# A SERME Control in the Hydraulic or Electric Type Drive Application

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### **Abstract**

One relay control implementation to the servo control systepm is contrived mainly for the high power electric or electro hydraulic type drive system. Proposed SERME control is relay emulated software scheme accounting a relay producing maximum torque in both direction in the insufficient bandwidth control system. Possibility is proposed mathematically prior to emperical work out.

# 1. Introduction

This is a follow-up work to "The electric and electro-hydraulic type high power drive" (Ahn Some of practically encounted probllems during high power drive system development either by electric or by hydraulic measure are full utilization of power, minimum time control, consistent precision control, etc. Relay control has been applied diversely, and particularly used in case of insufficient bandwidth for precision position control in the past. But not much has been done in the servo control system with implementation. Proposed SERME (sign error multiplys root of magnitute of error) method (Landers 1983) is aiming to control the high newog system especially consideration for military purpose. Experimental

work is on the way and ensued later. Biernson developed a equation for a gain decision in the position servo control loop. Case studies have been done for the motor control and electro-hydraulic valve control of the same parameters given in (Ahn 1984). SERME is mathematically developed and gain substitue is shown instead of a simplified position and velocity feedback system together with Biernson's equation connection. Discussion is given with regard to the SERME effectiveness on the state of the art technology.

# 2. Satuation (Biernson)

When slewing it is reasonable to argue that for minimum time control the system must be saturated throughout. The velocity of the load would then be like in Fig.1.

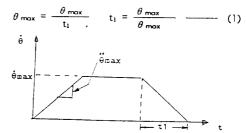


Fig.1. Minimum Time Control Profile

A simple double integration yields the error

$$\Theta_1 = \frac{1}{2} \theta_{\text{max}} t_1^2 \qquad ---- (2)$$

Consider now the following simplified position servo as In Fig.2.

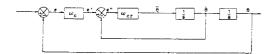


Fig.2. Simplified Conceptual Sero-loop

$$\frac{\theta}{e''} = \frac{\omega c_7}{S} \tag{3}$$

and

$$\frac{\theta}{e'} = \frac{\omega_{C_7}/S}{1+\omega_{C_7}/S} = \frac{\omega_{C_7}}{S+\omega_{C_7}}$$
(4)

$$\frac{\theta}{e} = \frac{\omega_{C}, \omega_{C}}{S(S + \omega_{C})} \simeq \frac{\omega_{C}}{S}$$
 (5)

If  $\omega_{C_7}$  is large

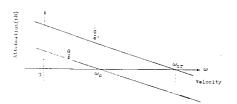


Fig.3. Attenuation Diagram

$$\theta = \omega_{C_7}$$
  $e'' = \omega_{C_7}(e' - \theta) = \omega_{C_7}(\omega_{C}e - \theta)$  ---- (6)

Therefore,

 $\theta$  change sign when  $e=\theta/\omega_C$  if  $\omega_{C7}$  is large

And therefore,

$$\frac{\theta_{\text{max}}}{\omega_{\text{C}}} = \frac{1}{2} \frac{\theta_{\text{max}}^2}{\theta_{\text{max}}} \text{ or } \omega_{\text{C}} = \frac{2 \theta_{\text{max}}}{\theta_{\text{max}}} \qquad -(7)$$

Acceleration limits result from current or pressure limits while velocity limits result from voltage or flow limits. Then it is easy enough to calculate the required gain  $\omega_{C}$ .

#### <Example>

Futher calculation shows that  $\omega_{C_7} > \omega_C$  is

sufficient and for this to be true

$$\frac{V_{\text{sot}}}{I_{\text{ext}}} > 4Q$$
 or  $\frac{Q_{\text{sot}}}{P_{\text{sot}}} > 4L$  (8)

The resistance associated with the armature of an electric motor including its power amplifier output stage is between  $1.0 \, \Omega$  and  $10 \, \Omega$ .

If  $V_{sat}=\pm 50V$ , then

$$I_{sot} < \frac{V_{sot}}{4R} = \frac{50}{1} = 50A$$
 (9)

This si not necessarily so, 100A might be a better estimate of  $I_{sot.}$  In the hydraulic case, a two stage electro-hydraulic valve may have the following parameters (at 1000 lbf/in<sup>2</sup>)

