

Frost Heaving in Artificial Ground Freezing

TAKAHIRO OHRAI

Seiken Co., Ltd., 2-11-16, Kawarayamachi,
Chuo-ku, Osaka, Japan

ABSTRACT

Artificial ground freezing methods have been applied to geotechnical construction projects for stabilizing earth materials and controlling water seepage into the ground. However, this can result in frost heaving and causes the same engineering problems as encountered with the natural freezing of soil.

In natural freezing, the ground freezes from the surface downward. When artificial ground freezing is applied at a deep location, however, freezing is limited locally. The soil condition differs between them as follows:

Natural freezing — unsaturated and without overburden pressure,

Artificial freezing -- saturated and under overburden pressure.

The authors investigated the practical application of artificial ground freezing and examined the frost behaviour of a saturated soil under overburden pressure. This paper presents the results obtained from experiments concerning frost heaving and discusses frost heaving at the freezing site.

CONTENTS

1. INTRODUCTION
2. DESCRIPTION OF ARTIFICIAL GROUND FREEZING
 - 2.1 Refrigeration Plant
 - 2.2 Thermal Analysis of Artificial Freezing of Ground
3. FROST HEAVING IN SATURATED SOIL
 - 3.1 Factors Affecting Frost Heaving
 - 3.2 Experimental Procedure
 - 3.3 Effect of Overburden Pressure on Frost Heave Ratio
 - 3.4 Effect of Penetration Rate on Frost Heave Ratio
 - 3.5 Data Deduced from the Experiment for Stress and Penetration Rate
 - 3.6 Effect of Pore Water Pressure on Frost Heave Ratio
 - 3.7 Maximum Heaving Pressure
 - 3.8 Long-term Frost Heaving in Partially Frozen Soil
4. PREDICTION OF FROST HEAVING AT FREEZING SITE
 - 4.1 Influence of Permeability of Unfrozen Soil on Frost Heave
 - 4.2 Consolidation of Unfrozen Soil During Freezing
 - 4.3 Change in Earth Pressure due to Freezing
 - 4.4 Uplift of Ground Surface
 - 4.5 Countermeasures Against Frost Heaving in Artificial Ground Freezing
5. CONCLUSIONS AND SUMMARY

1. INTRODUCTION

Cold regions, where permafrost prevails and the ground thaws only to a shallow depth in summer, occupy about 20 per cent of the world's land area. The depth that is subjected to a freeze-thaw cycle is called the active layer. Frost heaving in cold regions make development difficult and cause damage to construction including roads, railways, airports, and petroleum pipelines. Many scientists and technologists conducting research on permafrost have looked into its fundamentals as well as ways to prevent damage and take countermeasures against frost heaving.

In Japan, artificial ground freezing has been applied to geotechnical construction projects in order to stabilize earth materials and control ground water seepage. It is frequently applied to urban civil engineering in which buildings are crowded and underground structures abound, i.e., gas and sewage pipes, groups of buildings, etc.; so allowable displacement against them must not exceed 1 cm. Artificial ground freezing utilizes the strength and practical impermeability of frozen ground, but can result in frost heaving in the same way as in natural freezing.

When the ground freezes from the surface downward, the amount of frost heave appears mostly on the surface. When artificial ground freezing is applied at a deep location, however, freezing is limited locally, resulting in a compound behaviour; that is, the increase in pressure in the unfrozen soil around the frozen location is partly absorbed in the unfrozen soil and partly appears as a heave of the ground surface. Accordingly, much information should be available for predicting or estimating the displacement in an area of unfrozen soil apart from this location, the increase in soil pressure, and the heave amount at the surface. The principle of frost heaving remains the same whether it is caused by natural or artificial freezing of the ground. Therefore, when the ground is frozen artificially, analyzed data obtained from the results of research into the natural freezing of the ground can be applied to engineering. However, there are problems with some engineering applications which require more investigation.

The following are the three major differences between natural and artificial ground freezing:

- i) Saturation of soil
 - natural freezing : unsaturated.
 - artificial freezing: saturated because the soil to be frozen is below the ground water table.
- ii) Overburden pressure
 - natural freezing : $< 1 \text{ kgf/cm}^2$ because the active layer is above a depth of several meters.
 - artificial freezing : from zero to several tens of kgf/cm^2 .
- iii) Penetration rate
 - natural freezing : $< 2.5 \text{ mm/h}$
 - artificial freezing : $< 50 \text{ mm/h}$.

The authors are engaged in the practical application of artificial ground

... the behavior of a soil saturated with water and under pressure. This paper presents the results of their investigation. Section Two outlines artificial freezing and the freezing process in the ground; Section Three shows the results of laboratory experiments concerning frost heaving; and Section Four discusses frost heaving at the freezing site.

Most of the research described in this paper was conducted under the guidance of the late Dr. Takashi, whose assistance is gratefully acknowledged.

2. DESCRIPTION OF ARTIFICIAL GROUND FREEZING

2.1 Refrigeration Plant

The installation for artificial ground freezing consists of two parts: freeze pipes and a refrigeration system. The freeze pipe, which is a metal pipe usually 10 cm in diameter, is installed at appropriate spacings in the ground to be frozen, and is cooled to a temperature lower than the freezing point of the water in the soil. Soil surrounding the freeze pipe is cooled and then frozen gradually as the temperature of the soil water falls below the freezing temperature, thus forming a thick frozen soil column. As time goes by, the frozen soil columns merge with the adjacent ones, forming a solid frozen soil wall. Since the frozen soil wall is impermeable and mechanically strong, it can be used as a temporary logging for conducting civil engineering work. Upon completion of the work, the cooling of the freeze pipes is stopped, allowing the frozen soil wall to thaw gradually. When necessary, the frozen soil wall may be thawed rapidly by circulating hot water through the freeze pipes.

Two methods are available to cool the freeze pipes: the brine method and liquid nitrogen (LN_2) method, as shown in Figure 1.

The brine method employs a solution of brine, usually Calcium Chloride

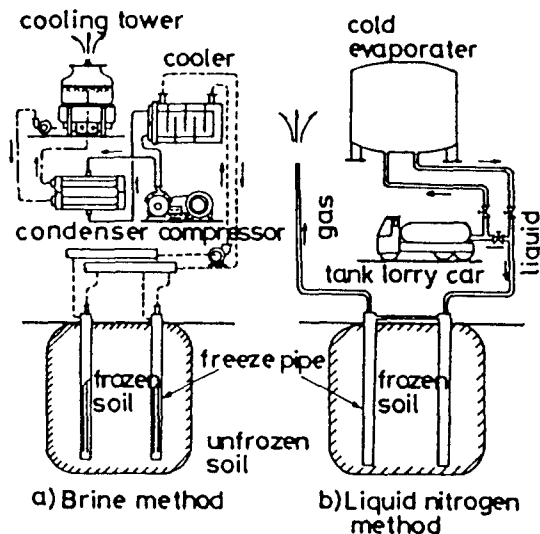


FIGURE 1. Freezing installations.

(CaCl₂). The solution is cooled from -20°C to -30°C by a refrigeration unit, and then circulated through the freeze pipes by a pumping system, which removes heat from the surrounding soil. The brine is returned by collector pipes to the refrigeration unit, cooled and then sent to the freeze pipe again. The refrigeration unit consists of a compressor, a condenser and a cooler; the evaporating liquid usually used in Japan is Freon Gas (R22). The volume of frozen soil per a refrigeration unit with 75 kw capacity, ranges from 400 m³ to 1000 m³. This system is widely applied in large-scale freezing work which requires from 200 m³ to 35000 m³ in volume of frozen soil (Ohrai et al., 1985).

The liquid nitrogen method, shown in Figure 1b, uses liquid nitrogen, which is supplied to the freeze pipes through a distributor pipe from an LN₂ tank or tank lorry loaded with LN₂. The soil surrounding the freeze pipes is frozen mainly by the latent heat (38.5 kcal/l) of LN₂ and partly by the sensitive heat of gas (boiling point=-196°C). Nitrogen gas from the freeze pipes is exhausted directly into the atmosphere. Mainly for economic reasons, this method is applied to small scale, short-term work, in which the volume of soil subjected to freezing is less than 200 m³.

2.2 Thermal Analysis of Artificial Freezing of Ground

Freezing of soil surrounding freeze pipes. The freezing of soil surrounding the freeze pipes advances along a cylindrical front until the frozen soil columns merge with adjacent ones, and then freezing advances along a plane. When the soil freezes, frost heaving or water migration or both should occur, whether the soil is frost-susceptible or not. Thermal analysis involving both water and heat presents a complex problem for the freezing of soil surrounding the freeze pipes. For many practical applications of artificial ground freezing, we use a model of heat conduction which neglects water migration, since there is little water migration relative to overburden pressure.

The equations of heat conduction, when expressed using the cylindrical coordinates as shown in Figure 2, are:

$$\frac{\partial \theta_1}{\partial t} = \kappa_1 \left(\frac{\partial^2 \theta_1}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_1}{\partial r} \right) \quad \text{for } a \leq r \leq R \quad (1)$$

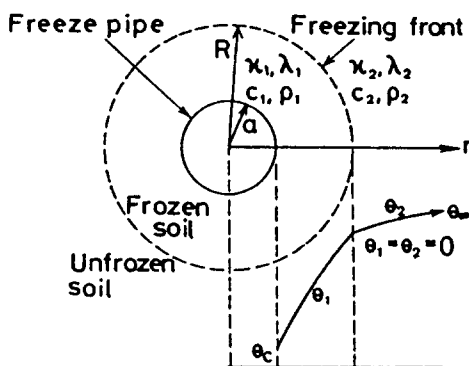


FIGURE 2. Calculation model.

$$\frac{\partial \theta_2}{\partial t} = \kappa_2 \left(\frac{\partial^2 \theta_2}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_2}{\partial r} \right) \quad \text{for } R \leq r \quad (2)$$

Initial condition is:

$$\theta_2 = \theta_\infty \quad \text{at } t = 0 \quad (3)$$

Boundary conditions are:

$$\theta_1 = \theta_c \quad \text{at } r = a \quad (4)$$

$$\theta_2 = \theta_\infty \quad \text{at } r = \infty \quad (5)$$

Boundary condition at the freezing front is:

$$\left[\lambda_1 \frac{\partial \theta_1}{\partial r} - \lambda_2 \frac{\partial \theta_2}{\partial r} \right]_{r=R} = L \rho_1 \frac{dR}{dt} \quad (6)$$

Where θ : temperature

r : radius coordinate,

R : radius of frozen soil,

t : time,

κ : coefficient of thermal diffusivity,

λ : thermal conductivity,

L : latent heat,

ρ : density of soil,

suffixes 1, 2 and ∞ : frozen soil, unfrozen soil and infinite, respectively.

