A STOCHASTIC EVALUATION METHOD OF ACOUSTIC SYSTEMS BASED ON EQUIVALENT ZERO-MEMORY TYPE NON-LINEAR SYSTEM

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ABSTRACT In this paper, a new method of statistically evaluating an output response probability distribution of a memory type non-linear system is practically derived based on a zero-memory type non-linear equivalent system. That is, first, the objective system is approximately and functionally separated into two functional parts, i.e., a zero-memory type non-linear part and a memory type linear part according to the well-known Wiener's idea. A whole mathematical frame of the output probability distribution is evaluated in an approximate but generalized form, based on the equivalent zero-memory type non-linear part. The memory effects between the input and the output of the system are reflected in the statistical parameters and the expansion coefficients.

1. INTRODUCTION

An environmental noise or vibration system must usually be considered as a memory type non-linear system, if taking a total system including sound (vibration) source, propagation mechanism and observation mechanism into the consideration[1]. However, the objective system is approximately and functionally separated into two functional parts, i.e., a zero-memory type non-linear part and a memory type linear part according to the well-known Wiener's idea[2] on a generalized analysis of a non-linear system. Moreover, the expression frame of output distribution form of a memory type non-linear system is dominantly affected by the zero-memory type non-linear part.

In this paper, first, based on these fundamental properties, a general expression of the output distribution of the equivalent system with memory is practically derived in a series expansion form connected with only the zero-memory type non-linear part of the system. Next, the memory effects between the input and the output of the system connected with the averaging operation can reasonably be reflected in the statistical parameters and the expansion coefficients[3], after a whole mathematical frame of response probability expression form is first established by the previous procedures. As a result, the output distribution of the environmental noise or vibration system with memory can be practically evaluated in a simplified form of reflecting both memory and zero-memory type non-linear effects.

Finally, the proposed method is confirmed experimentally too by applying it to the actual acoustic system.

2. ANALYSIS OF A ZERO-MEMORY TYPE NON-LINEAR SYSTEM

In general, an environmental noise or vibration system is considered as a memory type non-linear system. At that time, the relationship between an arbitrary random input ξ and its output z of the system can be expressed by following differential equation:

$$\xi = \Phi(z^{(n)}, z^{(n-1)}, \cdots, z^{(1)}, z; \beta_0), \tag{1}$$

where β_o $(=(\beta_{01}, \beta_{02}, \cdots, \beta_{0K}))$ is a system parameter with constant value. Usually, it is almost impossible to solve exactly the differential equation and evaluate the output probability density function of the system.

Now, we pay attention to the next two properties in order to evaluate the output response probability distribution of the actual non-linear system.

1. The objective system is approximately and equivalently separated into two functional parts, i.e., a zero-memory type non-linear part and a memory type linear part according to the well-known Wiener's idea.
2. The expression frame of output probability distribution function form of the memory type non-linear system is dominantly affected by the zero-memory type non-linear part.

Then, the relationship between the input and the output of the zero-memory non-linear part can be expressed as follows:

$$x = \Phi(0, 0, \cdots, 0, z; \beta_o) = h(z; \beta_o) \;\text{ or }\; z = F(x; \alpha_0). \tag{2}$$

2.1 General Expression of Output Response of a Zero-Memory Type Non-Linear System

In this section, we pay attension to an generalized zero-memory type non-linear system, $z = F(x, \alpha)$, having the system parameter $\alpha(=(\alpha_1, \alpha_2, \cdots, \alpha_K))$ which changes according to the time. And, the output distribution of the system is derived when an arbitrary stationary random input x is applied to the system.

Now, in order to focus our attention to non-stationary effects latent in the objective system, we introduce a quantity $g(x; \alpha)$ which means some deviation of the non-stationary effects from a well-known stationary non-linear function z_0 pre-established as the basis of the present analysis. Then, the output of the system can be expressed as,

$$z = z_0 + g(x; \alpha) \quad (z_0 \equiv a(\alpha_0) f(x)), \tag{3}$$

where $f(x)$ denotes r-valued non-linear function ($r < \infty$) of x and differentiable with respect to x. Here, we introduce an arbitrary function $\phi(z)$ where a value of the function and its differential value are zero at a boundary value of z, and its expectation value is defined as:

$$\langle \phi(z) \rangle_z \equiv \int_{-\infty}^{\infty} \phi(z) p(z) dz, \tag{4}$$

where $p(z)$ is a probability density function (abbr. p.d.f.) of the output fluctuation z. Since the output of the system fluctuates owing to the temporal variation of the input x and the system parameter α, the averaging operation in equation (4) with respect to z can be replaced by x and α as follows:

$$\langle \phi(z) \rangle_z = \left\langle \int_{-\infty}^{\infty} \phi(z) p_x(x|\alpha) dx \right\rangle_{\alpha}, \tag{5}$$

where $\langle * \rangle_\alpha$ denotes an averaging operation with respect to the system parameter α and $p_x(x)$ denotes a p.d.f. of the input. Now, substituting equation (3) into equation (5), and expanding $\phi(z_0 + g(x;\alpha))$ in the Taylor series form, we have:

$$\langle \phi(z) \rangle_z = \left\langle \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{g(x;\alpha)^n}{n!} \left\{ \left(\frac{d}{dz_0} \right)^n \phi(z_0) \right\} p_x(x|\alpha) dx \right\rangle_\alpha . \tag{6}$$

Since z_0 is an r-valued non-linear function with respect to x, the variable range of x can be divided by r-sections with each one-valued function (integration domain: from b_{i-1} to b_i $(i = 1, 2, \cdots, r)$), and using the relation:

$$\frac{d}{dz_0} = \frac{dx_i}{dz_0} \frac{d}{dx_i} = \frac{1}{a(\alpha_0) \cdot f'(x_i)} \frac{d}{dx_i}, \tag{7}$$

equation (6) can be calculated as follows:

$$\langle \phi(z) \rangle_z \equiv \left\langle \sum_{n=0}^{\infty} \frac{1}{n! \cdot a(\alpha_0)^n} \sum_{i=1}^{r} I_i \right\rangle_\alpha \tag{8}$$

with

$$I_i = \int_{b_{i-1}}^{b_i} g(x_i;\alpha)^n p_x(x_i|\alpha) \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \left(\cdots \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \phi(z_0(x_i)) \right) \right) \cdots \right) dx_i . \tag{9}$$

In the above equation, the integrations by parts can be repeated under the condition with respect to the function $\phi(z_0)$: then

$$I_i = (-1)^n \int_{b_{i-1}}^{b_i} \phi(z_0(x_i)) \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \left(\cdots \frac{d}{dx_i} \left[\frac{1}{f'(x_i)} g(x_i;\alpha)^n p_x(x_i;\alpha) \right] \right) \cdots \right) dx_i . \tag{10}$$

Here, we introduce a new function defined by the following notation in preparation to change the domain of integration in equation (10) as:

$$D(z_i;\lambda_{i-1},\lambda_i) \equiv \begin{cases} 1 & \min(\lambda_{i-1},\lambda_i) \le z_i < \max(\lambda_{i-1},\lambda_i) \\ 0 & z_i < \min(\lambda_{i-1},\lambda_i), z_i \ge \max(\lambda_{i-1},\lambda_i) \end{cases} . \tag{11}$$

Furthermore, substituting equations (10) and (11) into equation (8), finally, $\langle \phi(z) \rangle_\alpha$ can be expressed as follows:

$$\langle \phi(z) \rangle_z = \left\langle \sum_{n=0}^{\infty} \frac{(-1)^n}{n! a(\alpha_0)^n} \int_{-\infty}^{\infty} \phi(z) \left[\sum_{i=1}^{r} \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \cdots \right.\right.\right.\right.$$

$$\left.\left.\left.\left. \cdot \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} g(x_i;\alpha)^n p_x(x_i|\alpha) \right) \right) \cdots \right) \left| \frac{1}{a(\alpha_0) f'(x_i)} \right| D(z;\lambda_{i-1},\lambda_i) \Big|_{x_i \to f^{-1}(z/a(\alpha_0))} \right] dz \right\rangle_\alpha . \tag{12}$$

The averaging operation $\langle * \rangle_\alpha$ and the integration operation $\int * dz$ can be exchanged, and by equalizing equation (12) to equation (4), then the p.d.f. $p(z)$ for the output z can be expressed in a series form as:

$$p(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! a(\alpha_0)^n |a(\alpha_0)|} \sum_{i=1}^{r} \frac{1}{|f'(x_i)|} \left\langle \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \frac{d}{dx_i} \left(\frac{1}{f'(x_i)} \cdots \right.\right.\right.$$

$$\cdot\frac{d}{dx_i}\left(\frac{1}{f'(x_i)}g(x_i;\alpha)^n p_x(x_i|\alpha)\right)\Big)\cdots\Big)\Big\rangle_{\alpha}\Big|_{x_i\to f^{-1}(z/a(\alpha_0))} D(z;\lambda_{i-1},\lambda_i). \qquad (13)$$