Table 1. Valve Parameters

How Rate m³/s(in³/s)	freq.Resp. (-3ato) Hz	Phase/Lag (40/90) Hz	internal Leakage In <sup>3</sup> /s	Compressed Oil Volume in <sup>3</sup>	Welght d I
+80×10⁴(5)	50	50.125	0.6	0.08	2.75
320×10 <sup>4</sup> (20)	60	50.125	0.8	0.09	2.75
500×10 <sup>4</sup> (30)	60	50.125	1.1	0.1	2.75
640×10* (40)	50	50.125	1.1	0.17	2.75
10-3 (60)	50	40.90	1.8	0.17	2.75
1.6×10 <sup>3</sup> (100)	45	25.50	4	0.52	15
2.6 × 10 <sup>-3</sup> (160)	20	15.40	6	1.35	25

$$P = 1000 \text{ lb}_{1}/\text{ln}^{2} = 6.895 \times 10^{6} \text{ N/m}^{2}$$

$$\frac{Q}{P} = \frac{80 \times 10^{-6}}{6.895 \times 10^{6}} \sim \frac{2.6 \times 10^{-3}}{6.895 \times 10^{6}}$$

$$= 11.6 \times 10^{-12} \sim 377.1 \times 10^{6} \left(\frac{\text{m}^{3}}{\text{S}}, \frac{\text{m}^{2}}{\text{N}}\right)$$
(10)

For motor cases, consler now motor data.

Table2. Motor Parameters

cl <sub>m</sub> in³/rev	Leakage in <sup>3</sup> min <sup>-1</sup> /psi	inito/sec²	Compressed Oil Volume In <sup>3</sup>	Reference Data	
1	0.035	0.002	ı	Motor Displace- ment 2.5 × 10 <sup>4</sup> to 20 × 10 <sup>4</sup> m <sup>3</sup> /rod	
2.2	0.04	0.006	1.5	Compressed Vol 15 × 10 <sup>4</sup> to 8.4 × 10 <sup>3</sup>	
5	80.0	0.015	3.2	[""	
7.6	0.1	0.026	5.1	Motor thertia 0.0002 - 0.0026 Kgm²	

Losses are then  $0.035\sim0.1$  in  $^3$  min' /psl or  $0.01386\times10^{-10}\sim0.0396\times10^{-10}$  m $^5$ /N sec — (11)

Valve losses 
$$0.036 \sim 0.36 \text{ in}^3 \text{min}^{-1}/\text{psl}$$
 or  $1.4256 \times 10^{-12} \sim 1.4256 \times 10^{-11} \text{ m}^5/\text{N sec}$  — (12)

Total losses are then about 
$$2.5 \times 10^{-12}$$
 to  $18.2 \times 10^{-12}$  m<sup>5</sup>/N sec ----- (13

Therefore,

11.6×10<sup>-12</sup> to 377.1×10<sup>-12</sup> m<sup>5</sup>/N sec is usually 
$$4 \times 2.5 \times 10^{-12}$$
 to  $4 \times 18.2 \times 10^{-12}$ m<sup>5</sup>/Nsec - (14)

But in neither case is this criterion necessarily met.

## 3. SERME

If we have insufficient bandwidth in the inner loop, then, a solution considered in the past involves a relay which produces maximum torque in either direction. The equation of motion under torque limited conditions are

$$S^2 \theta_o = \frac{T_{\text{max}}}{I} \tag{15}$$

$$S\theta_{o} = \frac{T_{\text{max}}}{J} + C_{1} \qquad (16)$$

$$\theta_0 = \frac{T_{\text{max}}}{J} + C_1 t + C_2$$
 (17)

Where  $C_1$  is the velocity with which the system enters the torque limited region.

Consider now the relay system. Let the voltage on the relay operating coil for error be  $K_1$ ; let the voltage on the relay operating coil for unit C/P shaft velocity be  $K_2$ . The net voltage on the relay operating coil is then

$$K_1\Theta - K_2S\theta_o$$
 (18)

The torque is then given by

$$T = -T_{max}Sgn(K_1\Theta - K_2S \theta_o)$$

$$= +T_{max}Sgn(K_1\theta_o + K_2S \theta_o) ----- (19)$$

\* Positive error must produce negative torque Where  $e = -\theta_0$ , note  $\theta_1$  assumed zero.

The torque change over condition is then

$$K_1 \theta_0 = -K_2 S \theta_0 \text{ or } \theta_0 = -\frac{K_2}{K_1} S \theta_0 \qquad (20)$$

A case of particular Interest is that (in which the magnitude of the initial displacement is set so that torque is reversed when the O/P shaft has moved exactly half way) we go from maximum acceleration to maximum deceleration just at the correct time to bring the load to a stop just where we want it.