Takashi (1961) gave an exact solution for these equations. The radius of the frozen soil, R , is given by the following equation:

$$\lambda_1 \theta_c \left[\frac{\frac{\partial F(\kappa_1, a; r, t)}{\partial r}}{F(\kappa_1, a; r, t)} \right]_{r=R} + \lambda_2 \theta_\infty \left[\frac{\frac{\partial F(\kappa_2, a; r, t)}{\partial r}}{1 - F(\kappa_2, a; r, t)} \right]_{r=R} = L \rho_1 \left[\frac{\frac{\partial F(\kappa_1, a; r, t)}{\partial t}}{\frac{\partial F(\kappa_1, a; r, t)}{\partial r}} \right]_{r=R} \quad (7)$$

where

$$F(\kappa, a; r, t) = -\frac{2}{\pi} \int_0^\infty e^{-\frac{(x/a)^2 t}{\kappa}} \frac{J_0(xr/a)Y_0(x) - J_0(x)Y_0(xr/a)}{J_0^2(x) + Y_0^2(x)} \frac{dx}{x} \quad (8)$$

where J_0 and Y_0 are Bessel functions of the order zero and the integral order zero, respectively.

An example of this calculation is shown in Figure 3. According to this, a freeze pipe 10.2 cm in diameter will form a frozen soil column about 50 cm in radius and will be formed after one month when $\theta_\infty = 14^\circ\text{C}$ and $\theta_c = -20^\circ\text{C}$.

When a frozen wall is formed by the merging of adjacent columns, it advances forward as a freezing front. A practical example was given by Miyoshi et. al. (1975) where a structure was built below the bottom of a river at times by freezing the soil into the shape of a board. The freezing of a semi-infinite material by a freezing front in the shape of

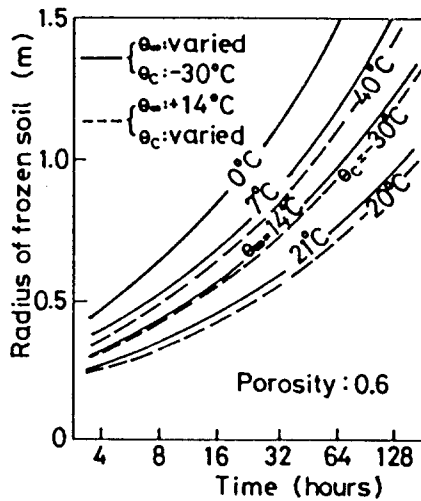


FIGURE 3. Change in radius of frozen column with time.

a board was examined by Neuman. He investigated the freezing of the still water surface of a lake in winter, and arrived at a solution now known as Neuman's solution (Carslow and Jaeger, 1959). In this case, the frost penetration length, X , is proportional to \sqrt{t} :

$$X = \alpha \sqrt{t} \quad (9)$$

where α is the constant determined from thermal properties of frozen and unfrozen soil, and temperature. For this equation, the soil type and water content must be taken into account. For example, when $\theta_c = -30^\circ\text{C}$ and $\theta_\infty = 14^\circ\text{C}$:

$$X = 0.0455 \sqrt{t} \quad [\text{m}] \quad (10)$$

That is, the soil freezes to the thickness of 1.2 m after about one month.

Freezing efficiency in soil freezing. We define the "freezing efficiency, η_f " as follows:

$$\eta_f = Q_L/Q_T \quad (11)$$

where Q_L : total quantity of latent heat (the quantity of latent heat used to freeze water contained in soil subtracted from the quantity of heat removed from the freezing front during a period from the beginning of cooling to the time of measurement)

Q_T : total quantity of heat consumed by the freezing of the soil.

It was found that η_f is independent of the cooling interval when the freezing surface is a flat plane (Takashi, 1965). Figure 4 shows the relation between η_f and the temperature of the cooling surface. From this figure, we can find the maximum value of η_f , which increases with the decreasing temperature of the ground, θ_∞ . Since the ground temperature at a shallow depth is from 14 to 18°C in Japan, η_f reaches its maximum at the temperature of the cooling surface ranging from -20 to -30°C. Therefore, the brine method is thermally more advantageous than the liquid Nitrogen method since the latter's freezing temperature is -196°C.

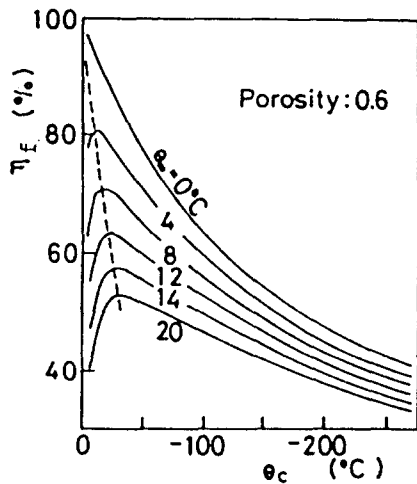


FIGURE 4. Freezing efficiency η_f as a function of temperature of cooling surface θ_c .

Influence of seepage flow on artificial freezing of soil. In artificial soil freezing, the presence of a seepage stream disturbs the development and the merging process of frozen soil columns around the freeze pipes, due to heat gain from the seepage stream. This phenomenon has frequently occurred, especially where there are a large number of sites where the ground has frozen artificially.

Takashi (1969) has demonstrated a criterion for determining whether the frozen soil columns can merge with one another. As shown in Figure 5, a finite number of freeze pipes were arranged in a row, and the seepage flow was perpendicular to the plane formed by the pipes. As the soil froze, the flow velocity of the seepage increased as a result of an increase in the hydraulic gradient caused by the phenomenon of dam-up. The following equation presents the critical value, $U_{\infty \text{ crit}}$, of the velocity of natural seepage in merging frozen soil columns, taking into account the dam-up phenomenon:

$$U_{\infty \text{ crit}} = \frac{6 \pi M^2 \lambda_1}{(n/100) l_f \gamma_w c_w} \frac{\theta_f - \theta_c}{\theta_{\infty} - \theta_f} F\left(\frac{2a}{P_i}, b_{\text{crit}}\right) \quad (12)$$

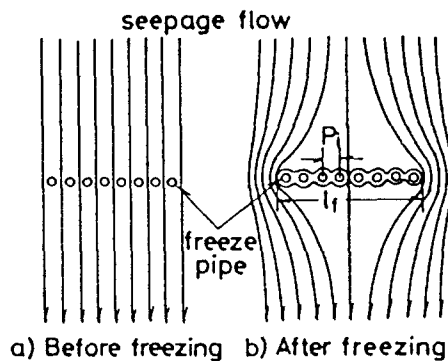


FIGURE 5. Model of seepage flow during freezing.

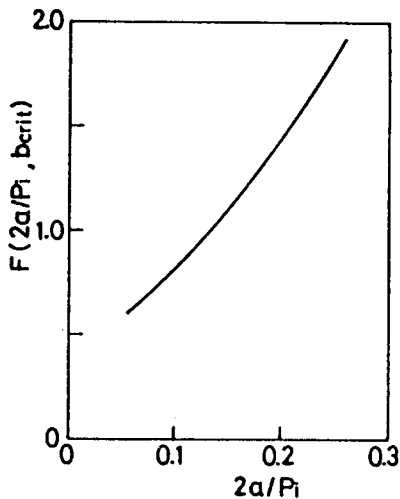


FIGURE 6. Relation between $F(2a/P_i, b_{crit})$ and $2a/P_i$

where $U_{\infty crit}$: critical flow velocity of seepage,
 M : number of rows of freeze pipes,
 λ_1 : thermal conductivity of frozen soil,
 n : porosity,
 l_f : width of a frozen soil wall perpendicular to the direction
of seepage flow,
 γ_w : unit weight of soil water,
 c_w : specific heat of soil water,
 θ_f : freezing temperature of soil water,
 θ_c : temperature of brine,
 θ_{∞} : temperature of soil water
 $F(2a/P_i, b_{crit})$: the function of radius, a , and spacing, P_i , of
the freeze pipes.

Figure 6 shows the relation between F and $2a/P_i$. Function F increases linearly with an increase in $2a/P_i$. In Japan, $U_{\infty crit} = 2$ m/day for the brine method and 10 m/day for the liquid nitrogen method.

3. FROST HEAVING IN SATURATED SOIL

3.1 Factors Affecting Frost Heaving

The following are the factors which affect frost heaving.

(A) Internal factors

- (1) physical properties of soil --- texture, structure, specific surface area, permeability, saturation, etc.
- (2) chemical properties of soil particles and soil water
- (3) history of stress and freeze-thaw

(B) External factors

- (1) overburden pressure
- (2) penetration rate and/or temperature gradient
- (3) pore water pressure

While scientists and technologists engaged in cold region research have investigated natural freezing (group A), the authors have experimentally investigated frost heaving with artificial freezing (group B), since this will determine the effects of external factors on frost heaving. An

understanding of these effects is necessary before artificial ground freezing can be applied to civil engineering. This Section shows the results of their investigation.

The following are the three main definitions for describing the results of the quantities obtained from the experiments:

(1) Frost heave ratio, ξ :

$$\xi = \Delta V/V = \Delta h/H \quad (13)$$

where ΔV :frost expanded volume during freezing,
 V :volume of soil before freezing,
 Δh :length of frost heave,
 H :length of soil before freezing.

(2) Water intake or discharge ratio, ξ_w :

$$\xi_w = \Delta V_w/V = \Delta h_w/H \quad (14)$$

where ΔV_w = total volume of water intake or discharge during freezing,
 $\Delta h_w = \Delta V_w/A$ (A is the cross-sectional area of specimen).

(3) Frost penetration rate, U :

Provided that unfrozen soil of ΔH freezes during unit time, Δt :

$$U = \Delta H/ \Delta t \quad (15)$$

On the other hand, the thickening rate of the frozen soil, U_f , is:

$$U_f = (\Delta H+ \Delta h)/ \Delta t = U + v_h \quad (16)$$

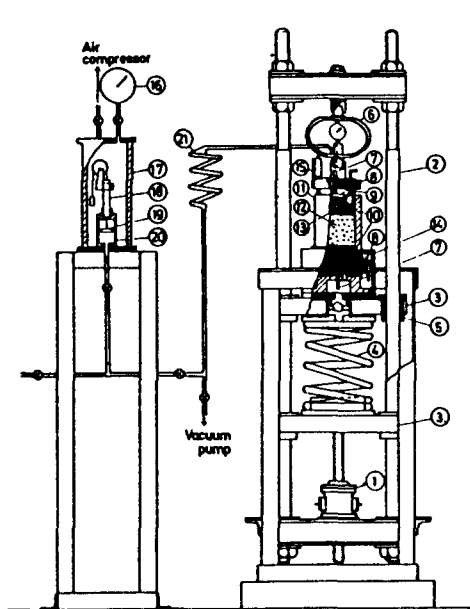
where v_h is the heave rate.

Although U and U_f have been frequently used without distinction, in this paper U defined by equation 15 is always used as the penetration rate or freezing rate.

3.2 Experimental Procedure

Apparatus. The experimental apparatus for determining the factors of group (B) which control frost heaving, must allow the overburden pressure, the penetration rate and the pore water pressure to take given values, and must also satisfy conditions which offer an open system to water migration.

