

UBET Analysis of the Combined Extrusion Using Shape Function

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Abstract

The main purpose of this study is constructing new velocity fields on the base of shape function used in finite element method and showing the possibility of application it to metal forming processes. Utilizing the 8-node quadratic rectangular element, we expressed the velocity within the deformation region by interpolating the velocity of each nodal points. And the upper-bound formulation from this velocity fields was derived. In order to confirm the validity of this method we applied it to axisymmetric combined extrusion problem. The results of load show that this method is in better agreement with experiment than the conventional UBET, and also the flow pattern and profile of extruded part are reasonable.

1. Introduction

The upper-bound method has been widely used to predict the forming load and deformation pattern in the analysis of the metal forming processes. In the upper-bound analysis the problem is solved by looking for the kinematically admissible velocity field which can predict the deformation pattern and load properly and by minimizing the energy consumption rate. But as it is almost impossible to describe somewhat complex deformation pattern, UBET (Upper-bound elemental technique) in which method the velocity of whole deforming part is expressed by combining the simple velocity elements was developed.

In recent years, the study to make it easy to compose the velocity fields which is still difficult to find out has been made by several researchers. T.Shimizu et al^[1] proposed stream function velocity fields which can be derived by the incompressibility condition of 2-dimensional flow. He applied it to the triangle element having 3 nodes and analyzed the plane-strain upsetting and axisymmetric forward extrusion problems.

J.P.Wang^[2] solved several problems by more general velocity fields based upon the of the stream function concept of incompressible flow and the shape function used

in finite element method.

Jiang Qin^[3] reported a paper in which the velocity fields interpolated by the nodal point velocities of triangle element was successfully applied to the plane-strain problems, showing the possibility of solving other problems by it.

In this study, as J.P.Wang developed a more general velocity field on the base of the stream function of T.Shimizu et al by introducing the concept of shape function, more general form of velocity fields than that of Jiang Qin is proposed. We expressed the velocity distribution in an element by interpolating the velocities of nodal points using shape function.

Because of the characteristics of the shape function, an element can take the shape of arbitrary triangle or rectangular, so we can apply it to the problems having more complex shapes. Besides, because the stream function can be defined in the 2-dimensional flow, the velocity fields by stream function can't be applied to 3-dimensional problems. But such difficulty may be overcome by the velocity fields in this study.

In this paper we applied this velocity fields to the combined extrusion problems, examined the load and deformation pattern, compared the results with

conventional UBET and experiments. The results show that it is in better agreement with experiment than the conventional UBET.

Therefore more favorable results can be acquired by using easier velocity fields than the conventional UBET, which proves that it can also be adopted to the analysis of other metal forming processes effectively.

2. Velocity fields expressed by using shape function

Generally parameters in a element can be expressed by approximating parameters of nodal points, and shape function is used in this interpolation. The form of shape function, if ϕ is parameter, can be defined as polynomial.

$$\phi = \sum_{m=0}^m a_m x^m \quad \dots\dots\dots (1)$$

or as interpolation function,

$$\phi = \sum_{n=1}^n N_n \phi_n \quad \dots\dots\dots (2)$$

The formulation (1) can be transformed into (2) through proper process. When the nodal values of a certain parameter X are $\{x\}$, and the set which defines the relationship between X and $\{x\}$ is $[N]$, then

$$X = [N]\{x\}$$

in which we call $[N]$ the shape function. If we use dimensionless coordinate system as the local coordinate, it is easy to define the shape function and to operate integration of distorted elements.

The most basic and easiest is rectangular element, so in this study as in Fig.(1) the eight-node quadratic rectangular element and shape function are used. According to this shape function we can express the coordinate and velocity in an element as follows.

$$\begin{aligned} r &= \sum_{k=1}^8 N_k r_k \\ z &= \sum_{k=1}^8 N_k z_k \\ \dots\dots\dots (3) \end{aligned}$$

$$U = \sum_{k=1}^8 N_k U_k$$

$$V = \sum_{k=1}^8 N_k V_k \quad \dots\dots\dots (4)$$

Where N_k are shape functions, k are nodal indices, r_k, z_k are coordinate component of r, z direction, and U_k, V_k are r, z direction velocities of the nodal points respectively.