Let this value be  $\theta_{d}$ ,

Then, 
$$\theta_o = \frac{T_{\text{max}}}{J} \frac{t^2}{2} - \theta_d$$
.  
Where  $C_2 = -\theta_d$  and  $C_1 = 0$  (21)

When,

$$\theta_o = \frac{\theta_d}{2}$$
, we have  $-\frac{\theta_d}{2} = \frac{T_{\text{max}}}{J} \frac{t^2}{2} - \theta_d$ 

Therefore,

$$\theta_d = -\frac{T_{\text{max}}}{J} t^2 \quad \text{or} \quad t = \sqrt{\frac{\theta_d J}{T_{\text{max}}}}$$
 (23)

And therefore,

$$S\theta_o = -\frac{T_{\text{max}}}{f} t = \frac{T_{\text{max}}}{f} \sqrt{\frac{\theta_d J}{T_{\text{max}}}} = \sqrt{\frac{\theta_d T_{\text{max}}}{T_{\text{max}}}}$$

But, 
$$\theta_o = -\frac{K_2}{K_1} S\theta_o = -\frac{K_2}{K_1} \sqrt{\frac{\theta_d T_{\text{mex}}}{I}}$$
 (25)

Therefore.

$$-\frac{\theta_d}{2} = -\frac{K_2}{K_1} \sqrt{\frac{\theta_d T_{\text{max}}}{J}} \tag{26}$$

And therefore

$$\frac{\theta^{2}_{o}}{4} = (\frac{K_{2}}{K_{1}})^{2} \theta_{d} \frac{T_{\text{max}}}{J}$$
 (27)

Lastly

$$\theta_d = \frac{4(K_2)^2 T_{\text{max}}}{(K_1)^2 I}$$
 (28)

For a particular value of  $\theta_{d}$ , we can obviously choose  $K_1$  and  $K_2$  so that this condition is satisfied. But the required values of  $K_1$  and  $K_2$  will vary for different disturbances. We would like this condition, which is the condition for constant response to disturbance, to be satisfied for any disturbance amplitude. This can be done if the value of the velocity feedback gain  $K_2$  is varied automatically so that the relay changes over exactly at

$$e = -\frac{\theta_d}{2} \,\forall \, \theta, \qquad ----- \tag{29}$$

Nov

$$S\theta_o = \frac{\sqrt{\theta_d T_{\text{max}}}}{J}$$
 and  $\theta_d = 4 \frac{K_2^2}{K_1^2} \frac{T_{\text{max}}}{J}$  - (30)

Eliminating  $\theta_d$ , we have

$$(S\theta_o)^2 \frac{I}{T_{\text{max}}} = 4 \frac{K_2^2}{K_1^2} \frac{T_{\text{max}}}{I}$$
 (31)

Therefore.

$$K_2^2 = \frac{K_1^2}{4} \frac{J_2}{T_{--}} (S\theta_o)^2$$
 (32)

And

$$K_2 = +\frac{K_1 J}{2T_{--}} \left[ \omega_o \right] \quad \text{when} \quad \omega_o = S\theta_o \quad -- \quad (33)$$

 $\ensuremath{K_2}$  must remain positive so that the velocity feedback remains negative. Therefore, we have taken the positive root. We previously had

$$T = -T_{\text{max}} \operatorname{sgn}(K_1 e - K_2 \operatorname{s}\theta_o) \qquad - \quad (34)$$

Therefore, 
$$T = -T_{\text{max}} \operatorname{sgn}(K_1 e - K_2 s \omega_o)$$
  

$$= -T_{\text{max}} \operatorname{sgn}(K_1 e - \frac{K_1 J}{2 T_{\text{max}}} \lceil \omega_o \rceil \omega_o)$$
(35)

(This is exactly the formula In the Zanker paper)

The equation of the change over line is then

$$e = \frac{\int}{2T_{\text{max}}} \left[ \omega_o \right] \omega_o \quad or$$

$$\theta_o = -\frac{\int}{2T_{\text{max}}} \left[ \omega_o \right] \omega_o \qquad (36)$$

In the days when V stood for velocity this was called V mod V for obvious reason. It was also regarded as difficult to produce a velocity feedback gain of  $J/(2 T_{max})I \, \omega_{o}$ . So the square root of each of the bracketed quantities was considered. i.e.

$$\sqrt{K_1 e} - \sqrt{\frac{K_1 J}{2T_{\text{max}}}} \omega_o \qquad (37)$$

This quantity changes sign whenever the original one does. We say then

$$T = -T_{\text{max}} \operatorname{sgn}(\operatorname{sgn}(e) \sqrt{\lceil e \rceil} - \omega_o \sqrt{\frac{1}{2T_{\text{max}}}}) - (38)$$

Where again we have carried through the sign of the variable when possessing the square root. This system is called SERME - Sign Error X Root of Magnitude of Error