Figure 7 shows the experimental apparatus which satisfies these conditions. The authors have conducted a series of experiments on frost heaving in a saturated soil using this apparatus. A soil specimen was contained in a transparent acrylite cylinder of 4~10 cm inside diameter. The lower end was fixed on a cooling plate (negative temperature side) and the upper end was in contact with a porous plate mounted on a piston (positive temperature side), movable in the cylinder. Thermoelectric cooling devices (Peltier batteries) mounted on the cooling plate and the piston, allowed the temperature of each end plate to be readily controlled. Water intake to or discharge from the specimen was measured by the change in the water level in the water-supply container connected to the piston through an access pipe. Total pressure was applied using a spring. Pore water pressure was applied using air pressure in a pressure tank. A recorder or computer was used to record the temperature at each end plate, the amount of frost heave and the amount of water intake or discharge.



- 1 Jack
- 2 Support
- 3 Guide arm
- 4 Spring
- 5 Slide bearing
- 6 Proving ring
- 7 Thermister conduit
- 8 Thermo electric module
- 9 Piston
- 10 O-ring
- 11 Cylinder
- 12 Specimen
- 13 Porous plate
- 14 Cooling plate
- 15 Differential transformer
- 16 Bourdon's gauge
- 17 Pressure container
- 18 Differential transformer
- 19 Float
- 20 Water container
- 21 Cu-pipe

Fig. 7 Schematic of test apparatus.

Control and confirmation of penetration rate. Frost heaving depends on the penetration rate, as will be described later. When the temperatures of the cooling plate and piston are constant but at different temperatures, the frost penetration rate dx/dt is obtained from equation 9:

$$dx/dt = \alpha / 2\sqrt{t} \quad (17)$$

In this case dx/dt changes with time. In an experiment studying the relation between frost heave ratio and penetration rate, it is desirable to make the penetration rate constant, since making the temperature at each end plate constant is not suitable for this experiment.

It was possible to make the penetration rate constant, upon analysis of the temperature conditions for this purpose, by changing θ_1 and θ_2 with the lapse of time according to the following equation, where θ_1 is the temperature at one end of the sample in contact with the cooling plate and θ_2 is the temperature at the other end of it (Takashi and Masuda, 1975):

$$\theta_1 = \frac{\kappa_1}{\lambda_1} \left(\frac{\lambda_2 U + v}{\kappa_2 U + v_h} \theta_\infty + \gamma_w L_w \frac{n_f U + v_w}{U + v_h} \right) \left[1 - \exp\left\{ -\frac{(U + v_h)^2}{\kappa_1} t \right\} \right] \quad (18)$$

$$\theta_2 = \theta_\infty \left[1 - \exp\left\{ -\frac{U + v}{\kappa_2} (H - Ut) \right\} \right] \quad (19)$$

where U:penetration rate,
 v_h :heave rate,
 v_w :water intake or discharge rate,
 λ :thermal conductivity,
 κ :thermal diffusivity,
 θ_∞ :initial temperature of soil,
H:specimen height,

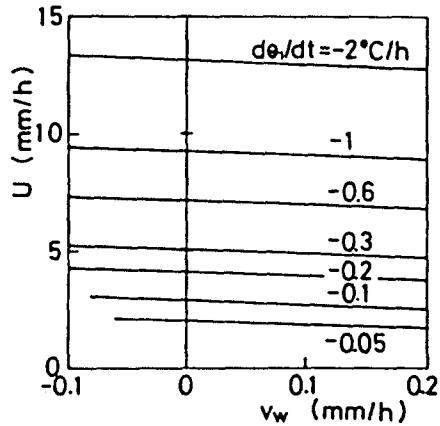


FIGURE 8. Penetration rate U as a function of water intake or discharge rate v_w .

$$v = (c_w \gamma_w / c_s \gamma_s) v_w, \text{ where } c = \text{specific heat and } \gamma = \text{unit weight} \\ \text{suffixes 1 and 2: frozen and unfrozen soil, respectively} \\ \text{suffixes w and s: pore water and saturated soil, respectively}$$

In the experiment, the value of U and H were made small and the temperature of the soil before freezing was adjusted to become equal to the freezing temperature of soil water; then $\theta_2 = 0$. As a result, equations 18 and 19 become approximately as follows:

$$\theta_1 = - \frac{\gamma_w L_w}{\lambda_1} (n_f U + v_w) (U + v_h) t \quad (20)$$

$$\theta_2 = 0 \quad (21)$$

Figure 8 shows the relation between v_w and U . Since the effect of U on v_w is relatively small and it is difficult to predict v_w before the experiment, we can control the temperature practically according to the following equation, ignoring v_w and v_h :

$$\theta_1 = - \frac{\gamma_w L_w n_f}{\lambda_1} U^2 t \quad (22)$$

Specimen's height in unidirectional freezing test of soil. Takashi et al. (1976) analyzed the effect on ξ of resistance in soil water migration through the unfrozen part within a freezing specimen, and found that the frost heave ratio, ξ , is a function of the specimen's height, H

On the other hand, it has recently been considered that ice segregation occurs within a certain limited zone in frozen soil (Dirksen and Miller, 1966; Hoekstra, 1966; Miller, 1972; Radd and Oertle, 1973; Loch and Kay, 1978; Takashi et al., 1979; Fukuda et al., 1980; Penner and Goodlich, 1980). The author's view on the position of the zone where ice segregation occurs is shown schematically in Figure 9, where θ_f is the freezing point of bulk water, θ_{init} is the temperature at which ice segregation is initiated and θ_{crit} is the critical temperature at which ice segregation is completed. The plane $\theta = \theta_f$ is defined as a freezing front. The zone $\theta_{init} < \theta < \theta_f$ is an "in situ water freezing zone" or "frozen

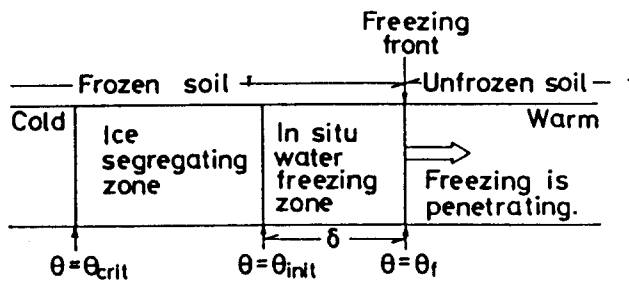


FIGURE 9. Schematic representation of unidirectional freezing of soil.

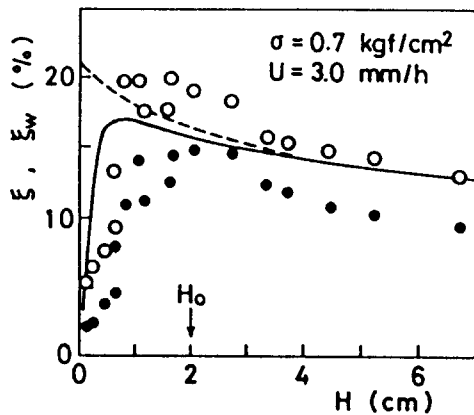


FIGURE 10. Dependence of frost heave ratio ξ and water intake or discharge ratio ξ_w and specimen height H (Manaitabashi clay).

fringe" defined by Miller (1972). According to this view point, the specimen's height has some influence on the frost heave ratio.

From the experiment using various heights of a specimen, Takashi et al. (1982) determined the relation between ξ , ξ_w and H , as shown in Figure 10. Here ξ reaches its maximum at a certain value H_0 of the specimen's height H . When $H > H_0$, ξ decreases gradually with increasing H . Meanwhile, ξ decreases rapidly with decreasing H when $H < H_0$. When $H > H_0$, it is assumed that the effect on ξ of resistance in soil water migration through the unfrozen soil appears as H becomes smaller than H_0 ; the reason for the decrease in ξ may be explained qualitatively from the existence of a frozen fringe.

From the experiment concerning various values of σ and U , it was found that the value of H_0 increased with increasing σ and with decreasing U . It is suggested from the experiment that the optimum H in laboratory frost heave tests is near H_0 . At this point both the resistance in soil water movement through the unfrozen soil and the existence of an in-situ water freezing zone within the frozen soil have the minimum compound effect on ξ .

3.3 Effect of overburden pressure on frost heave ratio.

It is not unusual that the frost heave ratio, ξ , exceeds 100 % in

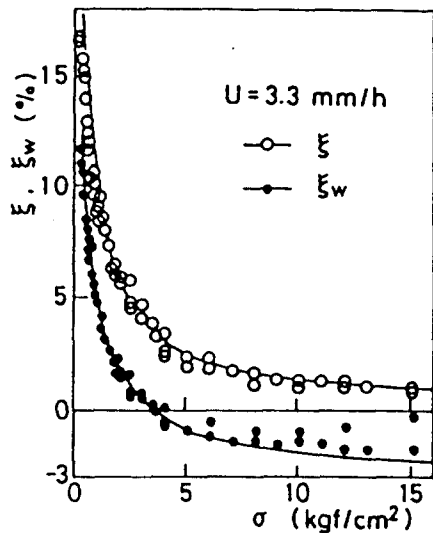


FIGURE 11. Dependence of frost heave ratio ξ and water intake or discharge ratio ξ_w on stress σ (Negishi silt).

natural freezing when the ground freezes from its surface downward. It is unusual, however, that ξ exceeds 10 % in artificial freezing applied to a fairly deep location underground. A reason that ξ differs between natural and artificial freezing is supposed to be caused by a difference in confining stress, σ .

The relation between frost heave ratio, ξ , water intake or discharge ratio, ξ_w , and confining stress, σ , for a Negishi silt are shown in Figure 11 (Takashi et al., 1978). Values of ξ and ξ_w decrease with increasing σ and are in inverse proportion to σ , as shown by the following relation:

$$\xi = \xi_0 + \frac{C}{\sigma} \quad (23)$$

where ξ_0 and C are the constants; ξ_0 may be considered to represent the amount of expansion caused by the gradual freezing of unfrozen water in the frozen soil; C depends on the penetration rate, as will be mentioned later. Accordingly, it is of interest to note that with increasing σ the nature of water migration changes from water intake to water discharge.

As seen from Figure 11, when σ is smaller than 1 kgf/cm², both ξ and ξ_w have somewhat smaller values than those obtained from equation 23. It has become almost obvious, from the analysis and experiment, that such phenomena are due to an increase in the effective stress caused by a drop in pore water pressure in unfrozen soil as a result of intense water intake occurring at the freezing front (Takashi et al, 1976).

3.4 Effect of Penetration Rate on Frost Heave Ratio

Taber (1929, 1930) suggested qualitatively that with a decrease in frost penetration rate, U , the frost heave ratio increases. Since his work, many researchers have pursued this subject, arriving at varying

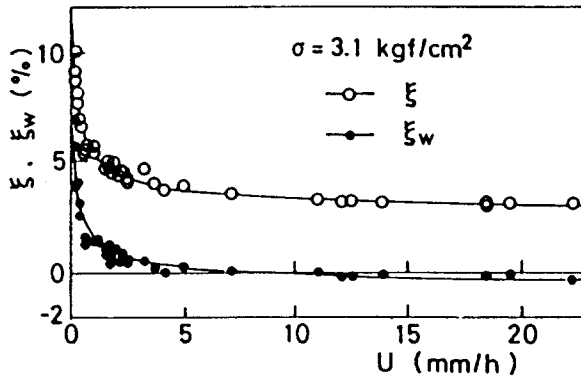


FIGURE 12. Dependence of frost heave ratio ξ and water intake or discharge ratio ξ_w on penetration rate U (Negishi silt).

conclusions, and in extreme cases, contradictory results.