2.2 Relationship between the Zero-Memory Type Equivalent System and an Actual Memory Type System

In equation (13), the input probability density function $p_x(x)$ can usually be expressed by using the following expression in an expansion form having an arbitrary probability distribution as the first expansion term:

$$p_x(x) = p_0(x)\left\{1 + \sum_{n=1}^{\infty} c_n\psi_n(x)\right\}, \qquad (14)$$

where c_n denotes an expansion coefficient and $\psi_n(x)$ denotes set of orthonormal functions. In accordance with the previous fundamental properties, first, equation (13), based on the equivalent zero-memory type non-linear system, can be adopted to the actual non-linear memory type system in order to evaluate the output probability distribution especially from the practical point of view. And, the memory effects between the input and the output of the system connected with an averaging operation can be approximately reflected in the averaging operation of statistical parameters and the expansion coefficients. By using the orthogonality condition of $\psi(x)$, the expansion coefficients in equation (14) can be calculated through the following definition:

$$c_n = \int \psi_n(x)p_x(x)dx = \langle\psi_n(x)\rangle_x. \qquad (15)$$

Furthermore, by considering the non-linear relationship expressed by equation (2), the expansion coefficient can be calculated by using the output of the system instead of the input as follows:

$$c_n = \langle\psi_n(h(z))\rangle_z. \qquad (16)$$

In the above equation, though z is an output of the equivalent zero-memory type non-linear system, the expansion coefficient c_n can be approximately calculated through the statistical quantities of the output in case of the objective memory type non-linear system. Thus, the memory effects between the input and the output of the system can be reflected in the expansion coefficients.

2.3 Application to an Acoustic System

In this section, we apply the proposed theory to the actual acoustic system. An echoic chamber is adopted as one example of the acoustic systems and its output response probability distribution is evaluated by using the proposed theory. As is well-known, the echoic chamber must be obviously considered as a memory type linear system on an energy scale. Furthermore, an environmental acoustic signal is usually measured on a dB scale. So, the whole acoustic system can be considered as a memory type non-linear system model as shown in Fig.1. Then, the relationship between the input and the output of the system can be expressed in the discrete time form as:

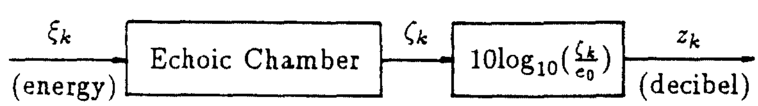

Fig.1 An acoustic system for the actual sound measurement in an echoic chamber.

$$\zeta_k = F_{k-1}\zeta_{k-1} + G_{k-1}\xi_{k-1}, \tag{17}$$

$$z_k = 10\log_{10}(\zeta_k/e_0), \quad e_0 \equiv 10^{-12}[\mathrm{W/m^2}], \tag{18}$$

where ζ denotes the output of the echoic chamber on an energy scale, F and G denote two system parameters and k denotes the sample time.

Originally, the equivalent zero-memory part means that the output z_k of the system responds to the input ξ_k momentarily. Therefore, by replacing $k-1$ with k and ξ_k with x_k, the following equation can be obtained

$$\zeta_k = \frac{G_k}{1-F_k}x_k. \tag{19}$$

Substituting equation (19) into equation (18), then, the relationship between the input and the output of the zero-memory non-linear part for the objective system is expressed by

$$x_k = x_0' e^{cz_k} = h(z_k); \quad x_0' \equiv \frac{1-F}{G}e_0 \text{ and } c \equiv \ln 10/10. \tag{20}$$

Furthermore, an input x on an energy scale is considered as a positive quantity and its probability density function $p_x(x)$ can be generally expressed in the form of the statistical Laguerre expansion series[4] as:

$$p_x(x) = \frac{1}{\Gamma(m)s^m}x^{m-1}e^{-x/s}\left\{1 + \sum_{n=1}^{\infty} B_n L_n^{(m-1)}\left(\frac{x}{s}\right)\right\}, \tag{21}$$

where m and s denote two arbitrary distribution parameters directly connected with a mean value and a standard deviation of the input, B_n denotes an expansion coefficient and $L_n^{(m-1)}(*)$ denotes an associated Laguerre polynomial respectively.