For change over

$$sgn(e)\sqrt{\lceil e \rceil} = \sqrt{\frac{1}{2T_{\text{max}}}}$$

$$\omega_o = sgn(e)\sqrt{\lceil e \rceil} = \sqrt{\frac{2T_{\text{max}}}{J}} \qquad (39)$$

$$\omega_o^2 = \lceil e \rceil \frac{2T_{\text{max}}}{J} : \text{see for V mod V} \qquad (40)$$

### 4. Connection between Biernson and SERME



Fig.4. Simplified Position and Velocity
Feedback System

Consider  $\theta = K_v(K_de-\theta)$ . We switch to maximum deceleration when  $K_de=\theta$ . Biernson says that  $K_d=2a/\theta$  or  $e=\theta^2/2a$ . This is the switching line. But this is the switching line for SERME, and to obtain SERME we say

$$\theta = \sqrt{2ae}$$
 or  $\theta = \sqrt{\frac{2T_{\text{max}}}{J}} \operatorname{sgn}(e) \sqrt{|e|}$ 

We then have

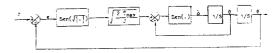


Fig.5. Servo Control System with Relay implementation

# 5. Objections to SERME

This method evolved when electrical systems didn't use control of field current of electric servo motors: i.e. there was no 'back emf' loop. There was therefore direct control of the motor torque.

$$T = \frac{P\Phi Z}{2\pi a} I_a = \frac{PZ}{2\pi a} K_F I_F I_a = K' I_F \qquad (41)$$

Electronic devices of those days(tubes) could not deal with armature currents.

But armature controlled electric motor, and flow controlled hydraulic motors are effectively velocity controlled devices. Maximum voltage or flow will not necessarily produce maximum torque because speed build up will tend to reduce the torque. It might be argued that if we use significant torque feedback then an input signal will control torque rather than velocity. From a linear point of view

this is certainly so.

Consider the electrical case using armature current feedback(which amounts to the same thing).

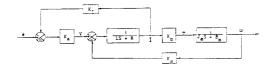


Fig.6. Current Feedback Servo Control System

For simplicity, the time constant of the power amplifier is ignored

$$T = J_{m} \frac{d\omega}{dt} + B_{m}\omega = K_{m}I \qquad (42)$$

and 
$$V = L \frac{dI}{dt} + RI + K_{m}\omega$$
 (43)

$$V = K_a(e - K_1 I) \tag{44}$$

Require the transfer function for T/e

Therefore,

$$K_{\sigma}(e - K_1 I) = L \frac{dI}{dt} + RI + K_{m} \omega \qquad (45)$$

$$K_{\mathbf{m}}\omega = K_{a}e - (K_{a}K_{I} + LS + R)I \qquad ---- \qquad (46)$$

Also

$$(J_m S + B_m)\omega = K_m I \quad \text{or} \quad \omega = K_m \frac{I}{J_m S + B_m} \quad - \quad (47)$$

Therefore

$$\frac{K_{m}^{2}I}{J_{m}S+B_{m}}=K_{a}e-(K_{a}K_{I}+LS+R)I \quad -- \quad (48)$$

And

$$[K_{\alpha}^{2} + (K_{\alpha}K_{l} + LS + R)(J_{m}S + B_{m})]I$$

$$= (J_{m}S + B_{m})K_{\alpha}e$$
(49)

Steady state gives

$$(K_m^2 + K_a K_l B_m + R B_m)I = B_m K_a e$$
 (50)

Therefore,

$$\frac{I}{e} = \frac{B_{m}K_{o}}{K_{m}^{2} + (K_{o}K_{I} + R)B_{m}}$$
 (51)

Let KaKi be large

$$\frac{I}{e} \simeq \frac{1}{K_I} \quad or \quad \frac{T}{e} \simeq \frac{K_{**}}{K_I} \qquad (52)$$

Therefore, e controls T.

But this is based on linearity. Suppose the SERME system switches e hard over one way. Then as  $\omega$  rises,  $K_m \omega$  will begin to cause I or T to reduce. This

is the original objection to SERME. However, it might be argued that the Ki loop tends to counter this. Certainly V will tend to rise if I drops and e remains constant (as it will). But if V was also saturated it could not rise further. Nevertheless this is a way to produce a SERME control in either the flow controlled hydraulic or armature controlled electric system.

#### 6. Discussion

A SERME control is discussed especially In relation to the flow controlled hydraulic or armature controlled electric system. In highly demaned trend of maximum power utilization, this control would be another substitue for the control sheme though not very well proved through the elabrate experiment. Further analytical and empirical work result will be presented including sallent features resulted from hardware advancement. None the less this control, hopely and surely, is one way of solution.

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