The authors conducted an experiment to systematically investigate the effect of U on ξ , keeping U constant during the freezing of a specimen (Takashi et al., 1979). Figure 12 represents the relation between ξ , ξ_w and U for $\sigma=3.1 \text{ kgf/cm}^2$ using a Negishi silt. Values of ξ and ξ_w decrease with increasing U . Especially, ξ_w changes from the water intake type to the water discharge type with increasing U . From this experiment, the following empirical equation was obtained:

$$\xi = A + \frac{B}{\sqrt{U}} \quad (24)$$

where A and B are the constants. The solid line in Figure 12 represents equation 24, which coincides with experimental results for a wide range of U .

When tests were conducted by changing U , it was found that ξ is inversely proportional to σ , but there was a difference in the value of C . It is predicted that C depends upon U , that is:

$$\sigma(\xi - \xi_0) = C = f(U) \quad (25)$$

All data of Figures 11 and 12 expressed by plotting $1/\sqrt{U}$ against $\sigma(\xi - \xi_0)$, as shown in Figure 13, lead to a proportional relation between them. Therefore, we obtain:

$$\sigma(\xi - \xi_0) = C_1 - C_2/\sqrt{U} \quad (26)$$

where C_1 and C_2 are the constants dependent upon a soil type, but not on σ and U . Arranging the equation by putting $C_1 = \sigma_0$ and $C_2 = \sigma_0\sqrt{U_0}$, equation 26 is rewritten as:

$$\xi = \xi_0 + \frac{\sigma_0}{\sigma} \left(1 + \sqrt{\frac{U_0}{U}} \right) \quad (27)$$

where ξ_0 , σ_0 and U_0 are the constants. The values of the soils used in the authors' experiments are given in Table 1.

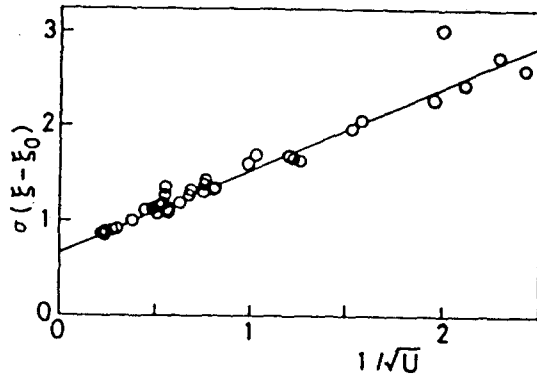


FIGURE 13. Relation between $\sigma(\xi - \xi_0)$ and $1/\sqrt{U}$.

TABLE 1. Properties of specimen.

	Nanao silt	Negishi silt	Manaitabashi clay
soil fraction			
sand (%)	28	27	0
silt (%)	65	50	33.5
clay (%)	7	23	66.5
frost heaving			
ξ_0 (-)	0.0055	0.003	0.01
σ_0 (kgf/cm ²)	0.0255	0.0667	0.0179
U_0 (mm/h)	2.941	1.71	137.02
n_f (-)	0.372	0.348	0.167
consolidation			
k (cm/s)	4×10^{-8}	3.3×10^{-8}	7×10^{-8}
m_v (cm ² /kgf)	3.3×10^{-3}	4×10^{-3}	13×10^{-3}
c_v (cm ² /s)	12.1×10^{-3}	8.25×10^{-3}	5.38×10^{-3}

3.5 Data Deduced from the Experiment for Stress and Penetration Rate

Relation between frost heave ratio and water intake or discharge ratio.

As mentioned above, soil has a tendency to heave during the freezing process, not only when it takes in water, but also when it discharges water, corresponding to the level of U and σ . For obtaining a certain relation between ξ and ξ_w , all points of Figures 11 and 12 were plotted against ξ and ξ_w , as shown in Figure 14. In this figure, all the points lie on a straight line having a gradient of $1+\Gamma$ (Γ is the volume expansion ratio of about 0.09 for water when its phase changes to ice). When $\xi_w=0$, the value of ξ results in the freeze expansion of the part of the soil water that became frozen and is given by the following, based on the results of the analysis:

$$\xi = \xi_0 + n_f \Gamma \quad (28)$$

where ξ_0 is the freeze expansion ratio of unfrozen water at a certain

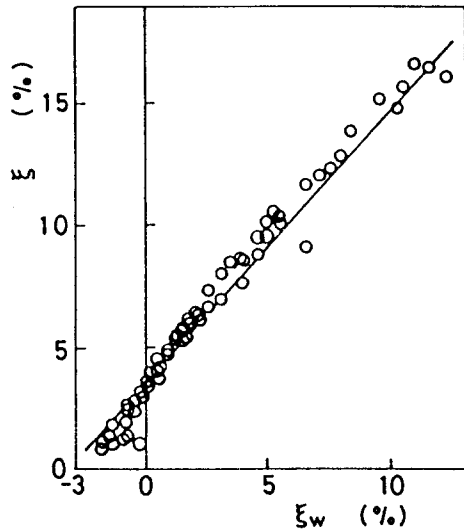


FIGURE 14. Relation between frost heave ratio ξ and water intake or discharge ratio ξ_w .

negative temperature, and n_f is the volumetric free water content which freezes at the freeze temperature. From these relations, ξ is equal to

$$\xi = \xi_0 + n_f \Gamma + (1 + \Gamma) \xi_w \quad (29)$$

Equation 29 shows the case of a soil saturated with water that ξ is the sum of ξ_w , the freeze expansion ratio of ξ_w , and the freeze expansion ratio of the water filling pores between soil particles which become frozen at the lowest temperature during the experiment. The principle of conservation of volume holds true for the case of saturated soil freezing. The maximum water discharge ratio, $\xi_w \text{ max}$, is obtained as follows when $\xi=0$:

$$\xi_w \text{ max} = - \frac{\xi_0 + n_f \Gamma}{1 + \Gamma} < n \frac{\Gamma}{1 + \Gamma} \quad (30)$$

where n is the porosity of the soil. When the soil consisting of sand or gravel freezes, ξ_w becomes around $\xi_w \text{ max}$.

We obtained the relation between ξ , σ and U from equations 27 and 29 as follows:

$$\xi_w = \frac{1}{1 + \Gamma} \frac{\sigma_0}{\sigma} \left(1 + \sqrt{\frac{U_0}{U}}\right) - n_f \frac{\Gamma}{1 + \Gamma} \quad (31)$$

Calculated results are shown in Figures 11 and 12. These results coincide well with the experimental results.

Effect of σ and U on heave rate and water intake or discharge rate. When dh denotes the heave amount during a lapse of time dt , then the heave rate is given by dh/dt . Meanwhile, dX denotes the distance that the freezing front advanced during dt , then the frost penetration rate is given by dX/dt . Accordingly, the following equation is derived from the causal fact that dh arises from the penetration of dX in time dt ,

together with the condition, maintained at constant during the experiment:

$$\frac{dh}{dt} = \frac{dh}{dX} \frac{dX}{dt} = \frac{dh}{dX} U = \zeta U \quad (32)$$

Substituting equation 27 into ζ of equation 32, we obtain the heave rate under σ and U :

$$\frac{dh}{dt} = \left(\zeta_0 + \frac{\sigma_0}{\sigma} \right) U + \frac{\sigma_0}{\sigma} \sqrt{U_0 U} \quad (33)$$

In the same way, we obtain the water intake or discharge rate, dw/dt , under σ and U :

$$\frac{dw}{dt} = \frac{U}{1 + \Gamma} \frac{\sigma_0}{\sigma} \left(1 + \sqrt{\frac{U_0}{U}} \right) - n_f \frac{\Gamma}{1 + \Gamma} U \quad (34)$$

Figures 15 and 16 show dh/dt and dw/dt , respectively. The heave rate, dh/dt , increases monotonically with increasing U when σ is constant. The water intake or discharge rate, dw/dt , increases monotonically with increasing U when σ is smaller than the critical value of σ , σ_c . When $\sigma > \sigma_c$, however, dw/dt has a peak. In order to examine the behaviour of dw/dt , we differentiate dw/dt as to U and equate this to zero:

$$\frac{d}{dU} \left(\frac{dw}{dt} \right) = \frac{1}{1 + \Gamma} \frac{\sigma_0}{\sigma} \left(1 + \frac{1}{2} \sqrt{\frac{U_0}{U}} \right) - n_f \frac{\Gamma}{1 + \Gamma} = 0 \quad (35)$$

Solving this equation with regard to U we obtain:

$$\sqrt{U_1} = \frac{1}{2} \frac{U_0 \sigma_0}{n_f \Gamma \sigma - \sigma_0} \quad (36)$$

Here U_1 is the penetration rate which represents the maximum value of dw/dt , dw/dt_{\max} , as shown in Figure 16 when σ is constant. Further, σ_c is obtained from equation 36 as:

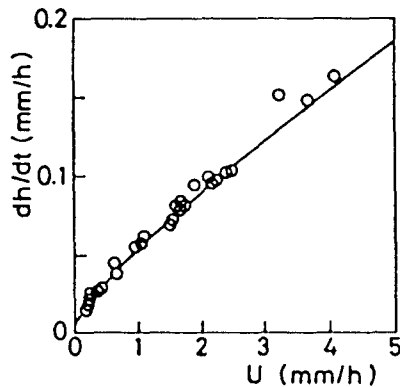


Figure 15. Heave rate dh/dt vs. penetration rate U .

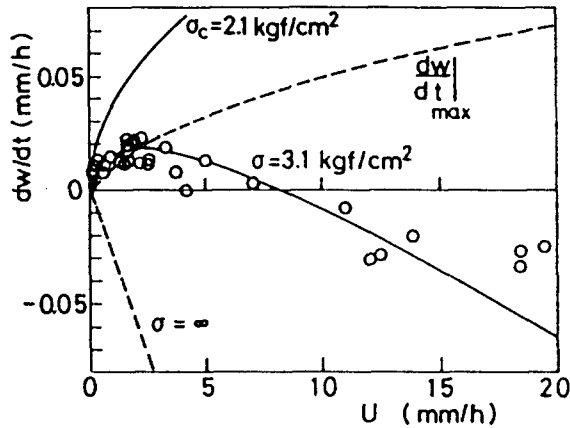


FIGURE 16. Water intake or discharge rate dw/dt penetration rate U .

$$\sigma_c = \frac{\sigma_0}{n_f \Gamma} \quad (37)$$

The following relation between the temperature gradient, α , of the frozen soil and U , the penetration rate, is kept constant by maintaining a constant falling rate of temperature at the cooling plate:

$$\alpha \propto U \quad (38)$$

Rearranging equation 34, we have:

$$\begin{aligned} \frac{dw}{dt} &= \frac{1}{1 + \Gamma} \left(\frac{\sigma_0}{\sigma} - n_f \Gamma \right) U + \frac{\sigma_0 \sqrt{U_0}}{\sigma(1 + \Gamma)} \sqrt{U} \\ &= AU + B\sqrt{U} \end{aligned} \quad (34')$$

Since dw/dt becomes the sum of the terms proportional to U and \sqrt{U} , we have the following relation between dw/dt and α :

$$\frac{dw}{dt} = A' \alpha + B' \sqrt{\alpha} \quad (39)$$

where A' and B' are the constants which depend on σ . The sign of A' may become negative when σ increases, but B' is always positive.