3. The upper-bound formulation

For the case of rectangular element using 8 nodes and axisymmetric deformation the upper-bound formulation can be expressed as follows.

The strain rate components from velocity components are

$$\dot{\epsilon}_r = \frac{\partial V_r}{\partial r} = \frac{\partial}{\partial r} (N_1 U_1 + N_2 U_2 + \dots\dots\dots + N_8 U_8)$$

$$\dot{\epsilon}_z = \frac{\partial V_z}{\partial z} = \frac{\partial}{\partial z} (N_1 V_1 + N_2 V_2 + \dots\dots\dots + N_8 V_8)$$

$$\dot{\epsilon}_\theta = \frac{V_r}{r} = \frac{1}{r} (N_1 U_1 + N_2 U_2 + \dots\dots\dots + N_8 U_8)$$

$$\begin{aligned} \dot{\epsilon}_{rz} &= \frac{1}{2} \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right) \\ &= \frac{1}{2} \left[\frac{\partial}{\partial z} (N_1 U_1 + \dots\dots\dots + N_8 U_8) \right. \\ &\quad \left. + \frac{\partial}{\partial r} (N_1 V_1 + \dots\dots\dots + N_8 V_8) \right] \\ \dots\dots\dots (5) \end{aligned}$$

Where $N_1, \dots\dots, N_8$ are the shape function in terms of natural coordinate components ξ, η .

According to the chain rule of partial derivative

$$\begin{bmatrix} \frac{\partial N_\alpha}{\partial r} \\ \frac{\partial N_\alpha}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial N_\alpha}{\partial \xi} \\ \frac{\partial N_\alpha}{\partial \eta} \end{bmatrix}$$

where, $\alpha = 1, \dots\dots, 8$ and

$$\frac{\partial r}{\partial \xi} = \frac{\partial N_1}{\partial \xi} r_1 + \frac{\partial N_2}{\partial \xi} r_2 + \dots\dots\dots + \frac{\partial N_8}{\partial \xi} r_8$$

$$\begin{aligned}\frac{\partial r}{\partial \eta} &= \frac{\partial N_1}{\partial \eta} r_1 + \frac{\partial N_2}{\partial \eta} r_2 + \dots + \frac{\partial N_8}{\partial \eta} r_8 \\ \frac{\partial z}{\partial \xi} &= \frac{\partial N_1}{\partial \xi} z_1 + \frac{\partial N_2}{\partial \xi} z_2 + \dots + \frac{\partial N_8}{\partial \xi} z_8 \\ \frac{\partial z}{\partial \eta} &= \frac{\partial N_1}{\partial \eta} z_1 + \frac{\partial N_2}{\partial \eta} z_2 + \dots + \frac{\partial N_8}{\partial \eta} z_8\end{aligned}$$

..... (6)

Using the above mentioned expression of strain rate components, we can derive the upper-bound formula which should be optimized for the unknown variables of velocities of r or z direction at nodal points.

$$\dot{W}_T = \dot{W}_i + \dot{W}_f$$

with

$$\begin{aligned}\dot{W}_i &= \int_V \bar{\sigma} \left[\frac{2}{3} (\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\theta^2 + 2 \dot{\epsilon}_{rz}^2) \right]^{\frac{1}{2}} dV \\ \dot{W}_f &= \int_s \frac{m \bar{\sigma}}{\sqrt{3}} |\Delta V_f| dS\end{aligned}$$

..... (7)

Where \dot{W}_T is the overestimated total power required for a metal forming process. \dot{W}_i is internal and \dot{W}_f represents the friction power losses at die-workpiece surfaces. ΔV_f is the velocity discontinuity on the die-workpiece interface, m is constant friction factor and $\bar{\sigma}$ is the effective stress.

4. The analysis of combined extrusion

The problem of combined (backward-forward) extrusion (Fig.(2)) involves the flow through two die openings, one in the direction of the ram travel with forward exit velocity and the other in a direction opposite to the ram travel with backward exit velocity. The magnitude and direction of these velocities are critically dependent on the die and ram geometry.

