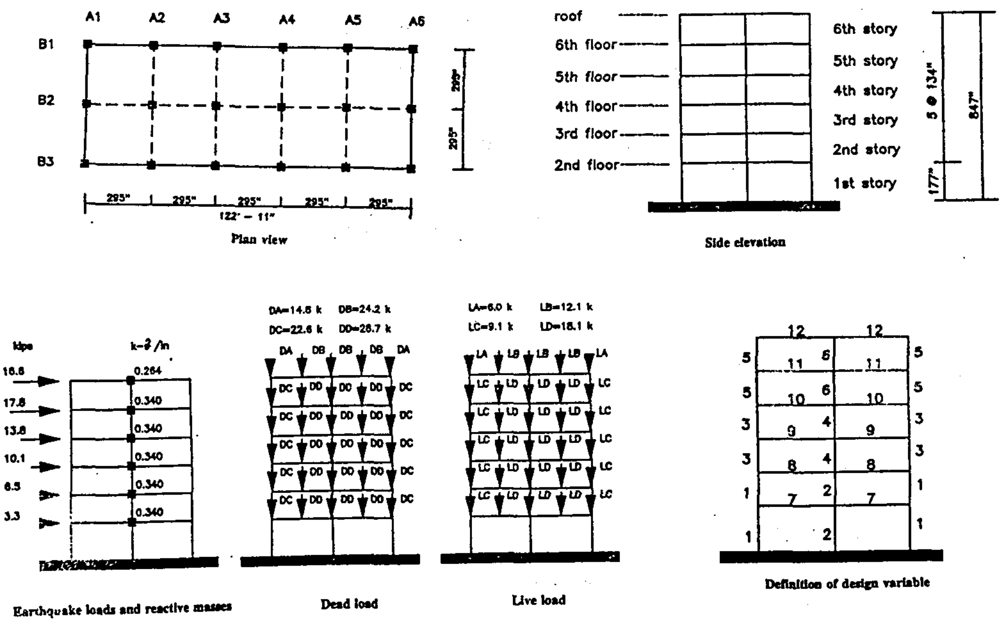

Figure 1 Geometry, load conditions and Design Variables

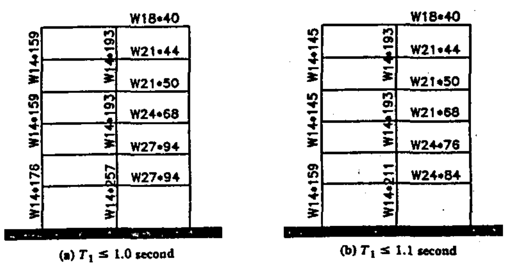

Figure 2 Two Optimum Designs

Table 1 Active constraints of Two Optimum Designs(Fig.2)

constraint		shadow value (in^4)	
type	content	design A ($T^* = 1.0$ s)	design B ($T^* = 1.1$ s)
period	$T_1 \le T^*$	99255	81905
mechanism*	$1.1\lambda_{p.c.} \le \lambda_1$	35124	29797
	$1.1\lambda_{p.c.} \le \lambda_2$		2202
	$1.1\lambda_{p.c.} \le \lambda_3$	36306	20938
	$1.1\lambda_{p.c.} \le \lambda_4$	21079	3764
	(B)** $1.1\lambda_{p.c.} \le \lambda_5$	276800	290750
moment***	$M_{11} \le 0.66F_yS = M_{allow}$	25081	30351
	(A)** $M_{12} \le 0.66F_yS$	24310	28040
column***	$A_3 \le A_1$		118
	$A_5 \le A_3$	9373	15241
	$A_6 \le A_4$	5116	7687

Table 2 Two Design Sensitivity Vectors

design var.*	1	2	3	4	5	6	7	8	9	10	11	12
$\frac{\partial\psi_A}{\partial b}$ ($\times 10^{-5}$)	0	0	0	0	-1	0	0	0	0	0	0	-126
$\frac{\partial\psi_B}{\partial b}$ ($\times 10^{-6}$)	3	1	0	0	-62	-30	3	3	4	4	87	94

Table 3 Reduction of Structural Volume by Release of Constraints

normalized constraint	volume of structure (in^3) [*reduction*]	
	actual	expected by shadow value
$\frac{1.1\lambda_{p.c.}}{\lambda_5} - 1 \le 0$	185670	185670
$\frac{1.1\lambda_{p.c.}}{\lambda_5} - 1 \le 0.05$	177330[8340]	171132[14537]
$\frac{1.1\lambda_{p.c.}}{\lambda_5} - 1 \le 0.1$	167520[18150]	156595[29075]

(a) Release of constraint $1.1\lambda_{p.c.} \le \lambda_5$ (shadow value = 290750 in^3)

normalized constraint	volume of structure (in^3) [*reduction*]	
	actual	expected by shadow value
$\frac{1.1\lambda_{p.c.}}{\lambda_1} - 1 \le 0$	185670	185670
$\frac{1.1\lambda_{p.c.}}{\lambda_1} - 1 \le 0.1$	184630[1040]	18[illegible]0[2980]

(b) Release of constraint $1.1\lambda_{p.c.} \le \lambda_1$ (shadow value = 29797 in^3)

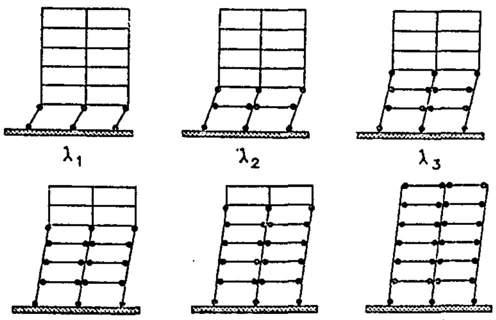

Figure 3 Related Plastic Mechnisms

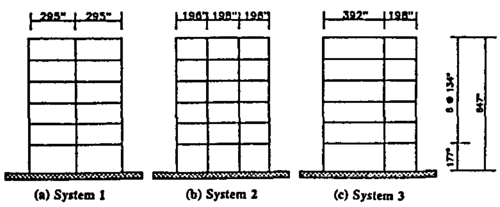

Figure 4 Different Geometry for System Analysis

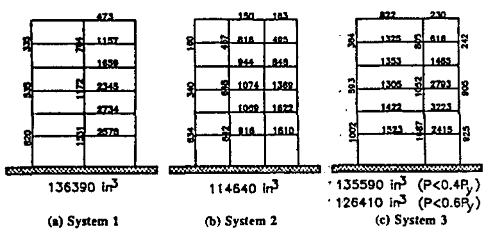

Figure 5 Moments of Inertia from Structural Optimization(in^4)