It is problematical whether or not the equations for ξ_w and dw/dt mentioned above can be applied to the case that $U \rightarrow 0$. In equation 31

$$\xi_w \rightarrow \infty \quad \text{when } U \rightarrow 0$$

On the other hand, from equation 34

$$dw/dt \rightarrow 0 \quad \text{when } U \rightarrow 0$$

These relations are incompatible. When $U \rightarrow 0$, frost heaving continues, eventually growing into an ice lens; that is, $dw/dt \neq 0$ and $\xi_w \rightarrow \infty$, since the freezing front does not penetrate into the unfrozen soil. Therefore,

it should be noted that the empirical formulas obtained in this paper must be applied for cases where the freezing front always propagates into the unfrozen soil, and may not be applied to the critical case when $U \rightarrow 0$.

The case where ice lenses form in the final stage as $U \rightarrow 0$ will be described later since this is related to the long-term freezing property of soil.

3.6 Effect of Pore Water Pressure on Frost Heave Ratio

In a saturated soil, the "effective stress" is defined as being equal to the total pressure, P , minus the pore water pressure, P_w :

$$\sigma = P - P_w \quad (40)$$

The mechanical behaviour of the soil is correlated with the effective stress instead of either total pressure or pore water pressure.

In the author's experiment, the pore water pressure is approximately equal to the atmospheric pressure. Therefore, the effective stress is always equal to the total pressure, that is:

$$\sigma = P$$

On the other hand, the pore water pressure of a soil to be frozen by artificial ground freezing is usually larger than the atmospheric pressure. This presents a problem as to whether or not the obtained results described in this paper can be applied to the case of $P_w \neq 0$.

The effect of the effective stress on the frost heave was examined by changing the combination of the total pressure and pore water pressure while maintaining the effective stress at constant, as shown in Figure 17. Experimental results, shown in Figure 18, suggest that the frost heave ratio, ζ , changed little or not at all when σ was constant, in spite of an increase in total pressure and pore water pressure. In particular, when $\sigma = 2 \text{ kgf/cm}^2$ little change occurred in ζ when P_w was less than the atmospheric pressure. It is clear then that ζ was dependent on σ but not on P or P_w when the penetration rate was kept constant.

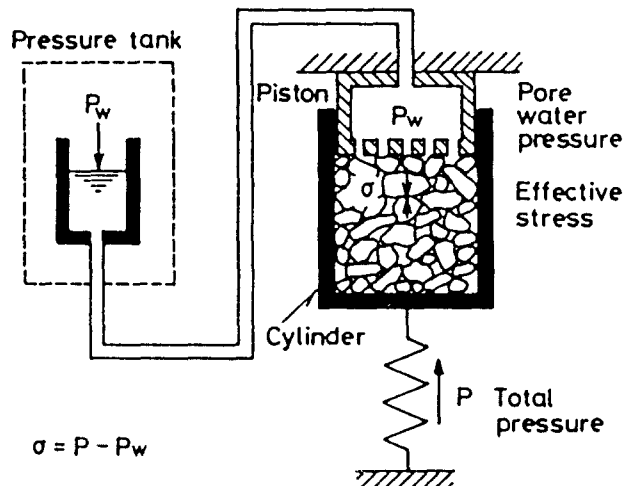


FIGURE 17. Schematic explanation of the pressure equilibrium in soil.

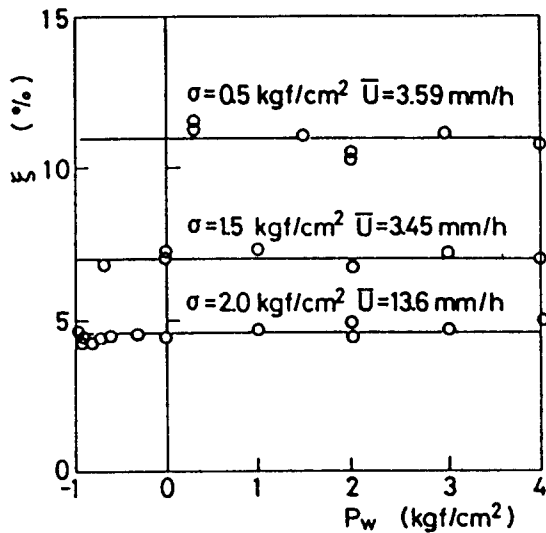


FIGURE 18. Dependence of frost heave ratio ξ on pore water pressure P_w (Negishi silt).

Therefore, equations given in this paper can be expressed by using the effective stress.

3.7 Maximum Heaving Pressure

Since frost heaving pressure causes damage to structures in cold regions, and in the vicinity of a site in a warm area to which artificial ground freezing has been applied, the understanding of its mechanism has been treated as an important subject of experimental and theoretical study. Experimental methods of studying the frost heaving phenomenon are divided into the following two groups (Sutherland and Gaskin, 1973):

1) Measurement of heaving pressure

A soil specimen is partially frozen in system open with respect to pore water so that the pore water pressure is atmospheric, not allowing frost heaving in order that the maximum heaving pressure developed can be observed.

2) Measurement of a drop in water pressure

A soil specimen is partially frozen by maintaining the overburden pressure at constant in a system closed with respect to pore water. By measuring a drop in pressure of pore water resulting from the freezing of the specimen, we can obtain water intake pressure (frost heaving pressure). Then, the maximum heaving pressure can be determined as the maximum effective stress as follows:

$$\sigma_{\max} = P - P_w$$

Measurement 1) can be considered to be a direct method, however, it has the disadvantage of causing the possibility of an undesirable influence on the result of the measurement. While stress increases, the perfect confinement of the specimen becomes difficult as a result of the consolidation of the specimen and the occurrence of a strain necessary for the detection of pressure. Meanwhile, although measurement 2) is an indirect method, it minimizes the shortcoming of measurement 1).

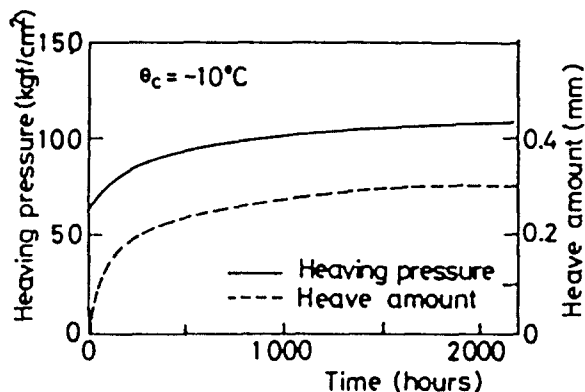


FIGURE 19. Heaving pressure and frost heave amount with time for Manaitabashi clay.

The authors conducted an experiment according to measurement 2) using four soils differing in frost-susceptibility. Typical curves of heaving pressure developed over a lapse of time, indicating that an equilibrium state was attained when P_w fell very slowly, as shown in Figure 19. It seems that the small quantity of water required to bring about a change in P_w migrated into the frozen part from the measuring system. Consequently, it became possible for the frozen part to attract water, even when its stress exceeded 100 kgf/cm^2 ; and frost heaving occurred, corresponding to the amount of water taken in. In fact, the authors were able to observe that an ice lens grew in contact with the cooling plate while the specimen was partially frozen. Since a freezing isotherm had to come to rest at the midway point of the specimen, it appears that the unfrozen water was allowed to migrate toward the ice front passing through the frozen soil. From this it was assumed that the frozen water continues to remain and builds up a network of veins in the frozen part, which combines the ice front with pore water in the unfrozen part, under a temperature gradient imposed on the partially frozen soil. When a change in σ ceased, the growth of the ice lens due to water intake also ceased, i.e. frost heaving was restrained. We define σ at this stage as the "upper limit of heaving pressure, σ_u ", under the temperature condition. Figure 20 shows σ_u vs. the temperature of the cooling plate, θ_c . Until a specific value of θ_c is reached depending on the soil type, σ_u increases linearly with decreasing θ_c . An empirical formula was then obtained as follows:

$$\sigma_u = -11.4 \theta_c \quad [\text{kgf/cm}^2] \quad (41)$$

which coincides with the result obtained by Radd and Oertle (1973). The relation between σ_u and θ_c expressed by equation 41 coincides with the generalized Clausius-Clapeyron equation from the analysis using thermodynamics as follows:

$$P = -\frac{L_w}{V_i} \frac{\Delta T}{T} = -11.4 \Delta T \quad [\text{kgf/cm}^2] \quad (42)$$

where L_w is the latent heat of water, V_i is the specific volume of ice, T is the Kelvin temperature and ΔT is the freezing point depression. Although σ_u depends linearly on θ_c , as shown in equation 41, it is difficult to believe that this relation holds unlimitedly with decreasing

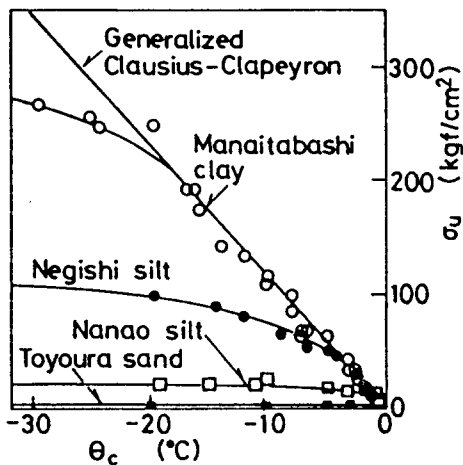


FIGURE 20. Dependence of upper limit of heaving pressure σ_u on temperature of cooling plate θ_c .

θ_c . There is a phenomenon that if $\theta_c < -25^\circ\text{C}$, then the σ_u obtained deviates from a straight line toward the lower side, as in case for a Manaitabashi clay, as shown in Figure 20 (Takashi et al. 1981a; 1981b). This tendency becomes more pronounced for a Negishi silt. If $\theta_c < 4^\circ\text{C}$, then σ_u deviates from the straight line and it appears that σ_u converges to its maximum value with decreasing θ_c . We will call it "maximum heaving pressure, $\sigma_u \text{ max}$ " that is:

$$\lim_{\theta_c \rightarrow \infty} \sigma_u = \sigma_u \text{ max}$$

It is assumed that $\sigma_u \text{ max}$ is one of the constants which determines the frost susceptibility of soil.