In order to perform the analysis the total deforming region is divided into 6 elements as shown in Fig.(3-b). Fig.(3-a) is the element dividing of conventional UBET and

there are some different configuration in Fig.(3-b) from Fig.(3-a). But in actual simulation the dimension of element 2 and 5 in Fig.(3-b) is set so small that we can hardly find the difference between Fig.(3-a) and Fig.(3-b) in dimension due to element 2, 5 in Fig.(3-b). The diameter of extrusion die is 14.5mm, that of punch is 21.75mm, and that of container is 27mm. The friction factor m is 0.13 which value we can obtain from ring compression test using lubricant MoS₂ Powder(10% wt.)+Calcium Grease(90% wt.).^[5]

And as the extrusion billet we use lead whose flow stress we set 3 kgf/mm². In fact there exists the strain hardening according to the forming process, but we assumed that lead is a rigid-perfect plastic material.

5. Constraints conditions and optimization

The energy consumption rate terms are evaluated by Gaussian Integration. To be accepted as kinematically admissible velocity field equation (4) must satisfy two constraints, i.e the boundary and volume constancy condition. But the form of equation (4) has no assurance that it satisfies the volume constancy condition, so we must take it into consideration by adding constraint condition at the optimization technique.

If we think of an element of 8 nodes, there must be two kinds of node. One is the node whose velocity is known and the other is unknown. And each node has r and z direction components of velocity. The velocities of nodes can be known in case it is constrained by container, die, or punch. All the unknown velocities are set to be unknown parameters which should be optimized. As optimization technique, FTM(Flexible tolerance method) which can minimize multi-variable functions having constraint (equality or nonequality) conditions was used.^[4] Fig(4) shows the flow chart for this numerical processes.

6. Results and discussion

Fig(5) shows the load according to punch stroke. The predicted extrusion load is lower and closer to the experimental results than the conventional UBET as shown in Fig(5). But the load of this study obtained from the initial cylinder billet to the punch stroke of 5mm is

higher than that of the conventional UBET, which is thought to be the result of non-steady effect, element dividing and the assumption that lead is a rigid-perfect plastic material. This may be improved if we use finer elements or divide the deformation region into more elements. And the effect of strain hardening should also be taken into consideration.

Fig(6-a) is the velocity distribution over the elements at the beginning point of deformation, and streamline at that part is shown in Fig(6-b). From this figure we can easily notice the neutral area of deformation.

Fig(7) is the velocity distribution at the punch stroke of 8mm, and Fig(8) shows that at the punch stroke 16mm and these are thought to be reasonable and in good agreement with prediction.

7. Conclusions

The kinematically admissible velocity field which is one of difficulties of UBET analysis can easily be composed and the results which are forging load and deformation pattern is more reasonable than the conventional UBET. Also in the study performed by J.P.Wang and T.Shimizu et al the application to the 3 dimensional problems seems to be impossible because stream function itself can be defined only in 2 dimensional flows, but the velocity field of this study isn't affected by such constraints, and so it can widen its range to 3 dimensional problems.

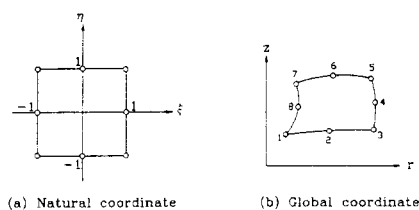
In this study, to analyze the combined extrusion 6 rectangular elements having 8 nodes each are used and there are 34 independent variables to be optimized. But to obtain more precise and reliable result, it is inevitable to increase the number of elements. But if we do so, the number of independent variables also increase, making it difficult to optimize them. This problem can be overcome by such efforts that we don't perform the optimization procedure over the total element but perform it for each element one by one, and after such procedure we combine the results over total deformation area.