As a result, after substituting equation (20) and (21) into equation (13), the output probability distribution of the acoustic system is obtained as follows:

$$p(z) = \frac{c}{\Gamma(m)s^m}x_0'^m \exp(czm - x_0' e^{cz}/s)\left\{1 + \sum_{n=1}^{\infty} B_n L_n^{(m-1)}\left(\frac{x_0' e^{cz}}{s}\right)\right\}. \tag{22}$$

3. EXPERIMENTAL CONSIDERATIONS

Our experiment is carried out according to the block diagram of the experimental system as shown in Fig.2. A traffic noise and a music sound data are previously measured and recorded in Hiroshima city, Japan. The input sound is recorded in the data recorder 1 and radiated through the loudspeaker, and the output sound is recorded in the data recorder 2. The experiments are carried out two times respectively, i.e., in the free sound field of outdoors necessary for getting an input of the objective system measured on an energy scale, and in the reverberation room[5] necessary for getting an output of the objective system measured on a dB scale. Then, the output probability distribution of this system (reverberation room) is estimated by the proposed method (equation (22)).

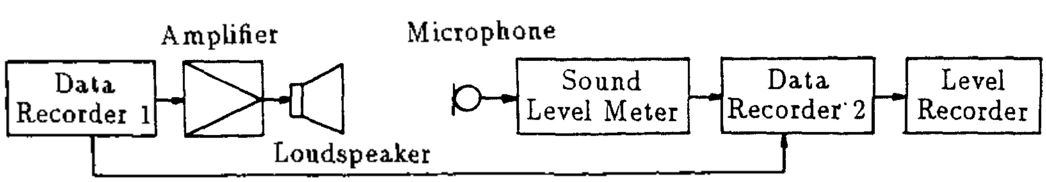

Fig.2 A block diagram of the experimental measurement system.

Some of the resuls are shown in Figures 3 and 4. The figures show that a compraison of the cumulative distribution function of the output level fluctuation between the experimentally obtained values and the theoretically estimated curves by the proposed method. In these experiments, the theoretical curves are estimated by using only the output data evaluated by a computer calculation based on the system equation. In these figures, the theoretically obtained curves approach to the experimentally sampled values well as the expansion terms increase. The estimation results in the low level range are not so good as compared with in the high level range. However, in the high level range, usually used in the environmental noise field as the evaluation quantities, L_5, L_{10}, L_{50}, are successfully estimated and are close enough for practical use.

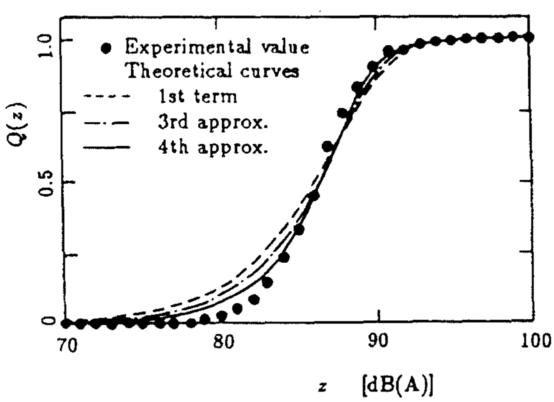

Fig.3 An estimation for the cumulative distribution function of the output noise level caused by the road traffic noise.

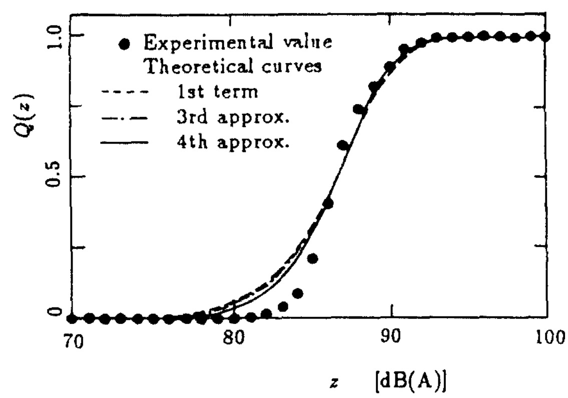

Fig.4 An estimation for the cumulative distribution function of the output noise level caused by the music.

4. CONCLUSIONS

In this paper, first, a general expression for the output response probability distribution of the equivalent zero-memory non-linear system has been newly derived in a series expansion form especially by employing some generalized form of a moment generating function connected with the probability density function. Next, it has been shown that the memory effects between the input and the output of the system connected with an averaging operation can be reflected in these parameters and expansion coefficients. As a reslt, a new method of statistically evaluating an output response probability distribution of a memory type non-linear system has been equivalently derived especially on the basis of the proposed analysis on a zero-memory type non-linear equivalent system. In the experiments, an echoic chamber has been adopted as the acoustic system and its output probability distribution has been well estimated by using the proposed theory.

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