3.8 Long-term Frost Heaving in Partially Frozen Soil

In large-scale artificial ground freezing work and in the freezing of ground surrounding an underground LNG storage tank, it has been observed that the ground continues to freeze for a very long time and the temperature profile in the ground is maintained at almost constant. Takashi et al. (1981a) have shown experimentally that frost heaving continues for a fairly long time, even though the temperature profile attains an equilibrium state. It is important then to estimate how long the frost heaving will continue and how much it will amount to eventually.

The behaviour of ice lenses was observed in an experiment on long-term frost heaving for which a constant thermal regime was maintained in an open system, whereby the total heave, and the location and thickness of the ice lenses were measured for 6044 hours (Ohrai and Yamamoto, 1985). The temperatures at each end were -8°C and 4°C , respectively. No overburden pressure was applied. Figure 21 shows the elapsed change in frost heave amount, h , and water intake amount, w . Although h and w increased during the partial freezing, their rates decreased with time. It was observed that several ice lenses grew simultaneously in the frozen part of the specimen. The position and thickness of each ice lens and the height of the specimen vs. time are shown in Figure 22. A serial number

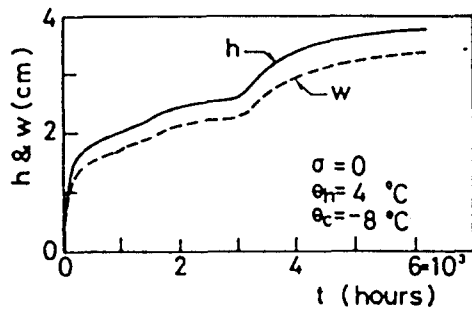


FIGURE 21. Total frost heave h and total water intake w with time for Manaitabashi clay.

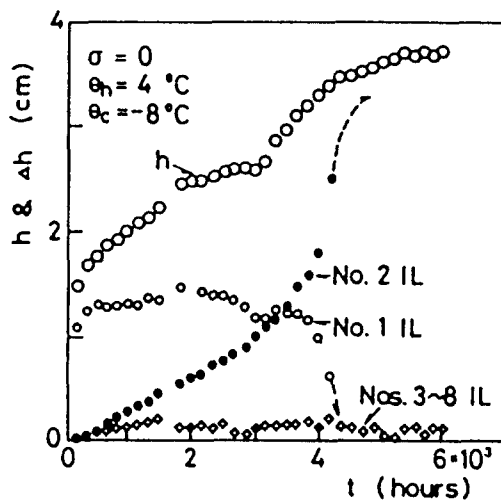


FIGURE 22. Total frost heave h ; and increment Δh of thickness of ice lens with time t for the first (No. 1 IL), second (No. 2 IL) and remaining ice lenses (Nos. 3~8).

was assigned to each ice lens beginning with the one nearest to the freezing front. Ice lenses Nos. 3 ~ 8 grew little because the cumulative h was 0.1 ~ 0.2 mm. However, ice lenses Nos. 1 and 2 displayed an interesting behaviour as follows: ice lens No. 1 attained a thickness of 13 mm after 200 hours after which no significant increase was noted. Meanwhile, after 20 hours ice lens No. 2 grew at a constant heave rate. After about 3000 hours passed, the thickness of ice lens No. 1 began to decrease as ice lens No. 2 grew; and disappeared after about 4000 hours.

When ice lens No. 1 grows to form a layer, water migrating through unfrozen soil should segregate on the higher temperature side of the ice lens, since water cannot migrate through an ice layer (Horiguchi and Miller, 1980). However, the thickness of ice lens No. 1 did not increase corresponding to the amount of water taken in, but decreased conversely. Following explanation for this result was considered: When ice lens No. 1 grows and becomes a layer, water through the unfrozen soil segregates on its higher temperature side. However, melting takes place at the bottom lower temperature side of ice lens No. 1, so that the quantity of melted water becomes equal to that of the frozen water which is necessary for ice lens No. 2 to grow. The melted water migrates through the frozen soil

between ice lenses Nos. 1 and 2, and then segregates at the top higher temperature side of ice lens No. 1. As a result, it appears as if water through the unfrozen soil passed the ice lenses. Considering ice lenses Nos. 1 and 2, the frozen soil between them may be likened to a wire in "wire regelation". The phenomenon where the frozen soil migrates through the ice lens may be regarded as "regelation" in frozen soil. The estimate based on the rate of regelation showed that the hydraulic conductivity of the frozen soil ranged from 2×10^{-11} to 1.5×10^{-12} cm/s at temperatures ranging from -1.5 to -2.2°C .

From this experiment, frost heaving accompanied by water intake occurred, and the frost heave amount did not converge to a constant value during the experiment, even though the temperature profile of the specimen attained an equilibrium state.

4. PREDICTION OF FROST HEAVING AT FREEZING SITE

The influence of frost heaving on the freezing of soil were examined in relation to artificial soil freezing and the preparation of a site for a shield machine from a shaft, as shown in Figure 23.

When the ground to be frozen consists of silty soil and/or clay soil, the soil may heave as water is taken in. The resistance against water migration is relatively small if the dimensions of the frozen soil are of the same magnitude as the lengths of specimens used in laboratory tests. However, at a construction site where the stratum exceeds several meters, the frost heave ratio is always smaller than the one obtained from the laboratory tests since the resistance against water migration is large and the supply of water is reduced.

Moreover, when the ground is soft and weak, a drop in pore water pressure as a result of the intake of water causes an increase in effective stress, leading to the discharge of water from the surrounding ground and the resultant consolidation of it. On the other hand, when the ground is

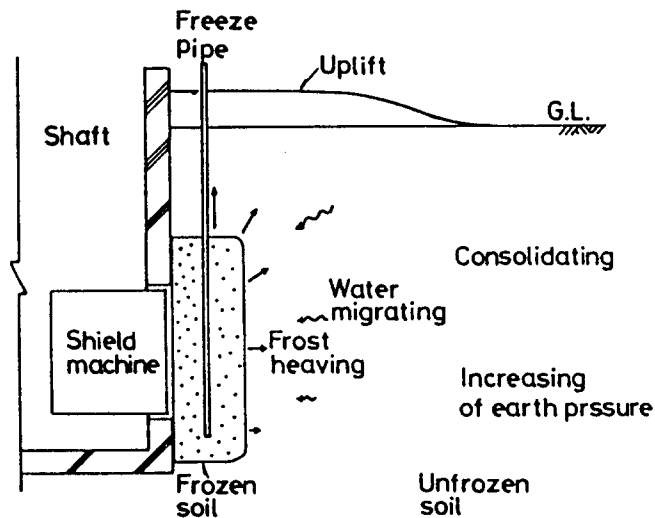


FIGURE 23. Schematic of phenomena resulting from frost heaving in ground.

densely compact as in the case of diluvial soil, soil pressure is increased by frost heaving.

As a consequence of these compound effects, the heave of the ground surface takes place with the unfrozen soil as a medium.

Discussed in this section are the "influence of permeability of unfrozen soil on frost heave", the "consolidation of unfrozen soil during freezing", the "change in earth pressure due to freezing" and the "uplift of the ground surface". A countermeasure against frost heaving during artificial freezing is briefly mentioned.

4.1 Influence of Permeability of Unfrozen Soil on Frost Heave

Expressing all data of Figure 11 by plotting $1/\sigma$, we obtain Figure 24.

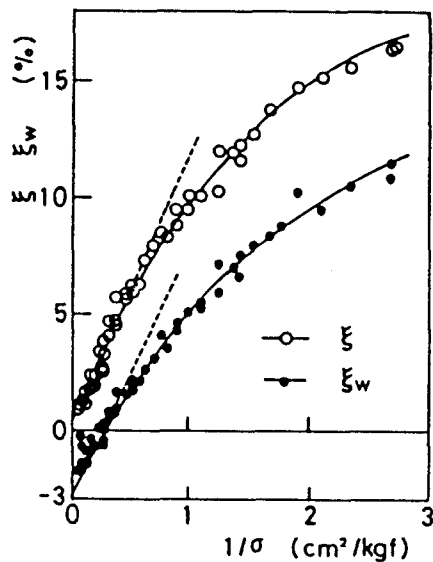


FIGURE 24. Relation between ξ , ξ_w and $1/\sigma$.

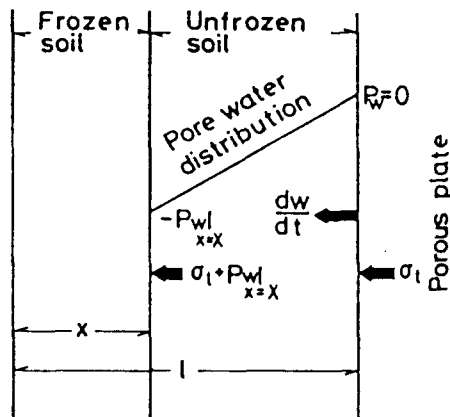


FIGURE 25. Freezing model when steady state is attained concerning water intake in unfrozen soil.

The experimental data is in good agreement with equation 27 for a wide range of large stresses, but does not provide good agreement for the range of small stresses ($\sigma < 1 \text{ kgf/cm}^2$). The authors attribute this behaviour to the increment of effective stress arising from the resistance of the water movement drawn to the freezing front from the unfrozen region.

When water is taken in during the freezing of saturated soil, the pore water pressure decreases and then the effective stress increases, as shown in Figure 25. Considering that the unfrozen soil zone cannot consolidate, the pore water pressure gradient becomes linear. The following equation was obtained on the basis of equations 27 and 31 by Takashi et al. (1976) as a result of an analysis of the effect on ξ of resistance in soil-water movement through the unfrozen part within a freezing soil specimen:

$$\xi = \xi_0 + n_f \Gamma + \frac{(1 + \Gamma)}{2U} \left\{ \sqrt{(K\sigma - B)^2 + 4AK} - K\sigma - B \right. \\ \left. + \frac{K(2A - B\sigma)}{B} \log \frac{B\sqrt{(K\sigma - B)^2 + 4AK} + B^2 + K(2A - B\sigma)}{2AK} \right. \\ \left. - K\sigma \log \frac{\sqrt{(K\sigma - B)^2 + 4AK} + (2A - B\sigma)/\sigma + K\sigma}{2K\sigma} \right\} \quad (43)$$

where

$$A = \frac{U}{1 + \Gamma} \sigma_0 \left(1 + \sqrt{\frac{U_0}{U}} \right) \quad (44)$$

$$B = n_f \frac{\Gamma}{1 + \Gamma} U \quad (45)$$

$$K = k / \gamma_w l \quad (46)$$

The calculated result is shown in Figure 24 by solid lines. Theoretically the difference between the experimental results and equations 27 and 31 were due to the effect on ξ of the resistance in soil water movement. The value of K in equation 43, which is expressed by equation 46, is the ratio of permeability to the specimens length. When the value of K is the same, the frost heave ratio is also the same. That is, for frost heaving, increasing the specimen's height 10 times is equivalent to increasing its permeability by one-tenth. Therefore, frost heaving due to the water intake can be reduced if it is possible to increase the permeability of the soil near the freezing front by some method, when the soil to be frozen is near the aquifer which supplies water to the freezing front passing through the unfrozen soil.