References

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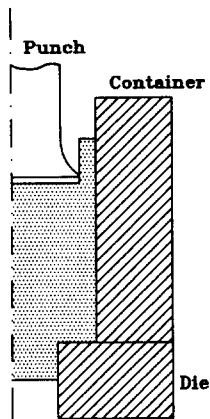
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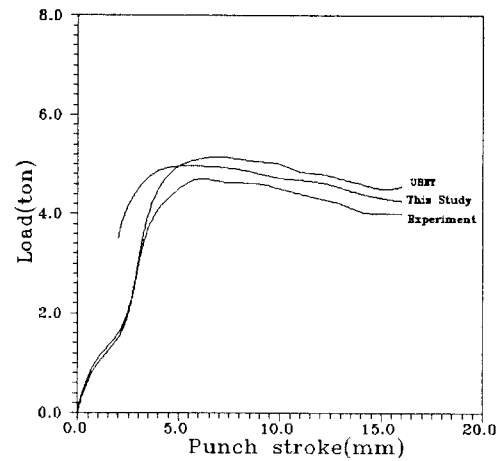
$$\begin{aligned}
 N_1 &= -\frac{1}{4}(1-\xi)(1-\eta)(\xi+\eta+1) & N_7 &= -\frac{1}{2}(1-\xi^2)(1-\eta) \\
 N_2 &= \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1) & N_8 &= \frac{1}{2}(1+\xi^2)(1-\eta) \\
 N_3 &= \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1) & N_6 &= \frac{1}{2}(1-\xi^2)(1+\eta) \\
 N_4 &= -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1) & N_5 &= \frac{1}{2}(1-\xi^2)(1+\eta)
 \end{aligned}$$

(c) The shape function of 8 node quadratic rectangular element

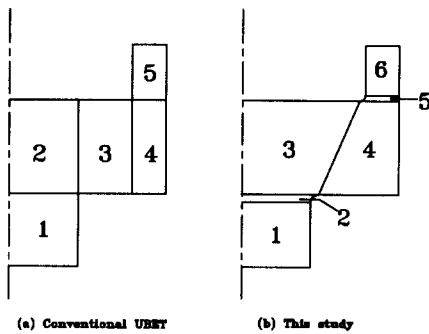
Fig(1). Coordinate system and shape function



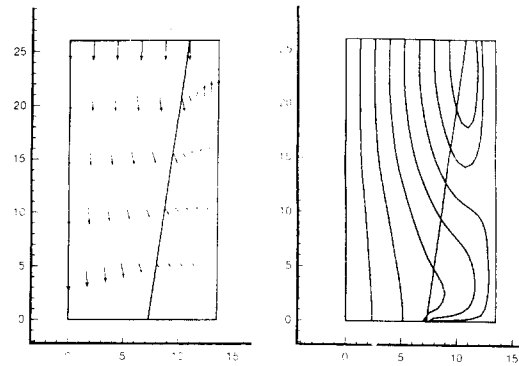
Fig(2). Combined extrusion



Fig(5). Stroke-Load Curve

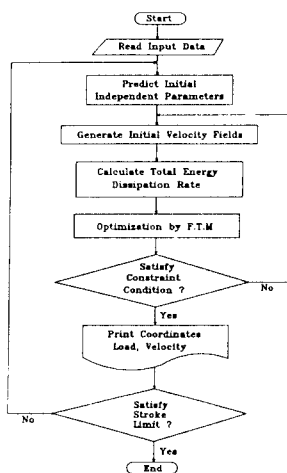


Fig(3). Layout of elements for combined extrusion

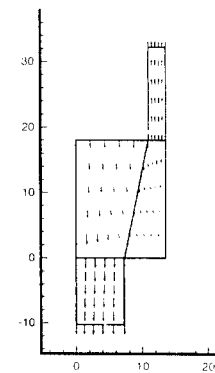


Fig(6-a). Velocity distribution of the initial deformation

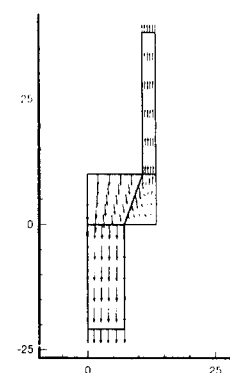
Fig(6-b). Streamline of the initial deformation flow



Fig(4). Flow chart for this simulation



Fig(7). Velocity and extruded profile at stroke 8mm



Fig(8). Velocity and extruded profile at stroke 16mm