On the Statistical Studies for Satellite Estimates of Rain Rate

Eunho Ha
Department of Statistics, Inje University

1. Introduction

Precipitation is probably the most crucial link in the hydrological cycle. Life on the earth's continents would be impossible without it. A significant fraction of the heating of the tropical atmosphere comes from latent heat released in precipitating clouds. The measurement and archival of precipitation across global scales with resolution of a few hundred kilometers is essential for advancement of our knowledge of the dynamics of the ocean and atmosphere. The large variability in precipitation makes it a particularly important scientific challenge in modelling the atmospheric circulation and climate. However, precipitation is one of the most difficult of all atmospheric variables to measure, especially over the tropical oceans. Tropical rainfall is the major source of global precipitation since more than two-thirds of the global precipitation falls in the tropical band lying between 30° North and 30° South. There have been many studies pointing to the importance of tropical rainfall (see Simpson et al., 1988, for references). Accurate estimates of the amount of rainfall in the tropics would substatially improve our knowledge of the workings of weather and climate, but tropical rainfall is not very well monitored, since much of the tropics is covered by ocean. Continuous coverage of these large oceanic expanses is probably only feasible from space. Several satellite missions are in the design and fabrication stage to gather data and form space-time smoothed maps of rain rates, especially over the tropical oceans (e.g., Simpson et al., 1988).

An important method of estimating rain rate is to make use of observables such as microwave brightness temperature, which is a covariate with rain rate, as measured from above the atmosphere. However, this method leads to a beam filling error. This is the error occurring when we estimate the area average rain rate by converting microwave temperature to rain rate where microwave temperature is related to rain rate nonlinearly. Due to the nonlinearity of the beam filling error, it is hard to derive statistical properties, such as mean and variance of the beam filling error, which are needed to correct the estimate of area average rain rate.

Another method of estimating area average rain rate is the so-called threshold method. This is based on the relationship between the area average rain rate and the fraction of the area covered by rain rate above a fixed threshold. One problem of this method is choosing an optimal threshold level.

2. Retrieval Method: Beam Filling Error Problem

The problem we pose to study here is that of the retrieval of rainrate statistics from a satellite based single channel radiometer which has a finite resolution. That is to say, the instrument measures an apparent microwave temperature averaged over a horizontal area usually of the order of 20 to 50 km across. If the realtionship between microwave temperature and rain rate were linear there would be no problem for this simple case of retrieval using a single microwave channel (taken here nominally to be the 19.6GHz channel, since this is the one used on most modern radiometers). The problem is that the relationship although one-to-one is nonlinear, having the shape of a saturating exponential which can be written approximately (Wilheit, 1986)

$$\begin{aligned} T(R) &= A+Be^{-cR} \\ \text{or} \quad R(T) &= -\frac{1}{c}\ln\frac{(T-A)}{B} \end{aligned} \tag{1}$$

where T is microwave temperature in degree Kelvin, R is rain rate in mm/hr. Since the radiometer measures an area average of T, referred to here as $[T(R)]$, the inversion to obtain an area average of R, $[R]$, is not so staraightforward, since

$$[R] \neq R([T]). \tag{2}$$

In fact, it can be shown that heterogeneities in the field of view (FOV) cause $[R] \geq R([T])$. It is easy to see that the retrieval is not unique; that is there are many values of $[R]$ that can correspond to a given measured value of $[T]$. This leads us to a pair of equations: 1) What is the bias,

$$\beta = \mathrm{E}(\delta R) = \mathrm{E}([R]-R([T])) \tag{3}$$

due to the finite footprint and its effect on lowering (incorrectly) our estimate of $[R]$? 2) What is the distribution of random errors associated with the estimate? In particular, what is the variance associated with the distribution:

$$\mathrm{Var}(\delta R) = \mathrm{E}((\delta R-\beta)^2). \tag{4}$$

The quantity δR is referred to as the beam filling error. It can be modelled as a random variable taking on different values with each realization (snapshot). Given the $T(R)$ relationship, the problem reduces to a study of the statistics of the rainfield within restricted areas the size of the field of view of the instrument.

We study a sequence of very simple model rainfields to see how the beamfilling error depends upon these field statistics.

3. Threshold Method: Choosing an Optimal Threshold

Recent studies of rain rate observations (Chiu, 1988; Rosenfeld, Atlas, and Short 1990) have demonstrated that the area average rain rate and the fraction of the area covered by a rain rate exceeding a given threshold $\tau \geq 0$ are highly correlated, provided that the averaging domain is sufficiently large. Exploration of this behavior is the basis of the so-called threshold method for measuring rainfall.

Several empirical studies support this method by showing that the sample correlation between the area average rain rate and the fraction of the area covered by rain rate exceeding a given threshold can easily exceed 95% and in some cases can even exceed 99% (Short, Wolff, Rosenfeld, and Atlas 1989). The effects of thresholding have been examined in data from a rain gauge network in the Darwin, Australia area (Short et al. 1989) and more recently in France (Braud, Creutin, Barancourt, 1991). From these empirical studies it has been found that the accuracy of the threshold improves as one moves away from the zero rain rate threshold, but decreases as the threshold is raised above a rain rate of 10 mm/hr. Chiu (1988) also empirically found that the threshold level $\tau = 5$ leads to a squared correlation of 98% and drops dramatically for other levels using the GATE data. For level $\tau = 0$, the squared correlation drops to below 80%.

Given the practicality of the method, a determination of the optimal threshold level from theoretical considerations is desirable.

Suppose that rain rate is observed over a given region A and througout a specific period $[0, T]$. Let X be the random variable that gives the value of rain rate. Then, X has a mixed distribution because it admits the value 0 (no rain) with positive probability, say, $1-p$. But conditional on rain, X has a continuous distribution. Using the statistical properties of mixed distribution, we can obtain the following relationship between $\mathrm{E}[X]$ and $\Pr(X > \tau)$:

$$\mathrm{E}[X] = \beta(\tau) \Pr(X > \tau), \qquad (5)$$

where

$$\beta(\tau) = \frac{\mathrm{E}[X \mid X > 0]}{\Pr[X > \tau \mid X > 0]}$$

depends on the distribution of rain rate and the *threshold level* τ. If a threshold level $\tau \geq 0$ is given, then via a random samples of rain rate $\{X_1, X_2, \cdots, X_n\}$, one would estimate the area average $<X>$ with the aid of the theoretical linear relationship (5) as

$$<X> \approx \beta(\tau) < I[X > \tau] >, \qquad (6)$$

where $I[\cdot]$ is an indicator function and thus $<I[X>\tau]>$ is the fractional area. The linear relationship (6) provides an explanation for the observed linear realtionship demonstrated in Chiu (1988) and in Rosenfeld et al. (1990). Now, (6) provides the clue we are seeking all along: an optimal level τ can be estimated to minimize the estimator of the variance of the maximum likelihood estimate of $\beta(\tau)$ under an appropriate choice of a distribution for rain rate (Procedure I). Another possibility is to choose τ that maximizes the correlation between the area average $<X>$ and the fractional area $<I[X>\tau]>$ (Procedure II).

We apply the procedures of estimating optimal threshold level for the mixed lognormal distribution, which is well known as the distribution of rain rate (for example, Kedem, Chiu and North 1990). We show that the optimal threshold level estimated by the above procedures agrees well with the best experimental value obtained by Chiu (1988) and compare the results of two procedures.

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