4.2 Consolidation of Unfrozen Soil During Freezing

Practical cases abound where it is necessary to freeze a silt or clay that is soft, weak and low in permeability. Shown in Figure 26 is a water content profile near the freezing front at a site subjected to artificial freezing. The water content decreases toward the freezing front in the

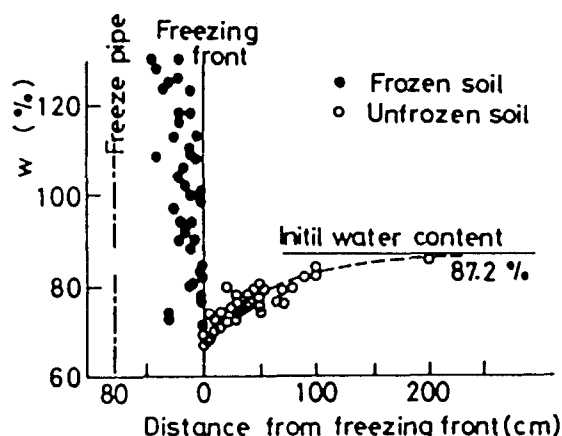


FIGURE 26. Distribution of water content, w , near the freezing front in an artificial frozen silty ground.

unfrozen soil because the consolidation of the soil occurs due to an increase in effective stress. However, the water content increases in the frozen soil before freezing when water is taken in. That is, from a macroscopic point of view, using the freezing front as a boundary, the soil expands on the side of the frozen soil and shrinks on the side of the unfrozen soil; then, if viewed from the side of the frozen soil, the result of these compound effects occurs on the side of the unfrozen soil.

Takashi et al. (1977) developed this equation theoretically. The pore water pressure was made to follow Terzaghi's consolidation equation within the unfrozen soil; and equation 34 was used to express the water intake rate at the freezing front, with σ as the effective stress at the freezing front. The following differential equation was obtained:

$$\frac{\partial P_w(x,t)}{\partial t} = c_v \frac{\partial^2 P_w(x,t)}{\partial x^2} \quad (47)$$

where P_w is the pore water pressure and c_v is the coefficient of consolidation:

$$c_v = \frac{k}{\gamma_w m_v} \quad (48)$$

where k and m_v are the permeability and coefficient of the volume change of unfrozen soil, respectively. It is difficult to obtain an analytical solution from this equation, since the question is concerned with a boundary condition of nonstationary movement. We will demonstrate a solution for the case in which the phenomenon becomes stationary after a lapse of sufficient time from the beginning of freezing. The pore water pressure in the stationary state in the unfrozen soil is given by:

$$P_w = \frac{(1 + \Gamma)m_v \sigma + n_f \Gamma - \sqrt{\{(1 + \Gamma)m_v \sigma - n_f \Gamma\}^2 + 4(1 + \Gamma)m_v \sigma_0 (1 + \sqrt{U_0/U})}}{2(1 + \Gamma)m_v} e^{-U \zeta / c_v} \quad (49)$$

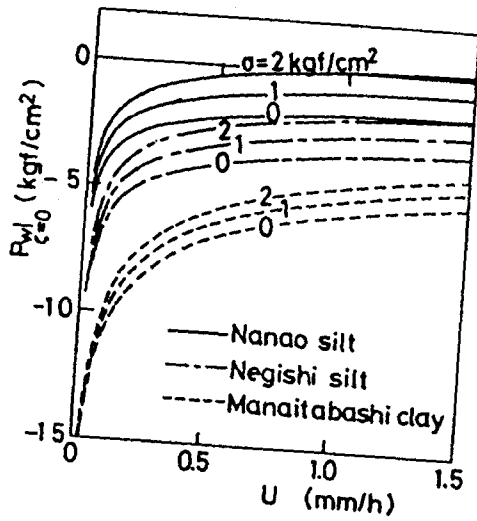


FIGURE 27. Pore water depletion $P_w|_{\xi=0}$ at freezing front as a function of penetration rate U .

where ξ is the distance from the freezing front. Shown in Figure 27 is the relation between pore water pressure at the freezing front, $P_w|_{\xi=0}$, and the frost penetration rate under the effective stress $\sigma=0, 1$ and 2 kgf/cm². It is known that a large drop generates in pore water pressure at the freezing front. Therefore, it is predicted that the drop in pore water pressure in the unfrozen soil leads to intense consolidation, increasing the effective stress when the soil is soft. The region where the consolidation occurs, δ_s , is:

$$\delta_s = c_v/U \quad (50)$$

Figure 28 shows the relation between δ_s and U . In the freezing site δ_s is unexpectedly large. Moreover, $\xi|_{\xi>\delta_s}$ which is observed at a point distant from δ_s is:

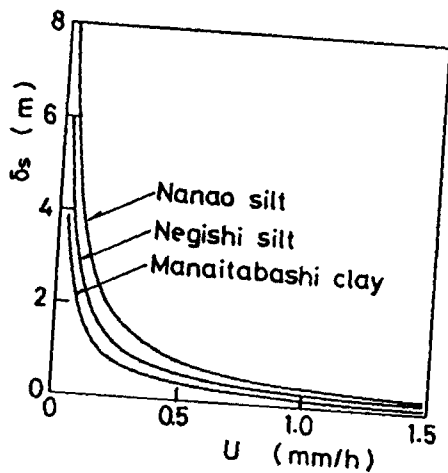


FIGURE 28. Consolidation region δ_s as a function of penetration rate U .

$$\left\{ \begin{array}{l} | \\ \delta_s \end{array} \right\} = \left\{ \begin{array}{l} \delta_0 \\ \delta_s \end{array} \right\} + n_f \Gamma \quad (51)$$

since at the freezing front the frost heave amount in the frozen soil cancels the amount of shrinkage resulting from water discharge and consolidation in the unfrozen soil. This equation shows that at a point distant from δ_s , the frost heave amount is approximately of the same magnitude as the amount of the expansion of water when frozen. This means that when the ground consists of a silt or clay that is low in permeability, besides being soft and weak, artificial freezing must be applied in a tightly closed system, differently from an open system used in laboratory experiments. It can be stated that the frost heave amount observed in the artificial freezing method is always smaller than that obtained from laboratory experiments as a result of the effects of consolidation and/or resistance against the water intake of the unfrozen soil.

4.3 Change in Earth Pressure due to Freezing

While soil is artificially frozen, the earth pressure increases in the unfrozen soil due to frost heaving when the soil is frost susceptible. A simple case in which the soil surrounding a vertical freeze pipe freezes was considered, as shown in Figure 29. When frost heaving occurred, the unfrozen soil surrounding the frozen column deformed by δ_1 at the outside, and the freeze pipe deformed by δ_0 at the inside. The deformation, δ_0 , which is caused by shrinkages due to decreasing temperature and increasing pressure, was less than δ_1 and may be calculated easily. Within the unfrozen region, the earth pressure may increase against the deformation, δ_1 . This antagonistic earth pressure becomes maximum at the freezing front and decreases with the increasing distance from the interface. Unfrozen soil behaves elastically when increments of earth pressure and deformation are relatively small, but when they increase a plastic yield or rupture occurs in the unfrozen soil zone. Takashi (1972) has shown this change in earth pressure theoretically, using equation 23 for frost heaving under overburden pressure. In the soft soil, the increment of earth pressure is small, but the deformation is large. On the other hand, in the hard soil the former is fairly large but the latter is small as shown in Figure 30. In designing artificial ground freezing, especially in the case of hard ground, the increase of earth pressure during freezing must be considered.

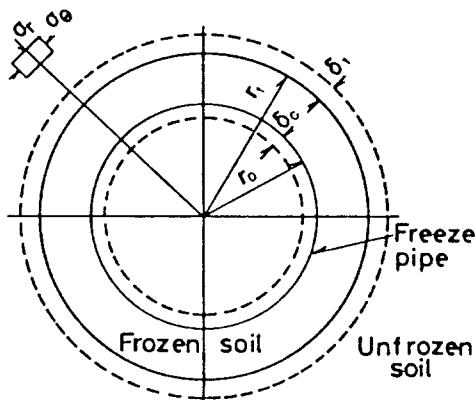


FIGURE 29. Freezing model.

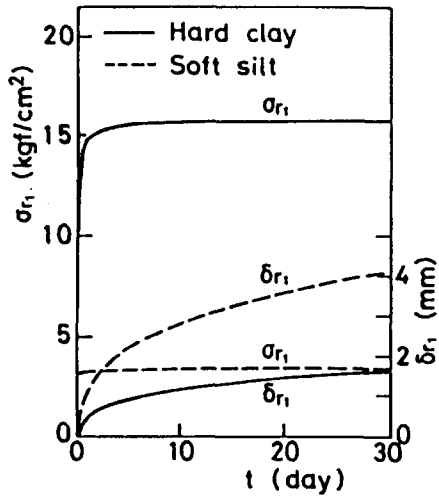


Figure 30. Stress change σ_{r1} and displacement δ_{r1} at freezing front due to frost heaving.

4.4 Uplift of Ground Surface

All frost heaving is not absorbed by horizontal displacement and/or consolidation of unfrozen soil, but appears partly as an uplift of the ground surface. The uplift presents a practical problem when artificial ground freezing is used in urban areas, where buildings are crowded and underground structures abound.

We were able to estimate the uplift distribution on the ground surface using calculations based on the assumption that the distribution may be approximated in the shape of the Gauss distribution curve. In the case of the model shown in Figure 31, it is given by the following equation (Tobe et al., 1979):

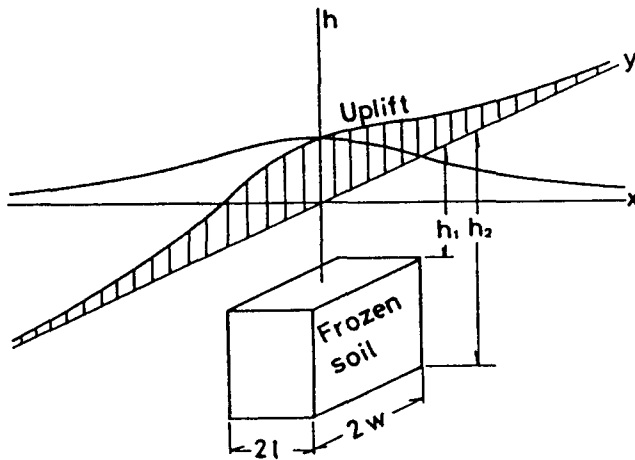


FIGURE 31. Model for the uplift of the ground surface

$$G(h,x,y) = \frac{\xi'}{4} \int_{h_1}^{h_2} \left[\left\{ \operatorname{erf}\left(\frac{l+x}{ah}\right) + \operatorname{erf}\left(\frac{l-x}{ah}\right) \right\} \left\{ \operatorname{erf}\left(\frac{w+y}{ah}\right) + \operatorname{erf}\left(\frac{w-y}{ah}\right) \right\} \right] dh \quad (52)$$

where $G(h,x,y)$: uplift amount,
 h_2-h_1 , l and w : lengths of the sides of a rectangular frozen soil,
 h : depth of the frozen zone,
 a : a constant concerned with the influence angle equal approximately to $a = \tan(45 + \Phi/2)$,
 Φ : friction angle,
 ξ' : modified frost heave ratio.

Equation 52 allows us to estimate in advance the amount of uplift at a freezing site with a maximum error of $\pm 30\%$.

4.5 Countermeasures Against Frost Heaving in Artificial Ground Freezing

In natural freezing, the supply of select granular soils and/or insulating materials are used generally as a countermeasure against frost heaving. However, it is difficult to use them for artificial ground freezing, since the area to be frozen is deep under the ground surface.

Countermeasures for artificial ground freezing, which the authors have used in practice, are briefly described as follows:

Increasing the viscosity of pore water The resistance of the unfrozen soil against water migration toward the freezing front may decrease the frost heave amount (see 4.1). That is, it is advantageous to try to decrease the permeability of the soil by some means. Takashi et al. (1980) considered that one can obtain some effect by increasing the viscosity of the pore water as a mechanical means of decreasing the permeability of the soil. Figure 32 shows the relation between the frost heave ratio, ξ , and the viscosity of pore water, η . Carboxy Methyl Cellulose (CMC) was used to increase η . As ξ decreases remarkably with increasing η , it is clear that the frost heave amount in a frost susceptible soil can be reduced by increasing the viscosity of the water attracted toward the freezing front during soil freezing.

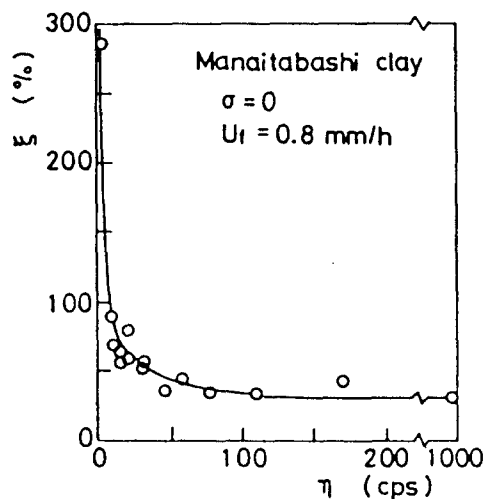


FIGURE 32. Dependence of frost heave ratio ξ on viscosity of pore water η .

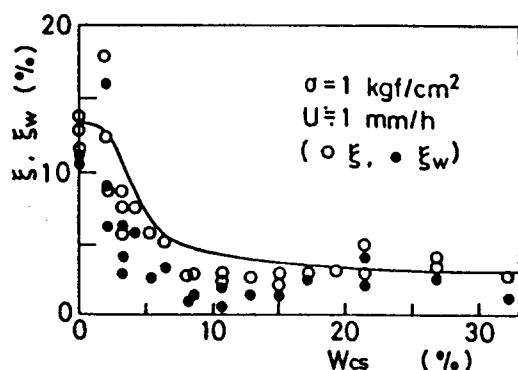


FIGURE 33. Dependence of frost heave ratio ξ and cement content W_{CS} .

Restraint of frost heaving by cement addition Thanks to the development of techniques of civil engineering techniques, it becomes easy to locally mix soil with cement underground. If the frost heave amount of the soil decreases from the addition of cement, this method is effective for artificial ground freezing. Accordingly, laboratory tests were conducted to investigate how much portland cement restrained the frost heave amount of soil, using a Fujinomori clay collected in Kyoto (Ohrai et al., 1984). As shown in Figure 33, the frost heave ratio, ξ , decreased 1/8 - 1/4 times that of cement free specimens when W_{CS} , which denotes that the weight ratio in the percentage of added cement to oven dried soil is 10%.

The decrease in ξ is small when $W_{CS} > 10\%$. Theoretical consideration shows that a decrease in the permeability of an unfrozen soil as a result of the addition of cement leads to little decrease in ξ . It is suggested that a decrease in ξ comes about from a decrease in the driving force to absorb water, due to the presence of cement in the soil.

It has been examined experimentally that thaw-settlement decreases and the mechanical strength of frozen soil increases when the proposed method is used.

5. CONCLUSIONS AND SUMMARY

This paper described the experimentally obtained effects of factors on frost heaving, and then discussed the prediction of frost heaving at the site of construction subjected to artificial ground freezing. Since these factors do not include all the factors which affect frost heaving, and analyses were made on assumptions as to the method of prediction, we must state that the results obtained give only approximate solutions. Nevertheless, in practical cases somewhat successful results have been obtained. This may be due to the fact that the experiments and analyses conducted by the authors and others on frost heaving have not deviated much from the fundamentals, since the results from practical applications of artificial ground freezing were taken into account.

The use of freezing as a ground stabilizing technique increases the range of applications to civil engineering, with the shape of soil frozen varied for each purpose. In these cases, it is impossible to rely on analytical approaches for a precise and quantitative prediction of frost heaving and its resultant phenomena. Fortunately, marked progress in computer technology is making such analytical approaches possible, since

we can now receive abundant input data, and calculate it with some speed. However, it is the authors' opinion that more established data on frost heaving and unfrozen soil must be collected before full advantage can be taken of computer technology. In order to predict frost heaving more precisely at the site of actual construction, we must continue to pursue experimental and theoretical research.

REFERENCES

Carslow, H.S. and J.C. Jager, 1959: Conduction of heat in solids, 2nd edition, Oxford at the Clarendon Press.

Dirksen, C. and R.D. Miller, 1966: Closed-system freezing of unsaturated soil. *Soil Sci. Soc. Am. Proc.*, 30, 163-173.

Fukuda, M., A. Orhaum and N. Luthin, 1980: Experimental studies of coupled heat and moisture transfer in soils during freezing, *Cold Regions Sci. and Tech.*, Vol. 3, 223-232.

Hoekstra, P., 1966: Moisture movement in soils under temperature gradients with the cold side temperature below freezing. *Water Resources Research Board Special Report*, 103, 78-90.

Horiguchi, K. and R.D. Miller, 1980: Experimental studies with frozen soil in "Ice sandwich" permeameter. *Cold Reg. Sci. Technol.* Vol. 3, 177-183.

Loch, J.P.G. and B.D. Kay, 1978: Water redistribution in partially frozen, saturated silt under several temperature gradients and overburden loads. *Soil Sci. Soc. Am. J.*, 42, 400-406.

Miller, R.D., 1972: Freezing and heaving of saturated and unsaturated soils. *Highway Res. Record*, 393, 1-11.

Miyoshi, M., T. Tsukamoto and S. Kiriya, 1975: Large-scale freezing work for subway construction in Japan. *Engineering Geology*, 13, 397-415.

Ohrai, T., H. Yamamoto, J. Okamoto and H. Izuta, 1984: Restraint of frost heaving and thaw settlement of soil by cement addition. *SEPPYO, J. Japanese Soc. Snow and Ice*, Vol. 46, No. 4, 189-197.

Ohrai, T. and H. Yamamoto, 1985: Growth and migration of ice lenses in partially frozen soil. *Proceedings of the 4th International Symposium on Ground Freezing*, 79-84.

Ohrai, T., Y. Ishikawa and Y. Kushida, 1985: Actual results of ground freezing in Japan. *Proceedings of the 4th International Symposium on Ground Freezing*, Vol. 2, 289-294.

Penner, E., 1960: The importance of freezing rate in frost action in soils. *Proceedings Am. Soc. Test. Mater.* 60, 1151-1165.

Penner, E. and L.E. Goodrich, 1980: Location of segregated ice in frost susceptible soil. *Proceedings of the 2nd Int. Symp. Ground Freezing*, 626-639.

Radd, F.J. and D.H. Oertle, 1973: Experimental pressure studies of frost

- heave mechanism and the growth fusion behavior of ice. Permafrost, 2nd Int. Conf., North Am. Contribu., Washington D.C., Nat. Acad. Sci., 377-384.
- Sutherland, H.B. and P.N. Gaskin, 1973: Pore water and heaving pressure developed in partially frozen soils. Permafrost, 2nd Int. Conf., North Am. Contribu., Washington D.C., Nat. Acad. Sci., 409-419.
- Taber, S., 1929: Frost heaving. J. Geol. 37, 428-461.
- Taber, S. 1930: The mechanics of frost heaving. J. Geol., 38, 303-317.
- Takashi, T. and S. Wada, 1961: The soil freezing method in engineering construction (I). 36, No. 408, 1-15.
- Takashi, T., 1965: On the freezing efficiency in soil freezing method. (in Japanese) Refrigeration, Japanese Association of Refrigeration, Vol. 40, No. 456, 1-7.
- Takashi, T., 1969: Influence of seepage stream on the joining of frozen soil zones in artificial soil freezing. Special Rep. 103, Highway Research Board, Washington, D.C., 273-286.
- Takashi, T., 1972: On the stress and displacement in unfrozen soil zone around artificial frozen soil. Proceedings of JSCE, No. 200, 49-62.
- Takashi, T. and M. Masuda, 1975: On an exact solution of heat transfer equation in freezing soil with constant speed, accompanying uniform form of suction water to the freezing front.
- Takashi, T., M. Masuda and H. Yamamoto, 1976: Influence of permeability of unfrozen soil on frost heave. (in Japanese) SEPPYO, Journal of the Japanese Society of Snow and Ice, Vol. 38, No. 1, 1-10.
- Takashi, T., T. Ohrai and H. Yamamoto, 1977: Pore water pressure and consolidation in unfrozen soil near the freezing front, SEPPYO, J. Japanese Soc. Snow and Ice, Vol. 39, No. 2, 53-64.
- Takashi, T., H. Yamamoto, T. Ohrai and M. Masuda, 1978: Effect of penetration rate of freezing and confining stress on the frost heave ratio of soil, Permafrost, 3rd Int. Conf., Vol. 1, 737-742.
- Takashi, T., T. Ohrai and H. Yamamoto, 1980: Decrease of frost heave amount by increasing the viscosity of pore water, Proceedings of JSCE, No. 298, 77-85.
- Takashi, T., T. Ohrai, H. Yamamoto and J. Okamoto, 1981a: Upper limit of heaving pressure derived by pore water pressure measurements of partially frozen soil, Eng. Geol., 18, 245-257.
- Takashi, T., T. Ohrai, H. Yamamoto and J. Okamoto, 1981b: An experimental study on maximum heaving pressure of soil, SEPPYO, J. Japanese Soc. Snow and Ice, Vol. 43, No. 4, 207-215.
- Takashi, T., T. Ohrai, H. Yamamoto and J. Okamoto, 1982: Effect of specimen height on frost heave ratio in unidirectional freezing test of soil. Proceedings of the 3rd Int. Symp. on Ground Freezing, 247-254.
- Tobe, N. and O. Akimoto, 1979: Calculating method of frost heaving. 34th Symp. of JSCE, III, 243-244.

Freezing and Melting Heat
Transfer in Engineering P.547~580
Edited by K.C.Cheng & N.Seki
Hemisphere Publishing Corporation